Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.5-Secant/121-4.5.2.1-a+b-sec-^m-c+d-sec-^n

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [241]. This is test number [121].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.59 (240)	0.41 (1)
Mathematica	97.10 (234)	2.90 (7)
Maple	90.46 (218)	9.54 (23)
Fricas	61.00 (147)	39.00 (94)
Maxima	40.66 (98)	59.34 (143)
Giac	31.54 (76)	68.46 (165)
Mupad	23.24 (56)	76.76 (185)
Sympy	2.49 (6)	97.51 (235)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

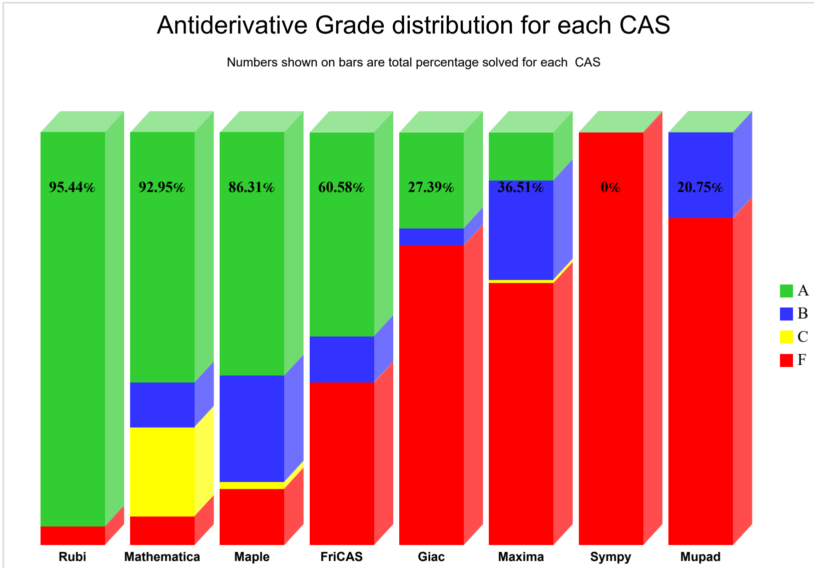
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

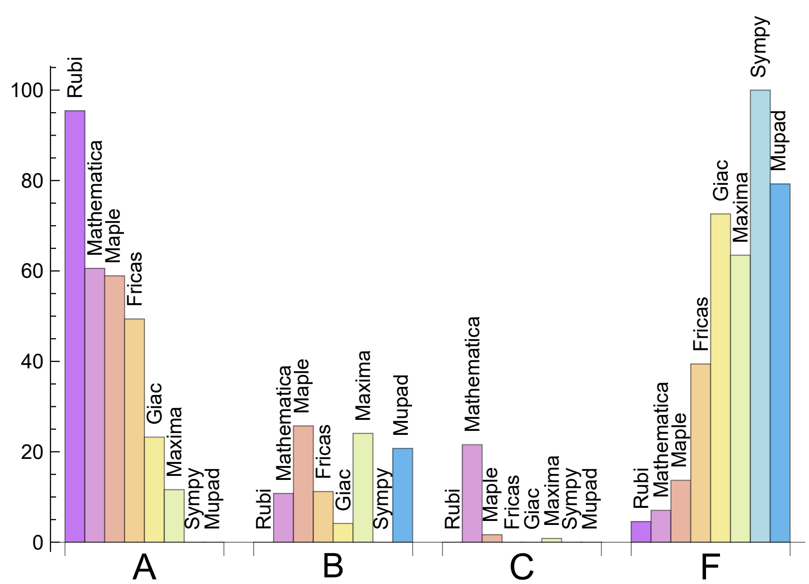
System	% A grade	% B grade	% C grade	% F grade
Rubi	95.436	0.000	0.000	4.564
Mathematica	60.581	10.788	21.577	7.054
Maple	58.921	25.726	1.660	13.693
Fricas	49.378	11.203	0.000	39.419
Giac	23.237	4.149	0.000	72.614
Maxima	11.618	24.066	0.830	63.485
Mupad	0.000	20.747	0.000	79.253
Sympy	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	7	100.00	0.00	0.00
Maple	23	100.00	0.00	0.00
Fricas	94	68.09	31.91	0.00
Maxima	143	72.03	15.38	12.59
Giac	165	73.33	0.00	26.67
Mupad	185	0.00	100.00	0.00
Sympy	235	85.11	14.89	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.69
Giac	0.92
Maxima	1.37
Fricas	3.57
Mathematica	7.19
Maple	7.91
Mupad	24.53
Sympy	38.33

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	26.83	0.94	27.00	0.93
Rubi	192.12	0.93	142.00	0.99
Giac	200.91	1.22	103.50	0.93
Fricas	568.29	3.15	405.00	2.86
Maxima	770.86	4.75	255.50	2.45
Mathematica	813.40	2.38	122.00	0.90
Mupad	1614.84	6.20	122.00	1.09
Maple	5191.79	10.18	211.00	1.37

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

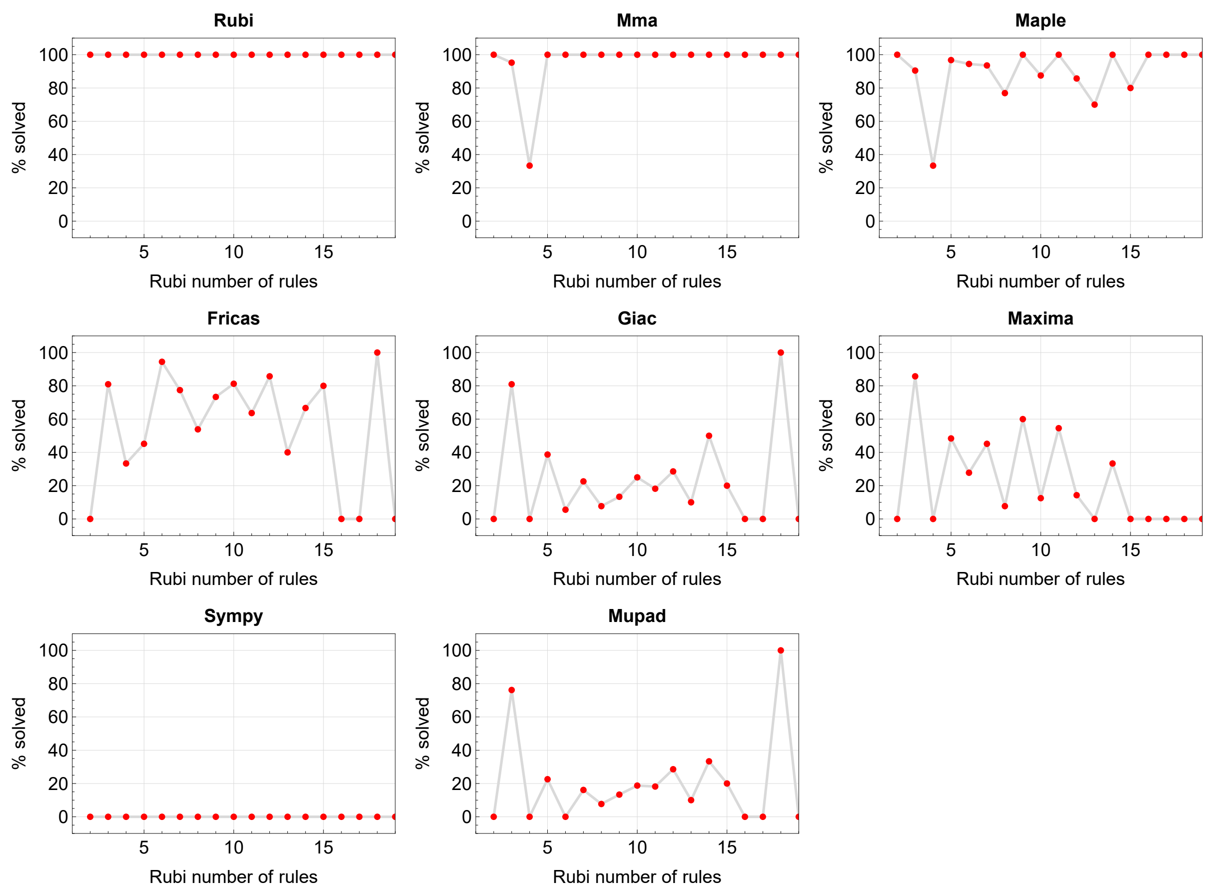


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

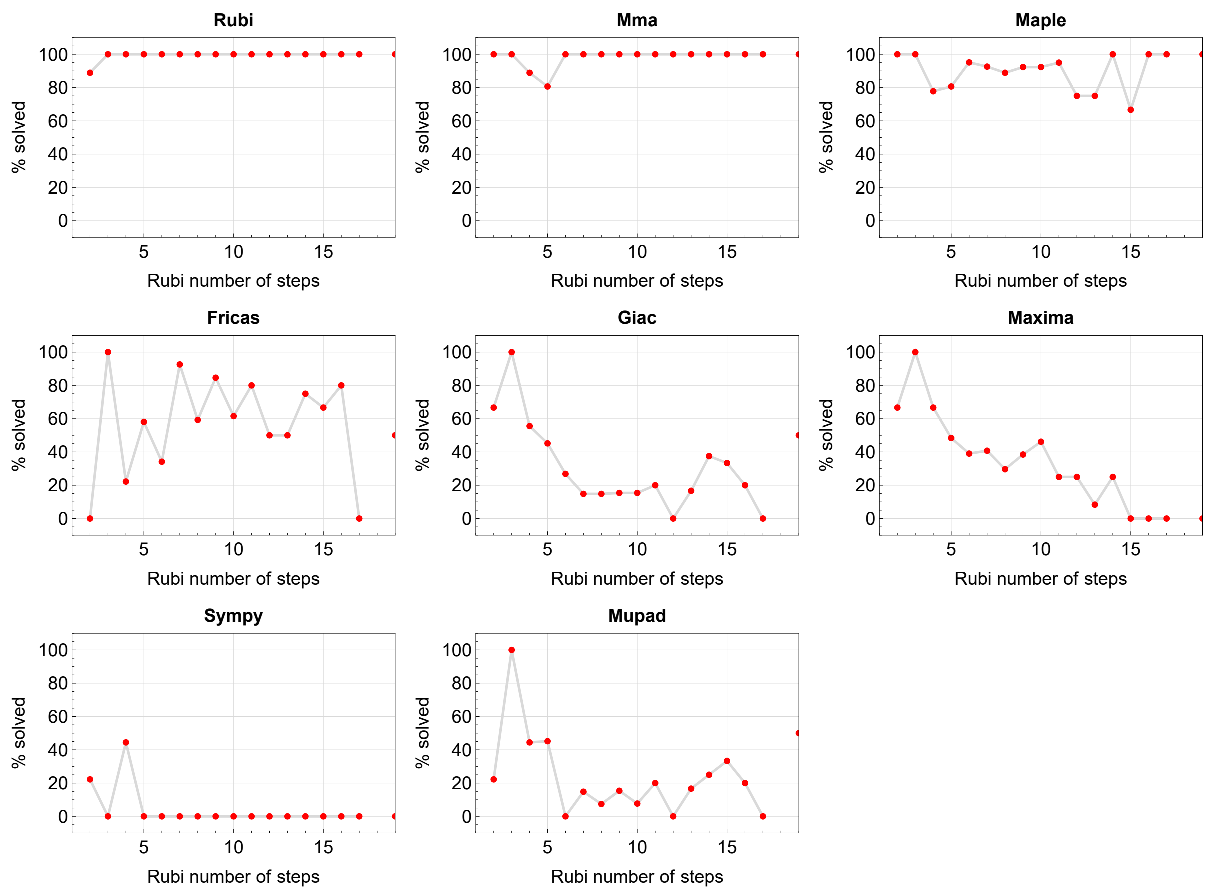


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

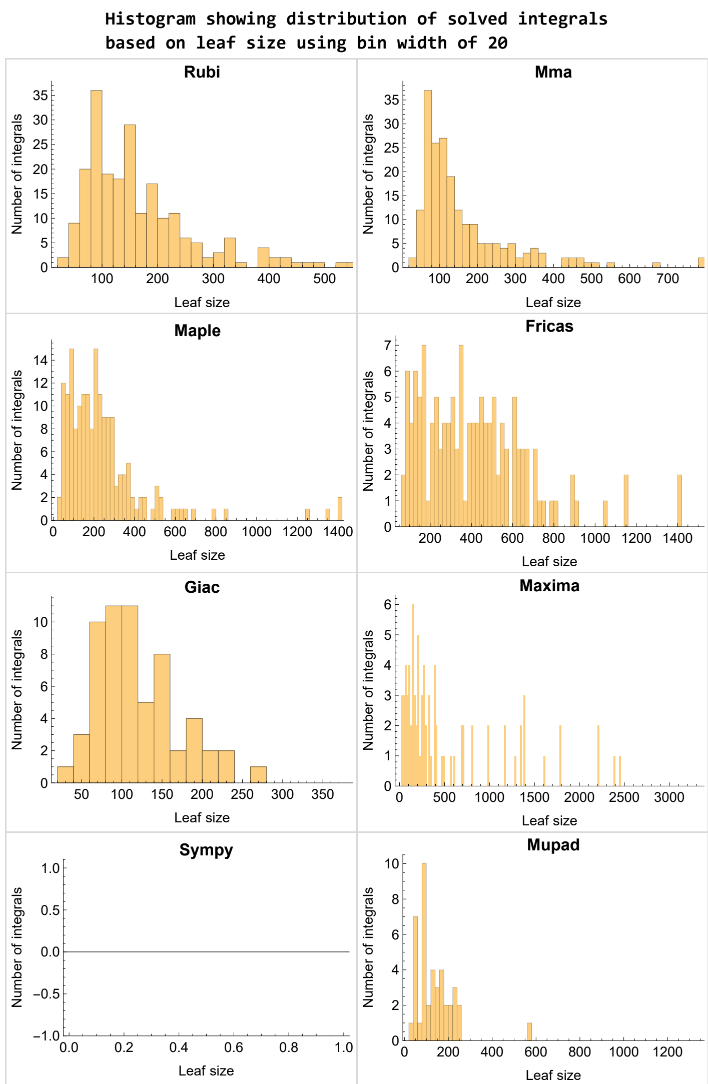


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

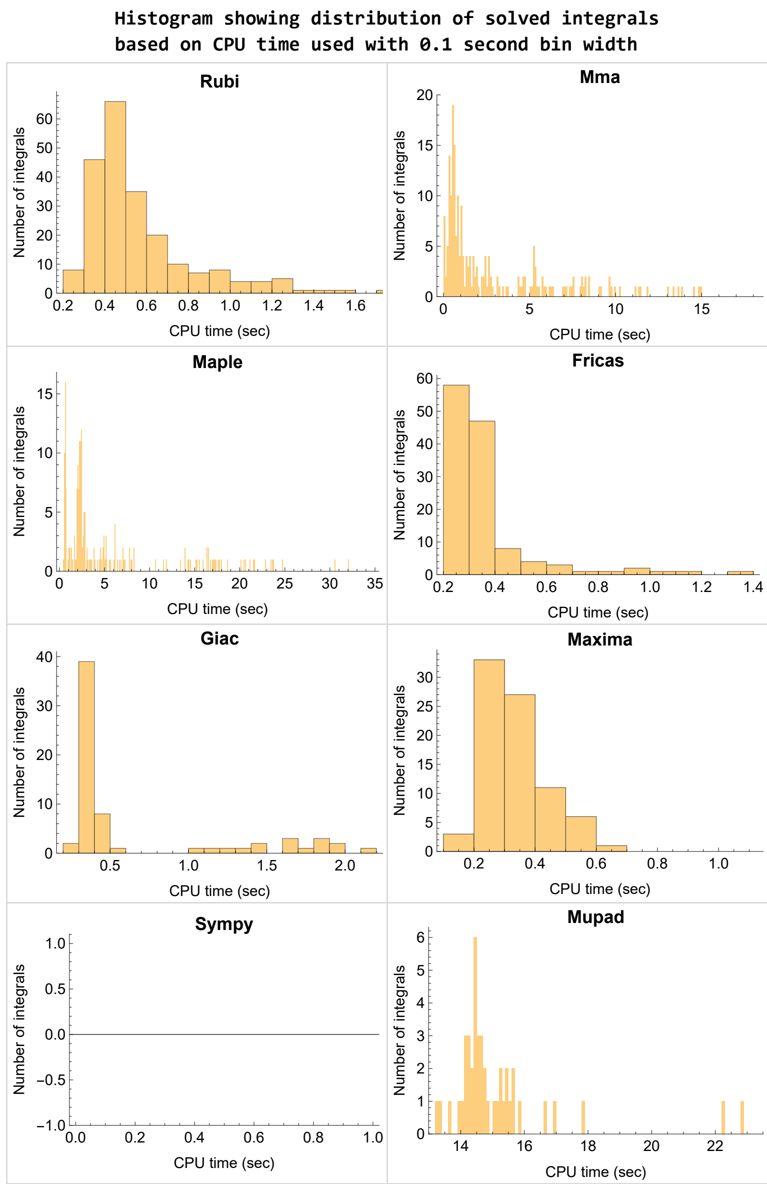


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

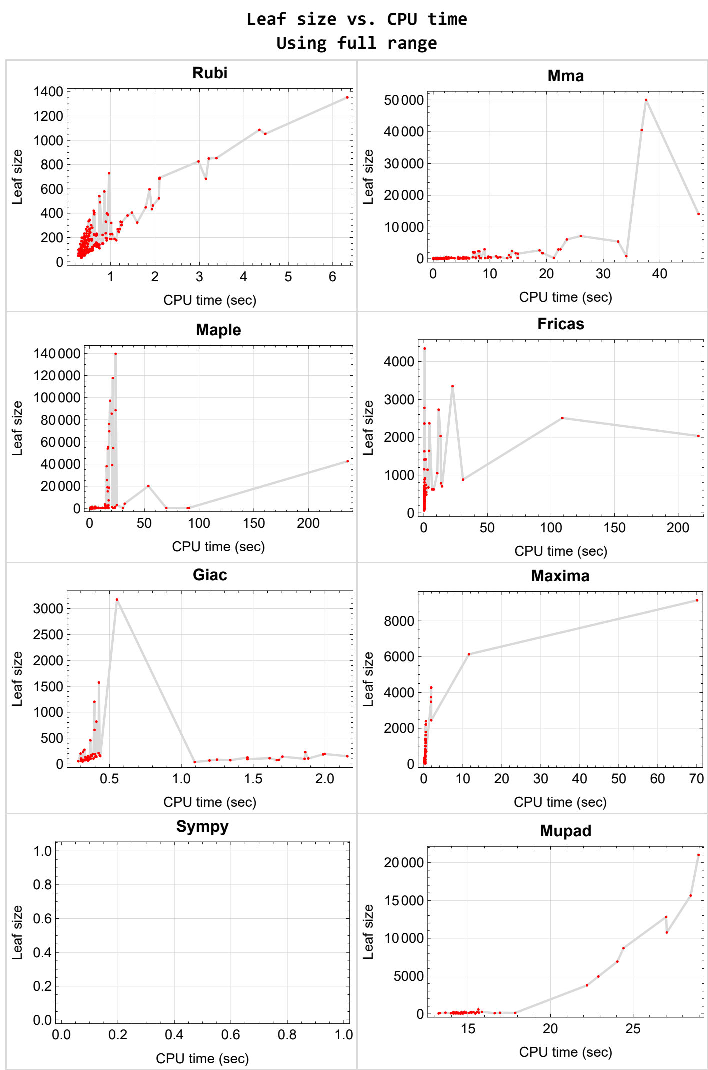


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{221, 222, 223, 224, 225, 226, 227, 228, 229, 236}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {147, 149, 152, 153, 158, 159, 163, 165, 166, 167, 170, 171, 172, 175, 176, 177, 181, 182, 183, 200, 202, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 230, 240, 241}

Maple {42, 43, 50, 59, 65, 66, 72, 73, 79, 80, 81, 82, 83, 84, 100, 107, 108, 109, 123, 124, 129, 151, 152, 153, 157, 158, 159, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 187, 188, 205, 209, 210, 211, 212, 213, 214, 215, 216, 220}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

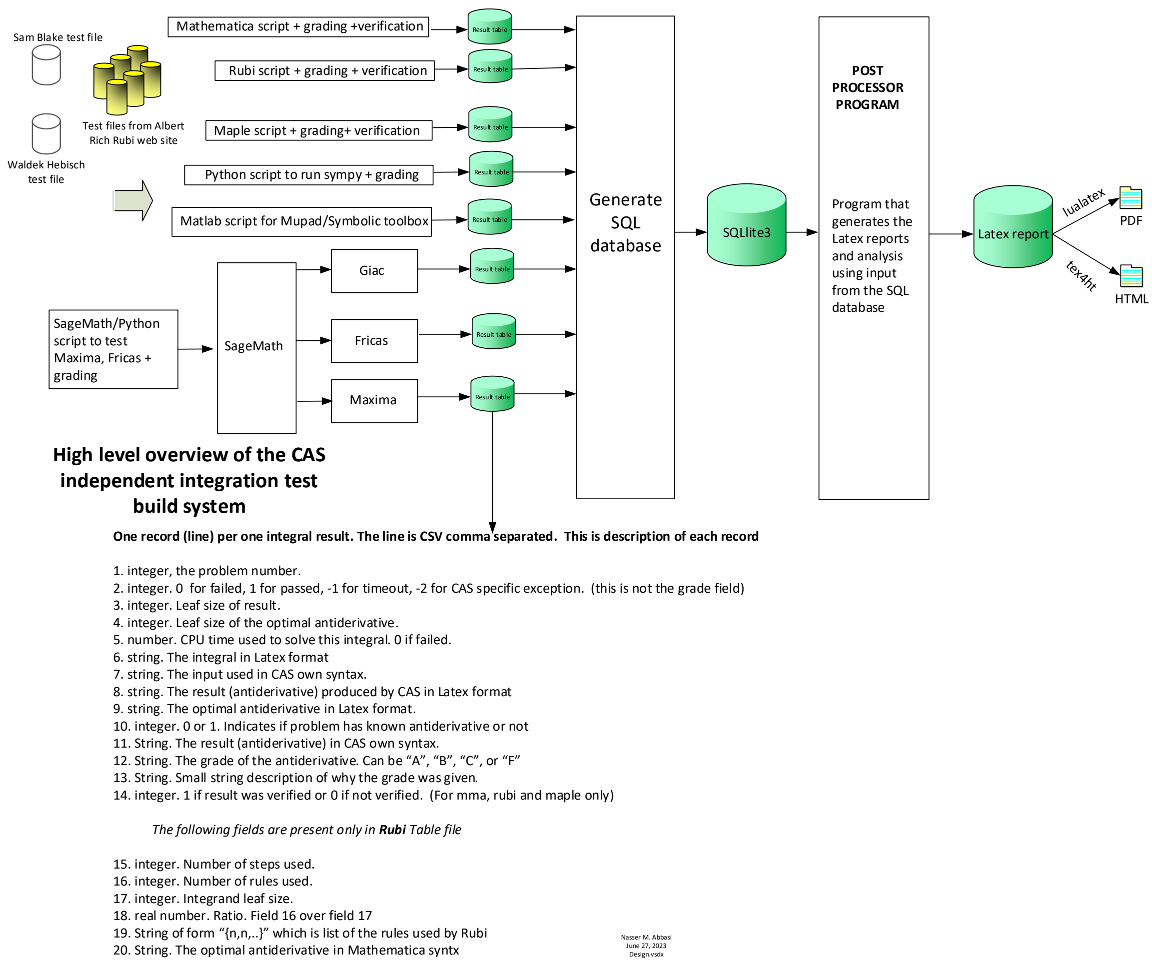
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

B grade { }

C grade { }

F normal fail { 217 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 8, 11, 12, 13, 14, 15, 19, 24, 33, 34, 42, 43, 44, 45, 50, 51, 52, 53, 57, 58, 59, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 138, 139, 140, 141, 142, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 168, 169, 170, 173, 174, 175, 176, 178, 179, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 196, 197, 198, 199, 201, 203, 204, 208, 218, 231, 232, 233, 234, 235, 237, 238, 239 }

B grade { 6, 17, 25, 144, 171, 177, 180, 183, 193, 195, 200, 202, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 220, 230, 240, 241 }

C grade { 7, 9, 10, 16, 18, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 46, 47, 48, 49, 54, 55, 56, 60, 61, 62, 63, 64, 69, 70, 71, 76, 77, 78, 84, 85, 143, 149, 163, 166, 167, 172, 207, 217, 219 }

F normal fail { 131, 132, 133, 134, 135, 136, 137 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 53, 54, 57, 58, 59, 60, 65, 66, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 143, 144, 145, 146, 147, 148, 149, 150, 154, 155, 160, 161, 162, 173, 178, 179, 189, 190, 191, 192, 193, 194, 195, 196, 199, 201, 203, 204, 207, 208, 217, 218, 219 }

B grade { 48, 49, 52, 55, 56, 61, 62, 63, 64, 67, 71, 75, 76, 84, 105, 106, 118, 151, 152, 153, 156, 157, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 186, 187, 188, 197, 198, 200, 202, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 220 }

C grade { 1, 11, 12, 13 }

F normal fail { 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 94, 95, 96, 101, 102, 103, 130, 143, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 175, 178, 179, 184, 185, 187, 188, 189 }

B grade { 5, 22, 23, 61, 62, 63, 74, 75, 83, 89, 105, 114, 118, 122, 153, 159, 174, 180, 186, 190, 191, 192, 193, 194, 195, 196, 197 }

C grade { }

F normal fail { 90, 91, 92, 93, 97, 98, 99, 100, 104, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 198, 200, 202, 205, 206, 208, 210, 211, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

F(-1) timedout fail { 146, 171, 176, 177, 181, 182, 183, 199, 201, 203, 204, 207, 209, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 4, 5, 13, 15, 26, 27, 28, 29, 30, 35, 36, 37, 38, 39, 40, 41, 89, 90, 97, 98, 106, 112, 113, 114, 119, 125 }

B grade { 3, 6, 7, 8, 9, 10, 11, 12, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 31, 32, 33, 34, 45, 52, 59, 86, 87, 88, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 107, 108, 109, 115, 116, 117, 120, 121, 122, 123, 126, 127, 128, 129, 130, 150, 156, 162 }

C grade { 68, 168 }

F normal fail { 42, 43, 44, 46, 47, 48, 50, 51, 53, 58, 60, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 154, 155, 157, 161, 163, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 184, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

F(-1) timedout fail { 49, 54, 55, 56, 57, 61, 62, 63, 64, 79, 80, 81, 153, 158, 159, 160, 164, 165, 177, 178, 182, 183 }

F(-2) exception fail { 104, 105, 110, 111, 118, 124, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 190, 192 }

B grade { 5, 27, 38, 189, 191, 193, 194, 195, 196, 197 }

C grade { }

F normal fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 184, 185, 186, 187, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

F(-1) timedout fail { }

F(-2) exception fail { 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 110, 111, 112, 117, 118, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 189, 190, 191, 192, 193, 194, 195, 196, 197 }

C grade { }

F normal fail { }

F(-1) timedout fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 226, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 112, 113, 114, 115, 116, 119, 120, 121, 122, 126, 127, 128, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 217, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

F(-1) timedout fail { 63, 64, 86, 87, 93, 94, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 117, 118, 123, 124, 125, 129, 130, 138, 139, 213, 214, 215, 216, 223, 226, 228, 229 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	176	165	206	334	179	0	191	228
N.S.	1	0.90	0.84	1.05	1.70	0.91	0.00	0.97	1.16
time (sec)	N/A	0.557	1.743	4.912	0.232	0.276	0.000	0.400	15.544

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	126	146	172	240	163	0	172	195
N.S.	1	0.90	1.04	1.23	1.71	1.16	0.00	1.23	1.39
time (sec)	N/A	0.463	1.160	3.126	0.214	0.269	0.000	0.373	15.150

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	89	122	133	203	147	0	153	163
N.S.	1	0.92	1.26	1.37	2.09	1.52	0.00	1.58	1.68
time (sec)	N/A	0.447	0.730	3.321	0.207	0.280	0.000	0.369	15.224

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	35	45	56	57	65	0	48	84
N.S.	1	0.74	0.96	1.19	1.21	1.38	0.00	1.02	1.79
time (sec)	N/A	0.318	0.025	1.699	0.205	0.248	0.000	0.313	16.617

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	52	97	84	95	103	0	103	91
N.S.	1	0.95	1.76	1.53	1.73	1.87	0.00	1.87	1.65
time (sec)	N/A	0.312	0.263	1.651	0.192	0.268	0.000	0.302	14.431

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	51	201	64	153	87	0	77	46
N.S.	1	0.91	3.59	1.14	2.73	1.55	0.00	1.38	0.82
time (sec)	N/A	0.352	1.609	0.490	0.279	0.260	0.000	0.313	14.136

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	66	53	41	174	88	0	57	40
N.S.	1	0.93	0.75	0.58	2.45	1.24	0.00	0.80	0.56
time (sec)	N/A	0.404	0.065	0.617	0.300	0.262	0.000	0.301	14.565

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	94	151	54	215	128	0	72	96
N.S.	1	0.92	1.48	0.53	2.11	1.25	0.00	0.71	0.94
time (sec)	N/A	0.499	1.356	0.598	0.296	0.253	0.000	0.324	14.427

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	122	119	67	294	172	0	88	124
N.S.	1	0.92	0.89	0.50	2.21	1.29	0.00	0.66	0.93
time (sec)	N/A	0.576	0.829	0.547	0.299	0.253	0.000	0.348	14.299

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	150	111	80	335	212	0	104	146
N.S.	1	0.91	0.68	0.49	2.04	1.29	0.00	0.63	0.89
time (sec)	N/A	0.684	0.867	0.686	0.296	0.255	0.000	0.370	14.414

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	169	189	217	356	195	0	210	259
N.S.	1	0.90	1.01	1.15	1.89	1.04	0.00	1.12	1.38
time (sec)	N/A	0.503	1.983	4.594	0.201	0.284	0.000	0.424	15.847

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	123	165	206	334	179	0	191	227
N.S.	1	0.93	1.25	1.56	2.53	1.36	0.00	1.45	1.72
time (sec)	N/A	0.587	1.162	3.828	0.201	0.282	0.000	0.384	15.336

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	52	61	81	94	81	0	65	122
N.S.	1	0.76	0.90	1.19	1.38	1.19	0.00	0.96	1.79
time (sec)	N/A	0.397	0.029	2.305	0.197	0.269	0.000	0.351	17.864

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	89	122	133	203	147	0	153	163
N.S.	1	0.92	1.26	1.37	2.09	1.52	0.00	1.58	1.68
time (sec)	N/A	0.443	0.464	2.742	0.197	0.266	0.000	0.350	15.664

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	76	101	77	107	118	0	104	104
N.S.	1	0.99	1.31	1.00	1.39	1.53	0.00	1.35	1.35
time (sec)	N/A	0.387	0.480	1.799	0.205	0.275	0.000	0.335	15.482

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	71	275	94	274	125	0	111	85
N.S.	1	0.91	3.53	1.21	3.51	1.60	0.00	1.42	1.09
time (sec)	N/A	0.441	4.392	0.650	0.285	0.253	0.000	0.343	14.380

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	80	226	66	274	156	0	80	45
N.S.	1	0.91	2.57	0.75	3.11	1.77	0.00	0.91	0.51
time (sec)	N/A	0.506	2.680	0.664	0.286	0.277	0.000	0.341	14.696

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	94	53	54	282	128	0	73	96
N.S.	1	0.92	0.52	0.53	2.76	1.25	0.00	0.72	0.94
time (sec)	N/A	0.589	0.062	0.548	0.294	0.269	0.000	0.334	14.503

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	122	161	67	383	172	0	88	122
N.S.	1	0.92	1.21	0.50	2.88	1.29	0.00	0.66	0.92
time (sec)	N/A	0.685	1.458	0.641	0.345	0.262	0.000	0.360	14.798

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	150	139	80	403	212	0	104	146
N.S.	1	0.91	0.85	0.49	2.46	1.29	0.00	0.63	0.89
time (sec)	N/A	0.818	1.452	0.718	0.358	0.261	0.000	0.389	16.938

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	351	137	603	242	0	153	145
N.S.	1	1.00	2.58	1.01	4.43	1.78	0.00	1.12	1.07
time (sec)	N/A	0.630	3.182	0.764	0.354	0.260	0.000	0.376	13.640

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	91	298	105	413	220	0	134	112
N.S.	1	0.89	2.92	1.03	4.05	2.16	0.00	1.31	1.10
time (sec)	N/A	0.532	2.443	0.682	0.321	0.296	0.000	0.333	13.307

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	77	189	58	268	173	0	80	46
N.S.	1	0.91	2.22	0.68	3.15	2.04	0.00	0.94	0.54
time (sec)	N/A	0.483	1.366	0.664	0.295	0.280	0.000	0.333	13.225

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	62	67	41	170	94	0	60	38
N.S.	1	0.93	1.00	0.61	2.54	1.40	0.00	0.90	0.57
time (sec)	N/A	0.396	0.045	0.543	0.289	0.247	0.000	0.308	14.535

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	56	139	37	119	86	0	53	41
N.S.	1	0.92	2.28	0.61	1.95	1.41	0.00	0.87	0.67
time (sec)	N/A	0.326	0.671	0.556	0.291	0.270	0.000	0.283	15.447

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	66	58	50	102	70	0	81	69
N.S.	1	0.96	0.84	0.72	1.48	1.01	0.00	1.17	1.00
time (sec)	N/A	0.425	0.759	0.554	0.293	0.246	0.000	0.304	13.963

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	34	39	38	46	81	0	95	58
N.S.	1	0.74	0.85	0.83	1.00	1.76	0.00	2.07	1.26
time (sec)	N/A	0.338	0.040	0.564	0.278	0.247	0.000	0.304	14.405

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	100	70	78	147	154	0	110	161
N.S.	1	1.02	0.71	0.80	1.50	1.57	0.00	1.12	1.64
time (sec)	N/A	0.559	1.032	0.575	0.278	0.250	0.000	0.340	14.175

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	146	81	91	167	166	0	122	185
N.S.	1	0.88	0.49	0.55	1.01	1.00	0.00	0.73	1.11
time (sec)	N/A	0.488	2.252	0.715	0.280	0.264	0.000	0.359	14.105

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	187	91	104	186	232	0	135	209
N.S.	1	0.89	0.43	0.50	0.89	1.10	0.00	0.64	1.00
time (sec)	N/A	0.566	5.911	0.737	0.288	0.261	0.000	0.409	14.298

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	143	372	118	562	289	0	154	134
N.S.	1	0.88	2.30	0.73	3.47	1.78	0.00	0.95	0.83
time (sec)	N/A	0.683	5.288	0.769	0.313	0.260	0.000	0.406	14.367

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	134	202	69	396	242	0	102	50
N.S.	1	0.91	1.36	0.47	2.68	1.64	0.00	0.69	0.34
time (sec)	N/A	0.732	2.657	0.620	0.299	0.265	0.000	0.352	14.267

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	88	90	54	277	138	0	80	93
N.S.	1	0.92	0.94	0.56	2.89	1.44	0.00	0.83	0.97
time (sec)	N/A	0.537	0.053	0.675	0.288	0.256	0.000	0.331	14.418

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	88	151	54	211	138	0	80	93
N.S.	1	0.92	1.57	0.56	2.20	1.44	0.00	0.83	0.97
time (sec)	N/A	0.467	1.272	0.624	0.294	0.261	0.000	0.323	14.468

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	80	79	50	159	124	0	71	85
N.S.	1	0.91	0.90	0.57	1.81	1.41	0.00	0.81	0.97
time (sec)	N/A	0.364	0.502	0.546	0.296	0.263	0.000	0.302	14.049

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	115	71	65	122	109	0	102	82
N.S.	1	0.91	0.56	0.52	0.97	0.87	0.00	0.81	0.65
time (sec)	N/A	0.451	0.623	0.618	0.303	0.243	0.000	0.302	14.700

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	101	70	78	146	154	0	116	161
N.S.	1	1.01	0.70	0.78	1.46	1.54	0.00	1.16	1.61
time (sec)	N/A	0.540	1.060	0.638	0.294	0.246	0.000	0.345	14.899

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	53	39	53	56	118	0	129	94
N.S.	1	0.79	0.58	0.79	0.84	1.76	0.00	1.93	1.40
time (sec)	N/A	0.401	0.067	0.648	0.282	0.254	0.000	0.349	15.053

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	126	81	104	187	232	0	142	209
N.S.	1	0.98	0.63	0.81	1.45	1.80	0.00	1.10	1.62
time (sec)	N/A	0.688	5.235	0.728	0.291	0.252	0.000	0.401	15.208

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	187	91	117	205	271	0	154	233
N.S.	1	0.89	0.43	0.56	0.98	1.29	0.00	0.73	1.11
time (sec)	N/A	0.535	6.333	0.738	0.291	0.259	0.000	0.436	14.610

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	220	787	130	227	310	0	169	257
N.S.	1	0.87	3.12	0.52	0.90	1.23	0.00	0.67	1.02
time (sec)	N/A	0.620	11.116	0.694	0.295	0.278	0.000	0.432	14.788

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	175	157	122	233	0	373	0	0	0
N.S.	1	0.90	0.70	1.33	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.438	5.732	7.131	0.000	0.291	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	140	128	112	211	0	347	0	0	0
N.S.	1	0.91	0.80	1.51	0.00	2.48	0.00	0.00	0.00
time (sec)	N/A	0.422	1.368	6.165	0.000	0.281	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	99	100	160	0	313	0	0	0
N.S.	1	0.94	0.95	1.52	0.00	2.98	0.00	0.00	0.00
time (sec)	N/A	0.409	0.821	4.938	0.000	0.278	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	68	91	93	147	234	0	0	0
N.S.	1	1.03	1.38	1.41	2.23	3.55	0.00	0.00	0.00
time (sec)	N/A	0.351	0.527	3.976	0.367	0.271	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	63	46	87	0	266	0	0	0
N.S.	1	0.91	0.67	1.26	0.00	3.86	0.00	0.00	0.00
time (sec)	N/A	0.364	0.365	2.256	0.000	0.322	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	96	59	180	0	339	0	0	0
N.S.	1	0.92	0.57	1.73	0.00	3.26	0.00	0.00	0.00
time (sec)	N/A	0.378	0.521	2.231	0.000	0.301	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	126	59	264	0	405	0	0	0
N.S.	1	0.91	0.42	1.90	0.00	2.91	0.00	0.00	0.00
time (sec)	N/A	0.396	0.654	2.653	0.000	0.336	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	156	59	348	0	475	0	0	0
N.S.	1	0.90	0.34	2.00	0.00	2.73	0.00	0.00	0.00
time (sec)	N/A	0.404	4.384	2.802	0.000	0.331	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	177	154	124	234	0	385	0	0	0
N.S.	1	0.87	0.70	1.32	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	0.413	1.704	8.283	0.000	0.264	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	125	114	189	0	355	0	0	0
N.S.	1	0.88	0.80	1.33	0.00	2.50	0.00	0.00	0.00
time (sec)	N/A	0.394	0.974	6.153	0.000	0.292	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	94	100	206	998	303	0	0	0
N.S.	1	0.93	0.99	2.04	9.88	3.00	0.00	0.00	0.00
time (sec)	N/A	0.350	0.588	1.109	0.436	0.271	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	65	78	90	0	269	0	0	0
N.S.	1	0.93	1.11	1.29	0.00	3.84	0.00	0.00	0.00
time (sec)	N/A	0.384	0.531	2.094	0.000	0.307	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	93	73	181	0	351	0	0	0
N.S.	1	0.91	0.72	1.77	0.00	3.44	0.00	0.00	0.00
time (sec)	N/A	0.392	0.644	2.067	0.000	0.303	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	126	73	265	0	417	0	0	0
N.S.	1	0.92	0.53	1.93	0.00	3.04	0.00	0.00	0.00
time (sec)	N/A	0.401	0.957	2.741	0.000	0.332	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	156	73	349	0	495	0	0	0
N.S.	1	0.91	0.42	2.03	0.00	2.88	0.00	0.00	0.00
time (sec)	N/A	0.403	4.602	2.461	0.000	0.349	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	181	134	223	0	441	0	0	0
N.S.	1	0.85	0.63	1.05	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.404	2.463	89.612	0.000	0.284	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	152	124	207	0	409	0	0	0
N.S.	1	0.86	0.70	1.17	0.00	2.31	0.00	0.00	0.00
time (sec)	N/A	0.405	1.096	22.837	0.000	0.295	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	132	119	110	190	1396	353	0	0	0
N.S.	1	0.90	0.83	1.44	10.58	2.67	0.00	0.00	0.00
time (sec)	N/A	0.412	0.622	7.990	0.418	0.280	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	90	66	100	0	291	0	0	0
N.S.	1	0.87	0.64	0.97	0.00	2.83	0.00	0.00	0.00
time (sec)	N/A	0.403	0.400	5.275	0.000	0.307	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	70	70	175	0	339	0	0	0
N.S.	1	0.95	0.95	2.36	0.00	4.58	0.00	0.00	0.00
time (sec)	N/A	0.396	0.607	13.920	0.000	0.323	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	92	73	267	0	441	0	0	0
N.S.	1	0.88	0.70	2.57	0.00	4.24	0.00	0.00	0.00
time (sec)	N/A	0.415	1.130	70.083	0.000	0.315	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	126	73	351	0	527	0	0	0
N.S.	1	0.90	0.52	2.51	0.00	3.76	0.00	0.00	0.00
time (sec)	N/A	0.409	4.699	1.636	0.000	0.345	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	153	73	435	0	601	0	0	0
N.S.	1	0.89	0.42	2.53	0.00	3.49	0.00	0.00	0.00
time (sec)	N/A	0.424	5.219	1.349	0.000	0.358	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	185	188	153	282	0	552	0	0	0
N.S.	1	1.02	0.83	1.52	0.00	2.98	0.00	0.00	0.00
time (sec)	N/A	0.508	5.428	6.491	0.000	0.701	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	152	155	166	260	0	518	0	0	0
N.S.	1	1.02	1.09	1.71	0.00	3.41	0.00	0.00	0.00
time (sec)	N/A	0.471	2.416	5.125	0.000	0.599	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	120	100	325	0	438	0	0	0
N.S.	1	1.01	0.84	2.73	0.00	3.68	0.00	0.00	0.00
time (sec)	N/A	0.401	0.685	5.185	0.000	0.382	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	85	102	137	699	298	0	0	0
N.S.	1	0.98	1.17	1.57	8.03	3.43	0.00	0.00	0.00
time (sec)	N/A	0.381	0.365	2.424	0.618	0.339	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	119	75	167	0	436	0	0	0
N.S.	1	0.98	0.62	1.38	0.00	3.60	0.00	0.00	0.00
time (sec)	N/A	0.412	0.433	2.476	0.000	0.354	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	151	83	202	0	520	0	0	0
N.S.	1	0.94	0.52	1.25	0.00	3.23	0.00	0.00	0.00
time (sec)	N/A	0.441	0.399	2.332	0.000	0.348	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	183	83	375	0	608	0	0	0
N.S.	1	0.93	0.42	1.91	0.00	3.10	0.00	0.00	0.00
time (sec)	N/A	0.480	0.533	2.847	0.000	0.365	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	203	213	196	282	0	634	0	0	0
N.S.	1	1.05	0.97	1.39	0.00	3.12	0.00	0.00	0.00
time (sec)	N/A	0.534	4.514	7.019	0.000	1.065	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	169	178	132	222	0	550	0	0	0
N.S.	1	1.05	0.78	1.31	0.00	3.25	0.00	0.00	0.00
time (sec)	N/A	0.446	2.757	5.666	0.000	0.560	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	142	119	200	0	542	0	0	0
N.S.	1	1.19	1.00	1.68	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.407	0.826	3.034	0.000	0.491	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	146	145	201	0	505	0	0	0
N.S.	1	1.29	1.28	1.78	0.00	4.47	0.00	0.00	0.00
time (sec)	N/A	0.384	0.640	2.430	0.000	0.387	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	175	102	339	0	514	0	0	0
N.S.	1	0.99	0.58	1.92	0.00	2.90	0.00	0.00	0.00
time (sec)	N/A	0.465	0.450	2.176	0.000	0.377	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	209	102	202	0	560	0	0	0
N.S.	1	0.98	0.48	0.94	0.00	2.62	0.00	0.00	0.00
time (sec)	N/A	0.481	0.470	2.693	0.000	0.372	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	241	102	232	0	714	0	0	0
N.S.	1	0.97	0.41	0.93	0.00	2.87	0.00	0.00	0.00
time (sec)	N/A	0.542	0.596	2.748	0.000	0.399	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	260	281	180	304	0	742	0	0	0
N.S.	1	1.08	0.69	1.17	0.00	2.85	0.00	0.00	0.00
time (sec)	N/A	0.572	8.452	7.706	0.000	1.695	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	247	164	246	0	655	0	0	0
N.S.	1	1.08	0.72	1.07	0.00	2.86	0.00	0.00	0.00
time (sec)	N/A	0.531	5.350	6.129	0.000	1.387	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	191	212	136	250	0	645	0	0	0
N.S.	1	1.11	0.71	1.31	0.00	3.38	0.00	0.00	0.00
time (sec)	N/A	0.443	2.652	3.888	0.000	1.122	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	189	203	145	245	0	645	0	0	0
N.S.	1	1.07	0.77	1.30	0.00	3.41	0.00	0.00	0.00
time (sec)	N/A	0.442	1.523	3.549	0.000	0.837	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	204	178	243	0	605	0	0	0
N.S.	1	1.38	1.20	1.64	0.00	4.09	0.00	0.00	0.00
time (sec)	N/A	0.408	1.490	2.662	0.000	0.564	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	230	230	117	500	0	608	0	0	0
N.S.	1	1.00	0.51	2.17	0.00	2.64	0.00	0.00	0.00
time (sec)	N/A	0.506	0.523	2.483	0.000	0.376	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	266	112	375	0	706	0	0	0
N.S.	1	0.99	0.42	1.39	0.00	2.62	0.00	0.00	0.00
time (sec)	N/A	0.523	0.530	2.705	0.000	0.393	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	187	84	167	1289	459	0	0	0
N.S.	1	1.01	0.45	0.90	6.97	2.48	0.00	0.00	0.00
time (sec)	N/A	1.012	1.825	2.492	0.463	0.334	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	140	72	157	710	425	0	0	0
N.S.	1	1.01	0.52	1.13	5.11	3.06	0.00	0.00	0.00
time (sec)	N/A	0.732	0.636	2.220	0.394	0.340	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	57	132	243	350	0	0	0
N.S.	1	1.00	0.61	1.42	2.61	3.76	0.00	0.00	0.00
time (sec)	N/A	0.512	0.502	2.338	0.398	0.350	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	87	39	200	0	0	0
N.S.	1	1.00	1.06	1.81	0.81	4.17	0.00	0.00	0.00
time (sec)	N/A	0.327	0.267	2.455	0.397	0.326	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	62	77	65	0	0	0	0
N.S.	1	1.00	1.22	1.51	1.27	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.356	2.112	0.308	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	72	148	399	0	0	0	0
N.S.	1	1.00	0.75	1.54	4.16	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	0.677	2.412	0.370	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	144	90	237	1173	0	0	0	0
N.S.	1	1.01	0.63	1.67	8.26	0.00	0.00	0.00	0.00
time (sec)	N/A	0.662	0.825	2.376	0.478	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	192	100	259	2444	0	0	0	0
N.S.	1	1.02	0.53	1.38	13.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.909	1.936	2.409	1.906	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	188	87	166	1356	467	0	0	0
N.S.	1	0.99	0.46	0.87	7.14	2.46	0.00	0.00	0.00
time (sec)	N/A	0.998	0.829	2.098	0.480	0.332	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	69	66	143	477	346	0	0	0
N.S.	1	0.67	0.64	1.39	4.63	3.36	0.00	0.00	0.00
time (sec)	N/A	0.425	0.578	2.298	0.403	0.321	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	68	118	243	347	0	0	0
N.S.	1	1.00	0.73	1.27	2.61	3.73	0.00	0.00	0.00
time (sec)	N/A	0.529	0.409	2.131	0.399	0.328	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	64	65	112	60	0	0	0	0
N.S.	1	0.62	0.62	1.08	0.58	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.370	1.990	0.385	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	76	145	95	0	0	0	0
N.S.	1	1.00	0.76	1.45	0.95	0.00	0.00	0.00	0.00
time (sec)	N/A	0.462	0.754	2.114	0.318	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	145	83	238	1786	0	0	0	0
N.S.	1	0.99	0.57	1.63	12.23	0.00	0.00	0.00	0.00
time (sec)	N/A	0.691	1.028	2.027	0.527	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	196	195	102	260	3480	0	0	0	0
N.S.	1	0.99	0.52	1.33	17.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.914	2.513	2.108	1.846	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	86	76	168	1619	405	0	0	0
N.S.	1	0.56	0.50	1.10	10.58	2.65	0.00	0.00	0.00
time (sec)	N/A	0.501	1.067	30.572	0.460	0.334	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	188	87	172	1356	467	0	0	0
N.S.	1	0.99	0.46	0.91	7.14	2.46	0.00	0.00	0.00
time (sec)	N/A	1.058	0.564	2.313	0.427	0.339	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	140	72	136	710	420	0	0	0
N.S.	1	1.01	0.52	0.98	5.11	3.02	0.00	0.00	0.00
time (sec)	N/A	0.736	0.309	2.237	0.398	0.341	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	72	69	155	0	0	0	0	0
N.S.	1	0.47	0.45	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.332	0.371	2.292	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	65	191	0	442	0	0	0
N.S.	1	1.00	0.68	1.99	0.00	4.60	0.00	0.00	0.00
time (sec)	N/A	0.535	0.613	2.231	0.000	0.335	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	76	240	139	0	0	0	0
N.S.	1	1.00	0.76	2.40	1.39	0.00	0.00	0.00	0.00
time (sec)	N/A	0.469	1.032	2.008	0.298	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	149	96	262	3738	0	0	0	0
N.S.	1	1.01	0.65	1.77	25.26	0.00	0.00	0.00	0.00
time (sec)	N/A	0.677	2.350	2.204	1.851	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	194	195	106	284	6134	0	0	0	0
N.S.	1	1.01	0.55	1.46	31.62	0.00	0.00	0.00	0.00
time (sec)	N/A	0.913	5.090	2.448	11.568	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	244	245	120	306	9150	0	0	0	0
N.S.	1	1.00	0.49	1.25	37.50	0.00	0.00	0.00	0.00
time (sec)	N/A	1.211	5.508	2.431	70.100	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	82	83	166	0	0	0	0	0
N.S.	1	0.40	0.41	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	1.906	2.118	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	68	69	146	0	0	0	0	0
N.S.	1	0.45	0.46	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	0.543	2.128	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	62	61	85	60	0	0	0	0
N.S.	1	0.61	0.60	0.83	0.59	0.00	0.00	0.00	0.00
time (sec)	N/A	0.310	0.364	2.021	0.369	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	57	52	34	0	0	36	0
N.S.	1	1.00	1.16	1.06	0.69	0.00	0.00	0.73	0.00
time (sec)	N/A	0.272	0.268	2.099	0.300	0.000	0.000	1.094	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	48	54	81	39	272	0	67	0
N.S.	1	1.04	1.17	1.76	0.85	5.91	0.00	1.46	0.00
time (sec)	N/A	0.333	0.325	1.981	0.360	0.432	0.000	1.196	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	95	89	151	818	0	0	106	0
N.S.	1	0.57	0.53	0.90	4.87	0.00	0.00	0.63	0.00
time (sec)	N/A	0.361	0.606	1.993	0.400	0.000	0.000	1.884	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	111	101	240	2206	0	0	141	0
N.S.	1	0.41	0.37	0.88	8.05	0.00	0.00	0.51	0.00
time (sec)	N/A	0.361	1.599	2.336	0.520	0.000	0.000	1.704	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	85	97	233	2393	0	0	0	0
N.S.	1	0.40	0.45	1.08	11.13	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	2.007	2.307	0.531	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	65	219	0	453	0	0	0
N.S.	1	1.00	0.68	2.28	0.00	4.72	0.00	0.00	0.00
time (sec)	N/A	0.539	0.555	1.958	0.000	0.342	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	80	97	70	0	0	73	0
N.S.	1	1.00	0.82	0.99	0.71	0.00	0.00	0.74	0.00
time (sec)	N/A	0.469	0.574	1.926	0.313	0.000	0.000	1.341	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	75	94	395	0	0	83	0
N.S.	1	1.00	0.80	1.00	4.20	0.00	0.00	0.88	0.00
time (sec)	N/A	0.469	0.262	2.344	0.371	0.000	0.000	1.249	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	93	108	136	818	0	0	99	0
N.S.	1	0.43	0.50	0.63	3.80	0.00	0.00	0.46	0.00
time (sec)	N/A	0.346	0.339	2.036	0.394	0.000	0.000	1.858	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	71	72	157	486	492	0	149	0
N.S.	1	0.70	0.71	1.55	4.81	4.87	0.00	1.48	0.00
time (sec)	N/A	0.440	0.463	2.263	0.379	0.489	0.000	2.156	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	347	128	118	262	4272	0	0	185	0
N.S.	1	0.37	0.34	0.76	12.31	0.00	0.00	0.53	0.00
time (sec)	N/A	0.372	0.991	1.999	1.882	0.000	0.000	1.987	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	220	89	85	216	0	0	0	93	0
N.S.	1	0.40	0.39	0.98	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.334	1.780	2.362	0.000	0.000	0.000	1.461	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	79	142	102	0	0	75	0
N.S.	1	1.00	0.81	1.45	1.04	0.00	0.00	0.77	0.00
time (sec)	N/A	0.466	0.725	1.956	0.313	0.000	0.000	1.664	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	143	86	154	1786	0	0	111	0
N.S.	1	0.99	0.60	1.07	12.40	0.00	0.00	0.77	0.00
time (sec)	N/A	0.709	0.591	2.242	0.509	0.000	0.000	1.615	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	142	89	129	1165	0	0	124	0
N.S.	1	1.01	0.64	0.92	8.32	0.00	0.00	0.89	0.00
time (sec)	N/A	0.658	0.416	2.077	0.486	0.000	0.000	1.461	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	107	101	195	2206	0	0	77	0
N.S.	1	0.40	0.37	0.72	8.17	0.00	0.00	0.29	0.00
time (sec)	N/A	0.348	1.016	2.516	0.507	0.000	0.000	1.679	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	345	126	118	262	4272	0	0	193	0
N.S.	1	0.37	0.34	0.76	12.38	0.00	0.00	0.56	0.00
time (sec)	N/A	0.355	0.894	2.262	1.898	0.000	0.000	1.997	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	88	84	222	1386	564	0	228	0
N.S.	1	0.58	0.56	1.47	9.18	3.74	0.00	1.51	0.00
time (sec)	N/A	0.518	0.749	2.317	0.499	0.635	0.000	1.863	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.291	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	172	154	101	0	0	0	0	0	0
N.S.	1	0.90	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.378	0.601	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	96	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.447	0.539	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	69	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	0.350	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	147	94	0	0	0	0	0	0
N.S.	1	1.06	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.360	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	194	122	0	0	0	0	0	0
N.S.	1	0.95	0.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.364	0.399	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	90	133	136	0	293	0	0	0
N.S.	1	0.99	1.46	1.49	0.00	3.22	0.00	0.00	0.00
time (sec)	N/A	0.424	1.001	1.822	0.000	0.346	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	810	257	0	0	0	0	0
N.S.	1	1.00	3.51	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.686	34.066	8.246	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	178	247	0	0	0	0	0
N.S.	1	1.00	0.79	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.643	7.529	5.050	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	325	182	287	0	0	0	0	0
N.S.	1	1.02	0.57	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.231	8.235	7.295	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	271	212	229	300	0	472	0	0	0
N.S.	1	0.78	0.85	1.11	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.412	8.411	6.826	0.000	0.302	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	167	192	223	0	392	0	0	0
N.S.	1	0.81	0.94	1.09	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.372	9.191	4.842	0.000	0.283	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	127	444	159	0	320	0	0	0
N.S.	1	0.88	3.08	1.10	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.338	6.983	4.575	0.000	0.277	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	76	96	147	235	0	0	0
N.S.	1	1.00	1.15	1.45	2.23	3.56	0.00	0.00	0.00
time (sec)	N/A	0.400	0.419	4.244	0.341	0.280	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	105	105	144	499	0	669	0	0	0
N.S.	1	1.00	1.37	4.75	0.00	6.37	0.00	0.00	0.00
time (sec)	N/A	0.600	2.893	15.146	0.000	0.497	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	219	195	244	38052	0	1413	0	0	0
N.S.	1	0.89	1.11	173.75	0.00	6.45	0.00	0.00	0.00
time (sec)	N/A	0.390	5.179	15.518	0.000	1.574	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	287	268	311	76269	0	2368	0	0	0
N.S.	1	0.93	1.08	265.75	0.00	8.25	0.00	0.00	0.00
time (sec)	N/A	0.454	7.574	17.656	0.000	4.365	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	232	219	297	0	482	0	0	0
N.S.	1	0.96	0.91	1.23	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.411	3.630	6.620	0.000	0.309	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	170	145	232	0	398	0	0	0
N.S.	1	0.97	0.82	1.32	0.00	2.26	0.00	0.00	0.00
time (sec)	N/A	0.354	1.515	4.699	0.000	0.278	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	108	102	206	998	316	0	0	0
N.S.	1	1.03	0.97	1.96	9.50	3.01	0.00	0.00	0.00
time (sec)	N/A	0.582	0.597	1.460	0.394	0.273	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	110	135	840	0	731	0	0	0
N.S.	1	1.00	1.23	7.64	0.00	6.65	0.00	0.00	0.00
time (sec)	N/A	0.602	0.865	14.277	0.000	0.908	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	229	206	293	25448	0	1640	0	0	0
N.S.	1	0.90	1.28	111.13	0.00	7.16	0.00	0.00	0.00
time (sec)	N/A	0.418	4.742	15.931	0.000	4.126	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	310	288	359	55536	0	2729	0	0	0
N.S.	1	0.93	1.16	179.15	0.00	8.80	0.00	0.00	0.00
time (sec)	N/A	0.473	5.862	16.912	0.000	11.665	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	255	286	367	0	620	0	0	0
N.S.	1	0.76	0.85	1.09	0.00	1.85	0.00	0.00	0.00
time (sec)	N/A	0.470	4.731	90.860	0.000	0.314	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	198	191	280	0	500	0	0	0
N.S.	1	0.77	0.74	1.09	0.00	1.94	0.00	0.00	0.00
time (sec)	N/A	0.415	2.398	21.676	0.000	0.295	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	150	128	211	1396	390	0	0	0
N.S.	1	1.06	0.90	1.49	9.83	2.75	0.00	0.00	0.00
time (sec)	N/A	0.817	0.905	7.703	0.429	0.306	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	203	162	343	1402	0	1140	0	0	0
N.S.	1	0.80	1.69	6.91	0.00	5.62	0.00	0.00	0.00
time (sec)	N/A	0.429	5.708	23.447	0.000	2.927	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	329	248	280	20138	0	2031	0	0	0
N.S.	1	0.75	0.85	61.21	0.00	6.17	0.00	0.00	0.00
time (sec)	N/A	0.533	3.260	53.694	0.000	13.167	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	536	405	551	42563	0	3351	0	0	0
N.S.	1	0.76	1.03	79.41	0.00	6.25	0.00	0.00	0.00
time (sec)	N/A	0.657	9.973	235.817	0.000	22.582	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	258	197	787	509	0	619	0	0	0
N.S.	1	0.76	3.05	1.97	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.427	8.070	5.597	0.000	7.795	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	183	142	295	348	0	481	0	0	0
N.S.	1	0.78	1.61	1.90	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.371	2.729	5.114	0.000	1.961	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	92	203	699	314	0	0	0
N.S.	1	1.00	1.01	2.23	7.68	3.45	0.00	0.00	0.00
time (sec)	N/A	0.417	0.537	2.401	0.598	0.685	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	166	177	243	659	0	1050	0	0	0
N.S.	1	1.07	1.46	3.97	0.00	6.33	0.00	0.00	0.00
time (sec)	N/A	1.155	3.439	15.288	0.000	10.595	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	416	301	337	54030	0	2508	0	0	0
N.S.	1	0.72	0.81	129.88	0.00	6.03	0.00	0.00	0.00
time (sec)	N/A	0.577	11.361	16.575	0.000	108.960	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	653	490	2940	97277	0	0	0	0	0
N.S.	1	0.75	4.50	148.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.756	22.452	18.669	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	324	233	856	587	0	701	0	0	0
N.S.	1	0.72	2.64	1.81	0.00	2.16	0.00	0.00	0.00
time (sec)	N/A	0.457	8.094	6.055	0.000	14.300	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	290	207	177	389	0	620	0	0	0
N.S.	1	0.71	0.61	1.34	0.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.430	5.217	3.462	0.000	6.467	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	133	230	289	0	548	0	0	0
N.S.	1	1.05	1.81	2.28	0.00	4.31	0.00	0.00	0.00
time (sec)	N/A	0.637	2.454	3.022	0.000	1.879	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	394	273	302	1419	0	2033	0	0	0
N.S.	1	0.69	0.77	3.60	0.00	5.16	0.00	0.00	0.00
time (sec)	N/A	0.525	6.105	16.332	0.000	215.922	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	560	391	470	69595	0	0	0	0	0
N.S.	1	0.70	0.84	124.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.661	11.849	17.956	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	802	579	2632	117747	0	0	0	0	0
N.S.	1	0.72	3.28	146.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.856	18.761	21.104	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	480	346	436	688	0	880	0	0	0
N.S.	1	0.72	0.91	1.43	0.00	1.83	0.00	0.00	0.00
time (sec)	N/A	0.534	9.677	4.859	0.000	30.821	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	468	334	262	523	0	782	0	0	0
N.S.	1	0.71	0.56	1.12	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.508	6.250	4.484	0.000	13.489	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	164	176	343	376	0	670	0	0	0
N.S.	1	1.07	2.09	2.29	0.00	4.09	0.00	0.00	0.00
time (sec)	N/A	0.915	7.472	2.893	0.000	3.610	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	592	420	360	2345	0	0	0	0	0
N.S.	1	0.71	0.61	3.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.631	11.406	16.762	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	756	540	465	85544	0	0	0	0	0
N.S.	1	0.71	0.62	113.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.756	13.651	20.180	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	999	729	2904	139613	0	0	0	0	0
N.S.	1	0.73	2.91	139.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.958	22.075	23.723	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	123	240	1496	0	806	0	0	0
N.S.	1	1.00	1.95	12.16	0.00	6.55	0.00	0.00	0.00
time (sec)	N/A	0.712	14.890	4.998	0.000	0.499	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	102	166	0	206	0	0	0
N.S.	1	1.00	1.67	2.72	0.00	3.38	0.00	0.00	0.00
time (sec)	N/A	0.279	0.687	2.899	0.000	0.338	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	135	609	0	517	0	0	0
N.S.	1	1.00	1.22	5.49	0.00	4.66	0.00	0.00	0.00
time (sec)	N/A	0.761	1.161	2.847	0.000	0.358	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	141	184	423	0	883	0	0	0
N.S.	1	1.00	1.30	3.00	0.00	6.26	0.00	0.00	0.00
time (sec)	N/A	0.752	13.066	2.717	0.000	0.575	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	141	171	367	0	913	0	0	0
N.S.	1	1.00	1.21	2.60	0.00	6.48	0.00	0.00	0.00
time (sec)	N/A	0.711	0.820	2.501	0.000	0.915	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	68	73	0	250	0	274	573
N.S.	1	1.00	1.01	1.09	0.00	3.73	0.00	4.09	8.55
time (sec)	N/A	0.401	0.201	0.541	0.000	0.280	0.000	0.326	15.629

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	151	155	168	0	561	0	201	3763
N.S.	1	1.23	1.26	1.37	0.00	4.56	0.00	1.63	30.59
time (sec)	N/A	0.722	1.065	0.634	0.000	0.306	0.000	0.299	22.207

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	250	267	287	0	1152	0	457	6909
N.S.	1	1.23	1.31	1.41	0.00	5.65	0.00	2.24	33.87
time (sec)	N/A	1.193	1.818	0.921	0.000	0.343	0.000	0.367	24.046

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	158	136	195	0	671	0	237	4934
N.S.	1	1.19	1.02	1.47	0.00	5.05	0.00	1.78	37.10
time (sec)	N/A	0.691	1.321	0.623	0.000	0.316	0.000	0.317	22.898

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	269	493	386	0	1409	0	658	8682
N.S.	1	1.14	2.08	1.63	0.00	5.95	0.00	2.78	36.63
time (sec)	N/A	1.188	2.694	1.008	0.000	0.360	0.000	0.396	24.417

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	433	438	635	0	2362	0	1201	12818
N.S.	1	1.15	1.16	1.68	0.00	6.27	0.00	3.19	34.00
time (sec)	N/A	1.947	4.424	1.095	0.000	0.450	0.000	0.395	27.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	302	517	458	0	1629	0	818	10759
N.S.	1	1.19	2.04	1.80	0.00	6.41	0.00	3.22	42.36
time (sec)	N/A	1.266	2.247	1.186	0.000	0.404	0.000	0.409	27.042

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	463	459	785	0	2776	0	1572	15647
N.S.	1	1.12	1.11	1.91	0.00	6.74	0.00	3.82	37.98
time (sec)	N/A	1.988	5.386	1.335	0.000	0.479	0.000	0.427	28.485

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	622	683	668	1345	0	4346	0	3173	21021
N.S.	1	1.10	1.07	2.16	0.00	6.99	0.00	5.10	33.80
time (sec)	N/A	3.196	9.725	2.379	0.000	0.688	0.000	0.552	28.964

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	379	1249	0	0	0	0	0
N.S.	1	1.00	1.18	3.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.018	10.202	20.479	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	225	400	0	0	0	0	0
N.S.	1	1.00	1.02	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.699	8.253	6.164	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	380	381	6063	2984	0	0	0	0	0
N.S.	1	1.00	15.96	7.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.389	23.573	24.793	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	330	230	532	0	0	0	0	0
N.S.	1	1.01	0.71	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.213	9.678	8.067	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	442	448	7138	4161	0	0	0	0	0
N.S.	1	1.01	16.15	9.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.795	26.039	32.018	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	145	182	0	0	0	0	0
N.S.	1	1.00	0.70	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.494	3.778	17.243	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	251	281	0	0	0	0	0
N.S.	1	1.00	1.16	1.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.661	21.285	7.082	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	376	405	1138	2883	0	0	0	0	0
N.S.	1	1.08	3.03	7.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.470	13.360	14.500	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	522	1589	7173	0	0	0	0	0
N.S.	1	1.05	3.21	14.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.116	14.926	17.337	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	40517	510	0	0	0	0	0
N.S.	1	1.00	104.16	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.952	36.774	11.872	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	336	322	0	0	0	0	0
N.S.	1	1.00	1.70	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	5.315	10.626	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	598	597	1708	2444	0	0	0	0	0
N.S.	1	1.00	2.86	4.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.899	14.547	14.380	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	899	826	1990	18680	0	0	0	0	0
N.S.	1	0.92	2.21	20.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.032	7.333	17.709	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	744	691	1750	3569	0	0	0	0	0
N.S.	1	0.93	2.35	4.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.044	19.215	13.973	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	919	850	1960	15314	0	0	0	0	0
N.S.	1	0.92	2.13	16.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.211	7.166	16.550	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1122	1053	2385	54551	0	0	0	0	0
N.S.	1	0.94	2.13	48.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.508	7.984	21.529	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	891	853	2026	18991	0	0	0	0	0
N.S.	1	0.96	2.27	21.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.438	7.027	16.321	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1150	1086	2344	39100	0	0	0	0	0
N.S.	1	0.94	2.04	34.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.332	8.150	20.623	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1428	1353	2979	88656	0	0	0	0	0
N.S.	1	0.95	2.09	62.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	6.430	9.058	23.698	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	652	0	50041	457	0	0	0	0	0
N.S.	1	0.00	76.75	0.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	37.554	17.069	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	325	322	0	0	0	0	0
N.S.	1	1.00	1.64	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.355	5.201	11.562	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	249	263	0	0	0	0	0
N.S.	1	1.00	0.63	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.912	3.141	13.405	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	622	683	1761	2867	0	0	0	0	0
N.S.	1	1.10	2.83	4.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.132	19.317	17.183	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	1.07
time (sec)	N/A	0.433	14.574	0.667	0.731	0.000	1.101	0.912	99.948

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	1.07
time (sec)	N/A	0.261	87.531	0.527	1.028	0.000	10.451	1.231	105.691

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	0	27	0
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.00	0.93	0.00
time (sec)	N/A	0.265	84.795	0.602	1.422	0.000	0.000	1.428	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	1.07
time (sec)	N/A	0.466	14.913	0.646	0.777	0.000	3.792	1.683	107.171

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	1.07
time (sec)	N/A	0.257	89.115	0.536	1.062	0.000	61.552	2.937	107.375

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	0	27	0
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.00	0.93	0.00
time (sec)	N/A	0.258	87.715	0.593	1.396	0.000	0.000	5.147	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	1.07
time (sec)	N/A	0.484	75.841	0.597	0.755	0.000	147.970	2.906	110.554

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	0	27	0
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.00	0.93	0.00
time (sec)	N/A	0.258	98.957	0.546	1.148	0.000	0.000	3.674	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	0	27	0
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.00	0.93	0.00
time (sec)	N/A	0.261	130.902	0.634	2.376	0.000	0.000	4.900	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	2425	0	0	0	0	0	0
N.S.	1	1.00	22.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.536	13.876	0.000	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	303	191	0	0	0	0	0	0
N.S.	1	1.10	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.254	1.789	0.000	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	227	152	0	0	0	0	0	0
N.S.	1	1.11	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.035	0.753	0.000	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	177	124	0	0	0	0	0	0
N.S.	1	1.13	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.577	0.135	0.000	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	208	235	159	0	0	0	0	0	0
N.S.	1	1.13	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.881	0.632	0.000	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	248	269	215	0	0	0	0	0	0
N.S.	1	1.08	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.227	1.516	0.000	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	26	29	33
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.96	1.07	1.22
time (sec)	N/A	0.356	6.636	1.290	4.191	0.313	5.086	0.878	15.968

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	323	278	0	0	0	0	0	0
N.S.	1	1.09	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.626	0.816	0.000	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	227	200	0	0	0	0	0	0
N.S.	1	1.08	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.013	0.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	177	125	0	0	0	0	0	0
N.S.	1	1.13	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.589	0.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	206	221	5411	0	0	0	0	0	0
N.S.	1	1.07	26.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.824	32.619	0.000	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	322	330	14108	0	0	0	0	0	0
N.S.	1	1.02	43.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.904	46.813	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [197] had the largest ratio of [.7199999999999999973]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	0.90	26	0.192
2	A	5	5	0.90	26	0.192
3	A	7	7	0.92	26	0.269
4	A	7	7	0.74	26	0.269
5	A	5	5	0.95	24	0.208
6	A	3	3	0.91	26	0.115
7	A	3	3	0.93	26	0.115
8	A	3	3	0.92	26	0.115
9	A	3	3	0.92	26	0.115
10	A	3	3	0.91	26	0.115
11	A	5	5	0.90	26	0.192
12	A	10	10	0.93	26	0.385
13	A	9	9	0.76	26	0.346
14	A	7	7	0.92	26	0.269
15	A	5	5	0.99	24	0.208
16	A	5	5	0.91	26	0.192
17	A	3	3	0.91	26	0.115
18	A	3	3	0.92	26	0.115
19	A	3	3	0.92	26	0.115
20	A	3	3	0.91	26	0.115
21	A	5	5	1.00	26	0.192
22	A	5	5	0.89	26	0.192

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
23	A	3	3	0.91	26	0.115
24	A	3	3	0.93	26	0.115
25	A	3	3	0.92	24	0.125
26	A	8	8	0.96	26	0.308
27	A	7	7	0.74	26	0.269
28	A	11	11	1.02	26	0.423
29	A	5	5	0.88	26	0.192
30	A	5	5	0.89	26	0.192
31	A	5	5	0.88	26	0.192
32	A	3	3	0.91	26	0.115
33	A	3	3	0.92	26	0.115
34	A	3	3	0.92	26	0.115
35	A	3	3	0.91	24	0.125
36	A	5	5	0.91	26	0.192
37	A	11	11	1.01	26	0.423
38	A	9	9	0.79	26	0.346
39	A	14	14	0.98	26	0.538
40	A	5	5	0.89	26	0.192
41	A	5	5	0.87	26	0.192
42	A	7	6	0.90	28	0.214
43	A	7	6	0.91	28	0.214
44	A	7	6	0.94	28	0.214
45	A	7	6	1.03	26	0.231
46	A	7	6	0.91	28	0.214
47	A	8	7	0.92	28	0.250
48	A	9	8	0.91	28	0.286
49	A	10	9	0.90	28	0.321
50	A	8	7	0.87	28	0.250
51	A	8	7	0.88	28	0.250
52	A	8	7	0.93	26	0.269
53	A	7	6	0.93	28	0.214
54	A	8	7	0.91	28	0.250
55	A	9	8	0.92	28	0.286
56	A	10	9	0.91	28	0.321

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
57	A	7	6	0.85	28	0.214
58	A	7	6	0.86	28	0.214
59	A	7	6	0.90	26	0.231
60	A	7	6	0.87	28	0.214
61	A	7	6	0.95	28	0.214
62	A	7	6	0.88	28	0.214
63	A	7	6	0.90	28	0.214
64	A	7	6	0.89	28	0.214
65	A	13	12	1.02	28	0.429
66	A	11	10	1.02	28	0.357
67	A	8	7	1.01	28	0.250
68	A	7	6	0.98	26	0.231
69	A	10	9	0.98	28	0.321
70	A	11	10	0.94	28	0.357
71	A	13	12	0.93	28	0.429
72	A	13	12	1.05	28	0.429
73	A	11	10	1.05	28	0.357
74	A	9	8	1.19	28	0.286
75	A	8	7	1.29	26	0.269
76	A	11	10	0.99	28	0.357
77	A	14	13	0.98	28	0.464
78	A	16	15	0.97	28	0.536
79	A	16	15	1.08	28	0.536
80	A	14	13	1.08	28	0.464
81	A	11	10	1.11	28	0.357
82	A	11	10	1.07	28	0.357
83	A	10	9	1.38	26	0.346
84	A	15	14	1.00	28	0.500
85	A	16	15	0.99	28	0.536
86	A	11	11	1.01	30	0.367
87	A	9	9	1.01	30	0.300
88	A	7	7	1.00	30	0.233
89	A	5	5	1.00	30	0.167
90	A	4	3	1.00	30	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	6	5	1.00	30	0.167
92	A	8	7	1.01	30	0.233
93	A	10	9	1.02	30	0.300
94	A	11	11	0.99	30	0.367
95	A	7	7	0.67	30	0.233
96	A	7	7	1.00	30	0.233
97	A	6	5	0.62	30	0.167
98	A	6	5	1.00	30	0.167
99	A	8	7	0.99	30	0.233
100	A	10	9	0.99	30	0.300
101	A	9	9	0.56	30	0.300
102	A	11	11	0.99	30	0.367
103	A	9	9	1.01	30	0.300
104	A	6	5	0.47	30	0.167
105	A	7	7	1.00	30	0.233
106	A	6	5	1.00	30	0.167
107	A	8	7	1.01	30	0.233
108	A	10	9	1.01	30	0.300
109	A	12	11	1.00	30	0.367
110	A	6	5	0.40	30	0.167
111	A	6	5	0.45	30	0.167
112	A	6	5	0.61	30	0.167
113	A	4	3	1.00	30	0.100
114	A	6	6	1.04	30	0.200
115	A	6	5	0.57	30	0.167
116	A	6	5	0.41	30	0.167
117	A	6	5	0.40	30	0.167
118	A	7	7	1.00	30	0.233
119	A	6	5	1.00	30	0.167
120	A	6	5	1.00	30	0.167
121	A	6	5	0.43	30	0.167
122	A	10	10	0.70	30	0.333
123	A	6	5	0.37	30	0.167
124	A	6	5	0.40	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
125	A	6	5	1.00	30	0.167
126	A	8	7	0.99	30	0.233
127	A	8	7	1.01	30	0.233
128	A	6	5	0.40	30	0.167
129	A	6	5	0.37	30	0.167
130	A	14	14	0.58	30	0.467
131	A	4	3	1.00	24	0.125
132	A	5	4	1.00	26	0.154
133	A	5	4	1.00	26	0.154
134	A	5	4	1.00	26	0.154
135	A	5	4	1.00	24	0.167
136	A	5	4	1.00	26	0.154
137	A	5	4	1.00	26	0.154
138	A	6	5	0.90	28	0.179
139	A	6	5	1.00	28	0.179
140	A	4	3	1.00	28	0.107
141	A	7	6	1.06	28	0.214
142	A	9	8	0.95	28	0.286
143	A	9	8	0.99	27	0.296
144	A	5	5	1.00	27	0.185
145	A	5	5	1.00	27	0.185
146	A	9	9	1.02	27	0.333
147	A	5	4	0.78	27	0.148
148	A	5	4	0.81	27	0.148
149	A	5	4	0.88	27	0.148
150	A	7	6	1.00	25	0.240
151	A	8	7	1.00	27	0.259
152	A	9	8	0.89	27	0.296
153	A	11	10	0.93	27	0.370
154	A	11	10	0.96	27	0.370
155	A	9	8	0.97	27	0.296
156	A	10	9	1.03	25	0.360
157	A	8	7	1.00	27	0.259
158	A	10	9	0.90	27	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	12	11	0.93	27	0.407
160	A	6	5	0.76	27	0.185
161	A	6	5	0.77	27	0.185
162	A	13	12	1.06	25	0.480
163	A	6	5	0.80	27	0.185
164	A	6	5	0.75	27	0.185
165	A	6	5	0.76	27	0.185
166	A	6	5	0.76	27	0.185
167	A	6	5	0.78	27	0.185
168	A	8	7	1.00	25	0.280
169	A	12	11	1.07	27	0.407
170	A	6	5	0.72	27	0.185
171	A	6	5	0.75	27	0.185
172	A	6	5	0.72	27	0.185
173	A	6	5	0.71	27	0.185
174	A	11	10	1.05	25	0.400
175	A	6	5	0.69	27	0.185
176	A	6	5	0.70	27	0.185
177	A	6	5	0.72	27	0.185
178	A	6	5	0.72	27	0.185
179	A	6	5	0.71	27	0.185
180	A	14	13	1.07	25	0.520
181	A	6	5	0.71	27	0.185
182	A	6	5	0.71	27	0.185
183	A	6	5	0.73	27	0.185
184	A	8	7	1.00	29	0.241
185	A	4	3	1.00	29	0.103
186	A	8	7	1.00	29	0.241
187	A	8	7	1.00	29	0.241
188	A	8	7	1.00	29	0.241
189	A	8	7	1.00	23	0.304
190	A	11	10	1.23	23	0.435
191	A	14	13	1.23	23	0.565
192	A	11	10	1.19	25	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
193	A	13	12	1.14	25	0.480
194	A	16	15	1.15	25	0.600
195	A	13	12	1.19	25	0.480
196	A	15	14	1.12	25	0.560
197	A	19	18	1.10	25	0.720
198	A	8	8	1.00	25	0.320
199	A	5	5	1.00	27	0.185
200	A	11	11	1.00	25	0.440
201	A	8	8	1.01	27	0.296
202	A	14	14	1.01	25	0.560
203	A	5	5	1.00	25	0.200
204	A	5	5	1.00	27	0.185
205	A	11	11	1.08	25	0.440
206	A	14	14	1.05	25	0.560
207	A	5	5	1.00	29	0.172
208	A	2	2	1.00	29	0.069
209	A	8	8	1.00	29	0.276
210	A	13	13	0.92	29	0.448
211	A	10	10	0.93	29	0.345
212	A	13	13	0.92	29	0.448
213	A	17	17	0.94	29	0.586
214	A	13	13	0.96	29	0.448
215	A	16	16	0.94	29	0.552
216	A	19	19	0.95	29	0.655
217	F	0	0	N/A	0.000	N/A
218	A	2	2	1.00	29	0.069
219	A	5	5	1.00	29	0.172
220	A	11	11	1.10	29	0.379
221	N/A	4	0	1.00	29	0.000
222	N/A	2	0	1.00	29	0.000
223	N/A	2	0	1.00	29	0.000
224	N/A	4	0	1.00	29	0.000
225	N/A	2	0	1.00	29	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
226	N/A	2	0	1.00	29	0.000
227	N/A	4	0	1.00	29	0.000
228	N/A	2	0	1.00	29	0.000
229	N/A	2	0	1.00	29	0.000
230	A	8	7	1.00	27	0.259
231	A	12	12	1.10	27	0.444
232	A	13	13	1.11	27	0.481
233	A	8	8	1.13	25	0.320
234	A	10	10	1.13	27	0.370
235	A	13	13	1.08	27	0.481
236	N/A	4	0	1.00	27	0.000
237	A	15	15	1.09	27	0.556
238	A	13	13	1.08	27	0.481
239	A	8	8	1.13	25	0.320
240	A	11	10	1.07	27	0.370
241	A	7	7	1.02	27	0.259

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^5 \, dx$

3.1.1 Optimal result

Integrand size = 26, antiderivative size = 196

$$\begin{aligned}
&\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^5 \, dx \\
&= a^2c^5x - \frac{19a^2c^5\text{arctanh}(\sin(e + fx))}{16f} - \frac{a^2c^5\tan(e + fx)}{f} \\
&\quad + \frac{17a^2c^5\sec(e + fx)\tan(e + fx)}{16f} + \frac{a^2c^5\sec^3(e + fx)\tan(e + fx)}{8f} + \frac{a^2c^5\tan^3(e + fx)}{3f} \\
&\quad - \frac{3a^2c^5\sec(e + fx)\tan^3(e + fx)}{4f} - \frac{a^2c^5\sec^3(e + fx)\tan^3(e + fx)}{6f} + \frac{3a^2c^5\tan^5(e + fx)}{5f}
\end{aligned}$$

output
```
a^2*c^5*x-19/16*a^2*c^5*arctanh(sin(f*x+e))/f-a^2*c^5*tan(f*x+e)/f+17/16*a
^2*c^5*sec(f*x+e)*tan(f*x+e)/f+1/8*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)/f+1/3*a
^2*c^5*tan(f*x+e)^3/f-3/4*a^2*c^5*sec(f*x+e)*tan(f*x+e)^3/f-1/6*a^2*c^5*se
c(f*x+e)^3*tan(f*x+e)^3/f+3/5*a^2*c^5*tan(f*x+e)^5/f
```

3.1.2 Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.84

$$\begin{aligned}
&\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^5 \, dx \\
&= \frac{a^2c^5\sec^6(e + fx)\,(1200e + 1200fx - 4560\text{arctanh}(\sin(e + fx))\cos^6(e + fx) + 1800(e + fx)\cos(2(e + f\ldots}{}
\end{aligned}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

output
```
(a^2*c^5*Sec[e + f*x]^6*(1200*e + 1200*f*x - 4560*ArcTanh[Sin[e + f*x]]*Co
s[e + f*x]^6 + 1800*(e + f*x)*Cos[2*(e + f*x)] + 720*e*Cos[4*(e + f*x)] +
720*f*x*Cos[4*(e + f*x)] + 120*e*Cos[6*(e + f*x)] + 120*f*x*Cos[6*(e + f*x
)] - 210*Sin[e + f*x] - 120*Sin[2*(e + f*x)] + 865*Sin[3*(e + f*x)] - 768*
Sin[4*(e + f*x)] + 435*Sin[5*(e + f*x)] - 88*Sin[6*(e + f*x)]))/(3840*f)
```

3.1.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(c-c\sec(e+fx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int (c-c\sec(e+fx))^3\tan^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cot\left(e+fx+\frac{\pi}{2}\right)^4\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4374}$$

$$a^2c^2\int \left(c^3\tan^4(e+fx)-c^3\sec^3(e+fx)\tan^4(e+fx)+3c^3\sec^2(e+fx)\tan^4(e+fx)-3c^3\sec(e+fx)\tan^4(e-$$

$$\Big\downarrow \text{2009}$$

$$a^2c^2\left(-\frac{19c^3\text{arctanh}(\sin(e+fx))}{16f}+\frac{3c^3\tan^5(e+fx)}{5f}+\frac{c^3\tan^3(e+fx)}{3f}-\frac{c^3\tan(e+fx)}{f}-\frac{c^3\tan^3(e+fx)\sec^3}{6f}\right.$$

input
```
Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

output
```
a^2*c^2*(c^3*x - (19*c^3*ArcTanh[Sin[e + f*x]])/(16*f) - (c^3*Tan[e + f*x]
)/f + (17*c^3*Sec[e + f*x]*Tan[e + f*x])/(16*f) + (c^3*Sec[e + f*x]^3*Tan[
e + f*x])/(8*f) + (c^3*Tan[e + f*x]^3)/(3*f) - (3*c^3*Sec[e + f*x]*Tan[e +
 f*x]^3)/(4*f) - (c^3*Sec[e + f*x]^3*Tan[e + f*x]^3)/(6*f) + (3*c^3*Tan[e
+ f*x]^5)/(5*f))
```

3.1.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.1.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.91 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.05

method	result
risch	$a^2c^5x - \frac{ic^5a^2\left(435\,e^{11i(fx+e)}-240\,e^{10i(fx+e)}+865\,e^{9i(fx+e)}+1200\,e^{8i(fx+e)}-210\,e^{7i(fx+e)}+1760\,e^{6i(fx+e)}+210\,e^{\cdots}\right.}{120f\left(1+e^{2i(fx+e)}\right)^6}$
parallelrisch	$-\frac{a^2c^5\left(\frac{19\left(-5-\frac{15\cos(2fx+2e)}{2}-3\cos(4fx+4e)-\frac{\cos(6fx+6e)}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8}+\frac{19\left(5+\frac{\cos(6fx+6e)}{2}+3\cos(4fx+4e)+\frac{15\,\mathrm{c}}{}\right.}{8}\right.}{}$
derivativedivides	$\frac{-c^5a^2\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)-3c^5a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^{\cdots}}{5}\right.}{}$
default	$\frac{-c^5a^2\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)-3c^5a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^{\cdots}}{5}\right.}{}$
parts	$a^2c^5x+\frac{a^2c^5\tan(fx+e)}{f}-\frac{3c^5a^2\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{5c^5a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+\cdots}{2}\right.}{f}$
norman	$\frac{a^2c^5x+a^2c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}-6a^2c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+15a^2c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-20a^2c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+15a^2c^5x\tan\left(\frac{fx}{2}\cdots\right.}{}$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^5*x-1/120*I*c^5*a^2*(435*exp(11*I*(f*x+e))-240*exp(10*I*(f*x+e))+865
*exp(9*I*(f*x+e))+1200*exp(8*I*(f*x+e))-210*exp(7*I*(f*x+e))+1760*exp(6*I*
(f*x+e))+210*exp(5*I*(f*x+e))+1440*exp(4*I*(f*x+e))-865*exp(3*I*(f*x+e))+1
296*exp(2*I*(f*x+e))-435*exp(I*(f*x+e))+176)/f/(1+exp(2*I*(f*x+e)))^6+19/1
6*c^5*a^2/f*ln(exp(I*(f*x+e))-I)-19/16*c^5*a^2/f*ln(exp(I*(f*x+e))+I)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.91

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$
$$=\frac{480\,a^2c^5fx\cos(fx+e)^6-285\,a^2c^5\cos(fx+e)^6\log(\sin(fx+e)+1)+285\,a^2c^5\cos(fx+e)^6\log(-\sin\cdots}{}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
1/480*(480*a^2*c^5*f*x*cos(f*x + e)^6 - 285*a^2*c^5*cos(f*x + e)^6*log(sin
(f*x + e) + 1) + 285*a^2*c^5*cos(f*x + e)^6*log(-sin(f*x + e) + 1) - 2*(17
6*a^2*c^5*cos(f*x + e)^5 - 435*a^2*c^5*cos(f*x + e)^4 + 208*a^2*c^5*cos(f*
x + e)^3 + 110*a^2*c^5*cos(f*x + e)^2 - 144*a^2*c^5*cos(f*x + e) + 40*a^2*
c^5)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

3.1.6 Sympy [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = -a^2c^5\left(\int(-1)\,dx+\int 3\sec(e+fx)\,dx+\int\left(-\sec^2(e+fx)\right)\,dx + \int\left(-5\sec^3(e+fx)\right)\,dx+\int 5\sec^4(e+fx)\,dx+\int\sec^5(e+fx)\,dx + \int\left(-3\sec^6(e+fx)\right)\,dx+\int\sec^7(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**5,x)
```

output
```
-a**2*c**5*(Integral(-1, x) + Integral(3*sec(e + f*x), x) + Integral(-sec(
e + f*x)**2, x) + Integral(-5*sec(e + f*x)**3, x) + Integral(5*sec(e + f*x
)**4, x) + Integral(sec(e + f*x)**5, x) + Integral(-3*sec(e + f*x)**6, x)
+ Integral(sec(e + f*x)**7, x))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.70

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = \frac{96\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^2c^5-800\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2c^{\cdots}}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
1/480*(96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^5
 - 800*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^5 + 480*(f*x + e)*a^2*c^5 +
 5*a^2*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(s
in(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x
+ e) + 1) + 15*log(sin(f*x + e) - 1)) + 30*a^2*c^5*(2*(3*sin(f*x + e)^3 -
5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x +
e) + 1) + 3*log(sin(f*x + e) - 1)) - 600*a^2*c^5*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 1440*a^2*c^
5*log(sec(f*x + e) + tan(f*x + e)) + 480*a^2*c^5*tan(f*x + e))/f
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.97

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^5 \, dx$$

$$= \frac{240\,(fx+e)a^2c^5 - 285\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + 1\right|\right) + 285\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - 1\right|\right) + \frac{2\left(525\,a^2 c\right.}{}}{}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
1/240*(240*(f*x + e)*a^2*c^5 - 285*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) +
1)) + 285*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(525*a^2*c^5*tan(
1/2*f*x + 1/2*e)^11 - 3135*a^2*c^5*tan(1/2*f*x + 1/2*e)^9 + 1746*a^2*c^5*t
an(1/2*f*x + 1/2*e)^7 - 366*a^2*c^5*tan(1/2*f*x + 1/2*e)^5 - 95*a^2*c^5*ta
n(1/2*f*x + 1/2*e)^3 + 45*a^2*c^5*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2
*e)^2 - 1)^6)/f
```

3.1.9 Mupad [B] (verification not implemented)

Time = 15.54 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.16

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^5 \, dx = a^2\,c^5\,x$$

$$- \frac{-\frac{35\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8} + \frac{209\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{8} - \frac{291\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{20} + \frac{61\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{20} + \frac{19\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{24} -}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12} - 6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10} + 15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8} - 20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6} + 15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4} - 6\,\mathrm{ta}\right.}$$

$$- \frac{19\,a^2\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

input
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5,x)
```

output
```
a^2*c^5*x - ((19*a^2*c^5*tan(e/2 + (f*x)/2)^3)/24 + (61*a^2*c^5*tan(e/2 +
(f*x)/2)^5)/20 - (291*a^2*c^5*tan(e/2 + (f*x)/2)^7)/20 + (209*a^2*c^5*tan(
e/2 + (f*x)/2)^9)/8 - (35*a^2*c^5*tan(e/2 + (f*x)/2)^11)/8 - (3*a^2*c^5*ta
n(e/2 + (f*x)/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2
- 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)
^10 + tan(e/2 + (f*x)/2)^12 + 1)) - (19*a^2*c^5*atanh(tan(e/2 + (f*x)/2)))
/(8*f)
```

3.2 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$

3.2.1 Optimal result

Integrand size = 26, antiderivative size = 140

$$\begin{aligned}\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx &= a^2c^4x-\frac{3a^2c^4\text{arctanh}(\sin(e+fx))}{4f}-\frac{a^2c^4\tan(e+fx)}{f}+\frac{3a^2c^4\sec(e+fx)\tan(e+fx)}{4f}\\&\quad+\frac{a^2c^4\tan^3(e+fx)}{3f}-\frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{2f}+\frac{a^2c^4\tan^5(e+fx)}{5f}\end{aligned}$$

output
```
a^2*c^4*x-3/4*a^2*c^4*arctanh(sin(f*x+e))/f-a^2*c^4*tan(f*x+e)/f+3/4*a^2*c
^4*sec(f*x+e)*tan(f*x+e)/f+1/3*a^2*c^4*tan(f*x+e)^3/f-1/2*a^2*c^4*sec(f*x+
e)*tan(f*x+e)^3/f+1/5*a^2*c^4*tan(f*x+e)^5/f
```

3.2.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx = \frac{a^2c^4\sec^5(e+fx)\left(600(e+fx)\cos(e+fx)-720\text{arctanh}(\sin(e+fx))\cos^5(e+fx)+300e\cos(3(e+fx)\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

output
```
(a^2*c^4*Sec[e + f*x]^5*(600*(e + f*x)*Cos[e + f*x] - 720*ArcTanh[Sin[e +
f*x]]*Cos[e + f*x]^5 + 300*e*Cos[3*(e + f*x)] + 300*f*x*Cos[3*(e + f*x)] +
 60*e*Cos[5*(e + f*x)] + 60*f*x*Cos[5*(e + f*x)] + 40*Sin[e + f*x] + 60*Si
n[2*(e + f*x)] - 220*Sin[3*(e + f*x)] + 150*Sin[4*(e + f*x)] - 68*Sin[5*(e
 + f*x)]))/(960*f)
```

3.2.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(c-c\sec(e+fx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int (c-c\sec(e+fx))^2\tan^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cot\left(e+fx+\frac{\pi}{2}\right)^4\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4374}$$

$$a^2c^2\int \left(c^2\tan^4(e+fx)+c^2\sec^2(e+fx)\tan^4(e+fx)-2c^2\sec(e+fx)\tan^4(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2c^2\left(-\frac{3c^2\text{arctanh}(\sin(e+fx))}{4f}+\frac{c^2\tan^5(e+fx)}{5f}+\frac{c^2\tan^3(e+fx)}{3f}-\frac{c^2\tan(e+fx)}{f}-\frac{c^2\tan^3(e+fx)\sec(e+}{2f}\right.$$

input
```
Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

output
```
a^2*c^2*(c^2*x - (3*c^2*ArcTanh[Sin[e + f*x]])/(4*f) - (c^2*Tan[e + f*x])/
f + (3*c^2*Sec[e + f*x]*Tan[e + f*x])/(4*f) + (c^2*Tan[e + f*x]^3)/(3*f) -
 (c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(2*f) + (c^2*Tan[e + f*x]^5)/(5*f))
```

3.2.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.2.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

method	result
parts	$a^2c^4x - \frac{c^4a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f} - \frac{a^2c^4\tan(fx+e)}{f} + \frac{2a^2c^4\sec(fx+e)\tan(fx+e)}{f} +$
risch	$a^2c^4x - \frac{ic^4a^2(75\,e^{9i(fx+e)}+60\,e^{8i(fx+e)}+30\,e^{7i(fx+e)}+360\,e^{6i(fx+e)}+320\,e^{4i(fx+e)}-30\,e^{3i(fx+e)}+280\,e^{2i(fx+e)}-}{30f\left(1+e^{2i(fx+e)}\right)^5}$
derivativedivides	$\frac{-c^4a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-2c^4a^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)}{8}\right.}{}$
default	$\frac{-c^4a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-2c^4a^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)}{8}\right.}{}$
parallelrisch	$\frac{a^2c^4\left(\left(\frac{15\cos(fx+e)}{2}+\frac{15\cos(3fx+3e)}{4}+\frac{3\cos(5fx+5e)}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\frac{15\cos(fx+e)}{2}-\frac{15\cos(3fx+3e)}{4}-\frac{3\cos(5fx+}{4}\right.\right.}{f(\cos}$
norman	$\frac{a^2c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-a^2c^4x+5a^2c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-10a^2c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+10a^2c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-5a^2c^4x\tan\left(\frac{fx}{2}+\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^4*x-c^4*a^2/f*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-
a^2*c^4*tan(f*x+e)/f+2*a^2*c^4*sec(f*x+e)*tan(f*x+e)/f+c^4*a^2/f*(-2/3-1/3
*sec(f*x+e)^2)*tan(f*x+e)-2*c^4*a^2/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))
*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.16

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$= \frac{120\,a^2c^4fx\cos(fx+e)^5 - 45\,a^2c^4\cos(fx+e)^5\log(\sin(fx+e)+1) + 45\,a^2c^4\cos(fx+e)^5\log(-\sin(f}{}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
1/120*(120*a^2*c^4*f*x*cos(f*x + e)^5 - 45*a^2*c^4*cos(f*x + e)^5*log(sin(
f*x + e) + 1) + 45*a^2*c^4*cos(f*x + e)^5*log(-sin(f*x + e) + 1) - 2*(68*a
^2*c^4*cos(f*x + e)^4 - 75*a^2*c^4*cos(f*x + e)^3 + 4*a^2*c^4*cos(f*x + e)
^2 + 30*a^2*c^4*cos(f*x + e) - 12*a^2*c^4)*sin(f*x + e))/(f*cos(f*x + e)^5
)
```

3.2.6 Sympy [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$= a^2c^4\left(\int 1\,dx + \int \left(-2\sec\left(e+fx\right)\right)\,dx + \int \left(-\sec^2\left(e+fx\right)\right)\,dx + \int 4\sec^3\left(e+fx\right)dx + \int \left(-\sec^4\left(e+fx\right)\right)\,dx + \int \left(-2\sec^5\left(e+fx\right)\right)\,dx + \int \sec^6\left(e+fx\right)dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**4,x)
```

output
```
a**2*c**4*(Integral(1, x) + Integral(-2*sec(e + f*x), x) + Integral(-sec(e
 + f*x)**2, x) + Integral(4*sec(e + f*x)**3, x) + Integral(-sec(e + f*x)**
4, x) + Integral(-2*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.71

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$= \frac{8\left(3\,\tan\left(fx+e\right)^5 + 10\,\tan\left(fx+e\right)^3 + 15\,\tan\left(fx+e\right)\right)a^2c^4 - 40\left(\tan\left(fx+e\right)^3 + 3\,\tan\left(fx+e\right)\right)a^2c^4 +}{}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
1/120*(8*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^4
- 40*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^4 + 120*(f*x + e)*a^2*c^4 + 1
5*a^2*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f
*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*
a^2*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log
(sin(f*x + e) - 1)) - 240*a^2*c^4*log(sec(f*x + e) + tan(f*x + e)) - 120*a
^2*c^4*tan(f*x + e))/f
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$= \frac{60\,(fx+e)a^2c^4 - 45\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) + 45\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(105\,a^2c^4\tan\right.}{}}{60\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/60*(60*(f*x + e)*a^2*c^4 - 45*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))
 + 45*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(105*a^2*c^4*tan(1/2*
f*x + 1/2*e)^9 - 530*a^2*c^4*tan(1/2*f*x + 1/2*e)^7 + 328*a^2*c^4*tan(1/2*
f*x + 1/2*e)^5 - 110*a^2*c^4*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c^4*tan(1/2*f
*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^5)/f
```

3.2.9 Mupad [B] (verification not implemented)

Time = 15.15 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.39

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx = a^2\,c^4\,x$$
$$+ \frac{\frac{7\,a^2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{2} - \frac{53\,a^2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{3} + \frac{164\,a^2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15} - \frac{11\,a^2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3} + \frac{a^2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10} - 5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8 + 10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6 - 10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 + 5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 - 1\right)}$$
$$- \frac{3\,a^2\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{2\,f}$$

input
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4,x)
```

output
```
a^2*c^4*x + ((164*a^2*c^4*tan(e/2 + (f*x)/2)^5)/15 - (11*a^2*c^4*tan(e/2 +
 (f*x)/2)^3)/3 - (53*a^2*c^4*tan(e/2 + (f*x)/2)^7)/3 + (7*a^2*c^4*tan(e/2
+ (f*x)/2)^9)/2 + (a^2*c^4*tan(e/2 + (f*x)/2))/2)/(f*(5*tan(e/2 + (f*x)/2)
^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)
/2)^8 + tan(e/2 + (f*x)/2)^10 - 1)) - (3*a^2*c^4*atanh(tan(e/2 + (f*x)/2))
)/(2*f)
```

3.3 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$

3.3.1 Optimal result

Integrand size = 26, antiderivative size = 97

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx = a^2c^3x-\frac{3a^2c^3\text{arctanh}(\sin(e+fx))}{8f}-\frac{a^2(8c^3-3c^3\sec(e+fx))\tan(e+fx)}{8f}+\frac{a^2(4c^3-3c^3\sec(e+fx))\tan^3(e+fx)}{12f}$$

output
```
a^2*c^3*x-3/8*a^2*c^3*arctanh(sin(f*x+e))/f-1/8*a^2*(8*c^3-3*c^3*sec(f*x+e
))*tan(f*x+e)/f+1/12*a^2*(4*c^3-3*c^3*sec(f*x+e))*tan(f*x+e)^3/f
```

3.3.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx = \frac{a^2c^3\sec^4(e+fx)\,(72e+72fx-72\text{arctanh}(\sin(e+fx))\cos^4(e+fx)+96(e+fx)\cos(2(e+fx))+24e$$

input
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

output
```
(a^2*c^3*Sec[e + f*x]^4*(72*e + 72*f*x - 72*ArcTanh[Sin[e + f*x]]*Cos[e +
f*x]^4 + 96*(e + f*x)*Cos[2*(e + f*x)] + 24*e*Cos[4*(e + f*x)] + 24*f*x*Co
s[4*(e + f*x)] - 18*Sin[e + f*x] - 32*Sin[2*(e + f*x)] + 30*Sin[3*(e + f*x
)] - 32*Sin[4*(e + f*x)]))/(192*f)
```

3.3.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 4392, 3042, 4369, 3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(c-c\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{ 4392}$$

$$a^2c^2\int (c-c\sec(e+fx))\tan^4(e+fx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$a^2c^2\int \cot\left(e+fx+\frac{\pi}{2}\right)^4\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 4369}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)(4c-3c\sec(e+fx))}{12f}-\frac{1}{4}\int (4c-3c\sec(e+fx))\tan^2(e+fx)dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)(4c-3c\sec(e+fx))}{12f}-\frac{1}{4}\int \cot\left(e+fx+\frac{\pi}{2}\right)^2\left(4c-3c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx\right)$$

$$\Big\downarrow \text{ 4369}$$

$$a^2c^2\left(\frac{1}{4}\left(\frac{1}{2}\int (8c-3c\sec(e+fx))dx-\frac{\tan(e+fx)(8c-3c\sec(e+fx))}{2f}\right)+\frac{\tan^3(e+fx)(4c-3c\sec(e+fx))}{12f}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$a^2c^2\left(\frac{1}{4}\left(\frac{1}{2}\left(8cx-\frac{3c\,\text{arctanh}(\sin(e+fx))}{f}\right)-\frac{\tan(e+fx)(8c-3c\sec(e+fx))}{2f}\right)+\frac{\tan^3(e+fx)(4c-3c\sec(e+fx))}{12f}\right.$$

input
```
Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

output
```
a^2*c^2*(((4*c - 3*c*Sec[e + f*x])*Tan[e + f*x]^3)/(12*f) + ((8*c*x - (3*c
*ArcTanh[Sin[e + f*x]])/f)/2 - ((8*c - 3*c*Sec[e + f*x])*Tan[e + f*x])/(2*
f))/4)
```

3.3.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.3.4 Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.37

method	result
parts	$a^2c^3x - \frac{a^2c^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f} - \frac{2a^2c^3\tan(fx+e)}{f} + \frac{a^2c^3\sec(fx+e)\tan(fx+e)}{f} - \frac{a^2c^3\left(-\left(-\mathrm{se}\right.\right.}{}$
risch	$a^2c^3x - \frac{ia^2c^3(15\,\mathrm{e}^{7i(fx+e)}+48\,\mathrm{e}^{6i(fx+e)}-9\,\mathrm{e}^{5i(fx+e)}+96\,\mathrm{e}^{4i(fx+e)}+9\,\mathrm{e}^{3i(fx+e)}+80\,\mathrm{e}^{2i(fx+e)}-15\,\mathrm{e}^{i(fx+e)}+32)}{12f\left(1+\mathrm{e}^{2i(fx+e)}\right)^4} +$
derivativedivides	$\frac{-a^2c^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-a^2c^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+2}{}$
default	$\frac{-a^2c^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-a^2c^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+2}{}$
parallelrisch	$\frac{a^2c^3\left(9(3+\cos(4fx+4e)+4\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+9(-\cos(4fx+4e)-4\cos(2fx+2e)-3)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.\right.}{24f(3+\cos(4fx+4e}$
norman	$\frac{a^2c^3x+a^2c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8-\frac{5a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{71a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{12f}-\frac{137a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{12f}+\frac{11a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}-4a}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^3*x-a^2*c^3/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a^2*c^3/f*tan(f*x
+e)+a^2*c^3/f*sec(f*x+e)*tan(f*x+e)-a^2*c^3/f*(-(-1/4*sec(f*x+e)^3-3/8*sec
(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.52

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^3\, dx$$
$$= \frac{48\, a^2c^3fx\cos\left(fx+e\right)^4 - 9\, a^2c^3\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right) + 9\, a^2c^3\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+\right.\right.}{48\, f\cos\left(f\right.}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/48*(48*a^2*c^3*f*x*cos(f*x + e)^4 - 9*a^2*c^3*cos(f*x + e)^4*log(sin(f*x
 + e) + 1) + 9*a^2*c^3*cos(f*x + e)^4*log(-sin(f*x + e) + 1) - 2*(32*a^2*c
^3*cos(f*x + e)^3 - 15*a^2*c^3*cos(f*x + e)^2 - 8*a^2*c^3*cos(f*x + e) + 6
*a^2*c^3)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

3.3.6 Sympy [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$= -a^2c^3\left(\int (-1)\,dx + \int \sec(e+fx)\,dx + \int 2\sec^2(e+fx)\,dx + \int \left(-2\sec^3(e+fx)\right)\,dx + \int \left(-\sec^4(e+fx)\right)\,dx + \int \sec^5(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**3,x)
```

output
```
-a**2*c**3*(Integral(-1, x) + Integral(sec(e + f*x), x) + Integral(2*sec(e
 + f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) + Integral(-sec(e + f*x)*
*4, x) + Integral(sec(e + f*x)**5, x))
```

3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(91) = 182.

Time = 0.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.09

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$= \frac{16\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2c^3+48(fx+e)a^2c^3+3a^2c^3\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3 + 48*(f*x + e)*a^2*c^3
+ 3*a^2*c^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin
(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 24
*a^2*c^3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 48*a^2*c^3*log(sec(f*x + e) + tan(f*x + e)) - 96*a^
2*c^3*tan(f*x + e))/f
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$

$$= \frac{24\,(fx+e)a^2c^3 - 9\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) + 9\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(33\,a^2c^3\tan\left(\frac{1}{2}\right.\right.}{}}{24\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/24*(24*(f*x + e)*a^2*c^3 - 9*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))
+ 9*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(33*a^2*c^3*tan(1/2*f*x
 + 1/2*e)^7 - 137*a^2*c^3*tan(1/2*f*x + 1/2*e)^5 + 71*a^2*c^3*tan(1/2*f*x
+ 1/2*e)^3 - 15*a^2*c^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)
^4)/f
```

3.3.9 Mupad [B] (verification not implemented)

Time = 15.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.68

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$

$$= \frac{\frac{11\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4} - \frac{137\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12} + \frac{71\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12} - \frac{5\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8 - 4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6 + 6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 - 4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + 1\right)}$$
$$+ a^2\,c^3\,x - \frac{3\,a^2\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

input
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3,x)
```

output
```
((71*a^2*c^3*tan(e/2 + (f*x)/2)^3)/12 - (137*a^2*c^3*tan(e/2 + (f*x)/2)^5)
/12 + (11*a^2*c^3*tan(e/2 + (f*x)/2)^7)/4 - (5*a^2*c^3*tan(e/2 + (f*x)/2))
/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x
)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) + a^2*c^3*x - (3*a^2*c^3*atanh(tan(e/2
 + (f*x)/2)))/(4*f)
```

3.3. $\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^3\,dx$

3.4 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$

3.4.1 Optimal result

Integrand size = 26, antiderivative size = 47

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = a^2c^2x - \frac{a^2c^2\tan(e+fx)}{f} + \frac{a^2c^2\tan^3(e+fx)}{3f}$$

output
```
a^2*c^2*x-a^2*c^2*tan(f*x+e)/f+1/3*a^2*c^2*tan(f*x+e)^3/f
```

3.4.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = a^2c^2\left(\frac{\arctan(\tan(e+fx))}{f} - \frac{\tan(e+fx)}{f} + \frac{\tan^3(e+fx)}{3f}\right)$$

input
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

output
```
a^2*c^2*(ArcTan[Tan[e + f*x]]/f - Tan[e + f*x]/f + Tan[e + f*x]^3/(3*f))
```

3.4.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 4392, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(c-c\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int \tan^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \tan(e+fx)^4dx$$

$$\Big\downarrow \text{3954}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)}{3f}-\int \tan^2(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)}{3f}-\int \tan(e+fx)^2dx\right)$$

$$\Big\downarrow \text{3954}$$

$$a^2c^2\left(\int 1dx+\frac{\tan^3(e+fx)}{3f}-\frac{\tan(e+fx)}{f}\right)$$

$$\Big\downarrow \text{24}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)}{3f}-\frac{\tan(e+fx)}{f}+x\right)$$

input
```
Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

output
```
a^2*c^2*(x - Tan[e + f*x]/f + Tan[e + f*x]^3/(3*f))
```

3.4.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.4.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

method	result
parts	$a^{2} c^{2} x -\frac{a^{2} c^{2} \left(-\frac{2}{3}-\frac{\sec \left(f x +e \right)^{2}}{3}\right) \tan \left(f x +e \right)}{f}-\frac{2 a^{2} c^{2} \tan \left(f x +e \right)}{f}$
derivativedivides	$\frac{-a^{2} c^{2} \left(-\frac{2}{3}-\frac{\sec \left(f x +e \right)^{2}}{3}\right) \tan \left(f x +e \right)-2 a^{2} c^{2} \tan \left(f x +e \right)+a^{2} c^{2} \left(f x +e \right)}{f}$
default	$\frac{-a^{2} c^{2} \left(-\frac{2}{3}-\frac{\sec \left(f x +e \right)^{2}}{3}\right) \tan \left(f x +e \right)-2 a^{2} c^{2} \tan \left(f x +e \right)+a^{2} c^{2} \left(f x +e \right)}{f}$
risch	$a^{2} c^{2} x -\frac{4 i a^{2} c^{2} \left(3 \,{\mathrm e}^{4 i \left(f x +e \right)}+3 \,{\mathrm e}^{2 i \left(f x +e \right)}+2\right)}{3 f \left(1+{\mathrm e}^{2 i \left(f x +e \right)}\right)^{3}}$
parallelrisch	$\frac{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6} x f -3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4} x f +2 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}+3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2} x f -\frac{20 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}-f x +2 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right) a^{2} c}{f \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{3} \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}$
norman	$\frac{a^{2} c^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}-a^{2} c^{2} x +\frac{2 a^{2} c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{f}-\frac{20 a^{2} c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 f}+\frac{2 a^{2} c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{f}+3 a^{2} c^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-3 a^{2}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^2*x-a^2*c^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a^2*c^2*tan(f*x+e
)/f
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{3\,a^2c^2fx\cos{(fx+e)}^3-\left(4\,a^2c^2\cos{(fx+e)}^2-a^2c^2\right)\sin{(fx+e)}}{3\,f\cos{(fx+e)}^3}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*c^2*f*x*cos(f*x + e)^3 - (4*a^2*c^2*cos(f*x + e)^2 - a^2*c^2)*s
in(f*x + e))/(f*cos(f*x + e)^3)
```

3.4.6 Sympy [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = a^2c^2\left(\int 1\,dx+\int \left(-2\sec^2{(e+fx)}\right)\,dx + \int \sec^4{(e+fx)}\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**2,x)
```

output
```
a**2*c**2*(Integral(1, x) + Integral(-2*sec(e + f*x)**2, x) + Integral(sec
(e + f*x)**4, x))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2c^2+3\,(fx+e)a^2c^2-6\,a^2c^2\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^2 + 3*(f*x + e)*a^2*c^2 - 6*a
^2*c^2*tan(f*x + e))/f
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{a^2c^2\tan\left(fx+e\right)^3+3\,(fx+e)a^2c^2-3\,a^2c^2\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(a^2*c^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2*c^2 - 3*a^2*c^2*tan(f*x + e)
)/f
```

3.4.9 Mupad [B] (verification not implemented)

Time = 16.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.79

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = a^2\,c^2\,x+\frac{2\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{20\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+2\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^3}$$

input
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2,x)
```

output
```
a^2*c^2*x + (2*a^2*c^2*tan(e/2 + (f*x)/2)^5 - (20*a^2*c^2*tan(e/2 + (f*x)/
2)^3)/3 + 2*a^2*c^2*tan(e/2 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^2 - 1)^3)
```

3.4. $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$

3.5 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

3.5.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = a^2cx+\frac{a^2c\,\text{arctanh}(\sin(e+fx))}{2f}-\frac{c(2a^2+a^2\sec(e+fx))\tan(e+fx)}{2f}$$

output
```
a^2*c*x+1/2*a^2*c*arctanh(sin(f*x+e))/f-1/2*c*(2*a^2+a^2*sec(f*x+e))*tan(f
*x+e)/f
```

3.5.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.76

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \frac{a^2c\sec^2(e+fx)\,(e+fx+6\arctan(\tan(e+fx))\cos^2(e+fx)+4\text{arctanh}(\sin(e+fx))\cos^2(e+fx)+e}{8f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

output
```
(a^2*c*Sec[e + f*x]^2*(e + f*x + 6*ArcTan[Tan[e + f*x]]*Cos[e + f*x]^2 + 4
*ArcTanh[Sin[e + f*x]]*Cos[e + f*x]^2 + e*Cos[2*(e + f*x)] + f*x*Cos[2*(e
+ f*x)] - 4*Sin[e + f*x] - 4*Sin[2*(e + f*x)]))/(8*f)
```

3.5.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 4392, 3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4392}$$

$$-ac\int (\sec(e+fx)a+a)\tan^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int \cot\left(e+fx+\frac{\pi}{2}\right)^2\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)dx$$

$$\Big\downarrow \text{4369}$$

$$-ac\left(\frac{\tan(e+fx)(a\sec(e+fx)+2a)}{2f}-\frac{1}{2}\int(\sec(e+fx)a+2a)dx\right)$$

$$\Big\downarrow \text{2009}$$

$$-ac\left(\frac{1}{2}\left(-\frac{a\text{arctanh}(\sin(e+fx))}{f}-2ax\right)+\frac{\tan(e+fx)(a\sec(e+fx)+2a)}{2f}\right)$$

input
```
Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

output
```
-(a*c*((-2*a*x - (a*ArcTanh[Sin[e + f*x]])/f)/2 + ((2*a + a*Sec[e + f*x])*
Tan[e + f*x])/(2*f)))
```

3.5.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.5.4 Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a^2c\tan(fx+e)+a^2c\ln(\sec(fx+e)+\tan(fx+e))+a^2c(fx+e)}{f}$
default	$\frac{-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a^2c\tan(fx+e)+a^2c\ln(\sec(fx+e)+\tan(fx+e))+a^2c(fx+e)}{f}$
parts	$a^2cx+\frac{a^2c\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{a^2c\tan(fx+e)}{f}-\frac{a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
risch	$a^2cx+\frac{ia^2c\left(e^{3i(fx+e)}-2e^{2i(fx+e)}-e^{i(fx+e)}-2\right)}{f\left(1+e^{2i(fx+e)}\right)^2}+\frac{a^2c\ln\left(e^{i(fx+e)}+i\right)}{2f}-\frac{a^2c\ln\left(e^{i(fx+e)}-i\right)}{2f}$
parallelrisch	$-\frac{\left((1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+(-1-\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-2fx\cos(2fx+2e)-2fx+2\sin(f\right.}{2f(1+\cos(2fx+2e))}$
norman	$\frac{a^2cx+a^2cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}-2a^2cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{3a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}-\frac{a^2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a^2*c*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-a^2*
c*tan(f*x+e)+a^2*c*ln(sec(f*x+e)+tan(f*x+e))+a^2*c*(f*x+e))
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(51) = 102$.

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.87

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))\,dx = \frac{4\,a^2cfx\cos\left(fx+e\right)^2 + a^2c\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right) - a^2c\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right.}{4\,f\cos\left(fx+e\right)^2}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*(4*a^2*c*f*x*cos(f*x + e)^2 + a^2*c*cos(f*x + e)^2*log(sin(f*x + e) +
1) - a^2*c*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*(2*a^2*c*cos(f*x + e)
 + a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^2)
```

3.5.6 Sympy [F]

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))\,dx = -a^2c\left(\int (-1)\,dx + \int \left(-\sec\left(e+fx\right)\right)\,dx + \int \sec^2\left(e+fx\right)dx + \int \sec^3\left(e+fx\right)dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

output
```
-a**2*c*(Integral(-1, x) + Integral(-sec(e + f*x), x) + Integral(sec(e + f
*x)**2, x) + Integral(sec(e + f*x)**3, x))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= \frac{4\,(fx+e)a^2c+a^2c\Big(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\Big)+4\,a^2c\log\left(\sec\left(fx+e\right.\right.}{4\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*(4*(f*x + e)*a^2*c + a^2*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(
sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 4*a^2*c*log(sec(f*x + e) + ta
n(f*x + e)) - 4*a^2*c*tan(f*x + e))/f
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(51) = 102.

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.87

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= \frac{2\,(fx+e)a^2c+a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.\right.}}{2\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
1/2*(2*(f*x + e)*a^2*c + a^2*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - a^2*c*
log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(a^2*c*tan(1/2*f*x + 1/2*e)^3 - 3*a
^2*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^2)/f
```

3.5.9 Mupad [B] (verification not implemented)

Time = 14.43 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.65

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= a^2\,c\,x - \frac{3\,a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right) - a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 - 2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + 1\right)} + \frac{a^2\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}$$

input
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x)),x)
```

output
```
a^2*c*x - (3*a^2*c*tan(e/2 + (f*x)/2) - a^2*c*tan(e/2 + (f*x)/2)^3)/(f*(ta
n(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1)) + (a^2*c*atanh(tan(e/2 +
 (f*x)/2)))/f
```

3.6 $\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$

3.6.1 Optimal result

Integrand size = 26, antiderivative size = 56

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx=\frac{a^2x}{c}-\frac{a^2\operatorname{arctanh}(\sin(e+fx))}{cf}-\frac{4a^2\tan(e+fx)}{cf(1-\sec(e+fx))}$$

output
```
a^2*x/c-a^2*arctanh(sin(f*x+e))/c/f-4*a^2*tan(f*x+e)/c/f/(1-sec(f*x+e))
```

3.6.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(56) = 112.

Time = 1.61 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.59

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx =$$
$$-\frac{a^{3/2}\tan(e+fx)\left(4\sqrt{c}\left(\sqrt{a}\sqrt{1-\sec(e+fx)}(1+\sec(e+fx))+\arcsin\left(\frac{\sqrt{a(1+\sec(e+fx))}}{\sqrt{2}\sqrt{a}}\right)\sec(e+fx)\sqrt{\ldots}\right.\right.}{c^{3/2}f(1-\sec(e+f\ldots}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x]),x]
```

output
```
-((a^(3/2)*Tan[e + f*x]*(4*Sqrt[c]*(Sqrt[a]*Sqrt[1 - Sec[e + f*x]]*(1 + Se
c[e + f*x]) + ArcSin[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqrt[a])]*Sec[e +
 f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sin[(e + f*x)/2]^2) - ArcTanh[Sqrt[-(a*c*
Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[1 - Sec[e + f*x]]*Sqrt[-(a*c*Tan[
e + f*x]^2)]))/(c^(3/2)*f*(1 - Sec[e + f*x])^(3/2)*(1 + Sec[e + f*x])))
```

3.6.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^2}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^2(e+fx)a^2}{1-\sec(e+fx)}+\frac{2\sec(e+fx)a^2}{1-\sec(e+fx)}+\frac{a^2}{1-\sec(e+fx)}\right)dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2\text{arctanh}(\sin(e+fx))}{f}-\frac{4a^2\tan(e+fx)}{f(1-\sec(e+fx))}+a^2x}{c}$$

input
```
Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x]),x]
```

output
```
(a^2*x - (a^2*ArcTanh[Sin[e + f*x]])/f - (4*a^2*Tan[e + f*x])/(f*(1 - Sec[
e + f*x])))/c
```

3.6.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.6.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{4a^2\left(\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{fc}$
default	$\frac{4a^2\left(\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{fc}$
parallelrisch	$\frac{a^2\left(4+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)fx+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fc\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$
risch	$\frac{a^2x}{c}+\frac{8ia^2}{fc(\mathrm{e}^{i(fx+e)}-1)}+\frac{a^2\ln(\mathrm{e}^{i(fx+e)}-i)}{cf}-\frac{a^2\ln(\mathrm{e}^{i(fx+e)}+i)}{cf}$
norman	$\frac{\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{c}-\frac{4a^2}{cf}+\frac{4a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{cf}-\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{cf}-\frac{a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{cf}$

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
4/f*a^2/c*(1/tan(1/2*f*x+1/2*e)+1/4*ln(tan(1/2*f*x+1/2*e)-1)+1/2*arctan(ta
n(1/2*f*x+1/2*e))-1/4*ln(tan(1/2*f*x+1/2*e)+1))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$$
$$=\frac{2\,a^2fx\sin\left(fx+e\right)-a^2\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)+a^2\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)+8\,a}{2\,cf\sin\left(fx+e\right)}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(2*a^2*f*x*sin(f*x + e) - a^2*log(sin(f*x + e) + 1)*sin(f*x + e) + a^2
*log(-sin(f*x + e) + 1)*sin(f*x + e) + 8*a^2*cos(f*x + e) + 8*a^2)/(c*f*si
n(f*x + e))
```

3.6.6 Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^2}{c - c\sec(e + fx)}\,dx = -\frac{a^2\left(\int \frac{2\sec(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{\sec^2(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{1}{\sec(e+fx)-1}\,dx\right)}{c}$$

input
```
integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

output
```
-a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(sec(e + f
*x)**2/(sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x) - 1), x))/c
```

3.6.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(54) = 108$.

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.73

$$\int \frac{(a + a\sec(e + fx))^2}{c - c\sec(e + fx)}\,dx$$
$$= \frac{a^2\left(\frac{2\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c} + \frac{\cos(fx+e)+1}{c\sin(fx+e)}\right) - a^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c} - \frac{\cos(fx+e)+1}{c\sin(fx+e)}\right) + \frac{2a^2(\cos(fx}{c\sin(fx}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
(a^2*(2*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c + (cos(f*x + e) + 1)/(c*
sin(f*x + e))) - a^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin
(f*x + e)/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e)))
 + 2*a^2*(cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$$
$$=\frac{\frac{(fx+e)a^2}{c}-\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c}+\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c}+\frac{4\,a^2}{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
((f*x + e)*a^2/c - a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c + a^2*log(abs(
tan(1/2*f*x + 1/2*e) - 1))/c + 4*a^2/(c*tan(1/2*f*x + 1/2*e)))/f
```

3.6.9 Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx=\frac{a^2\,x}{c}-\frac{a^2\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{4}{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{c\,f}$$

input
```
int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x)),x)
```

output
```
(a^2*x)/c - (a^2*(2*atanh(tan(e/2 + (f*x)/2)) - 4/tan(e/2 + (f*x)/2)))/(c*
f)
```

3.7 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx$

3.7.1 Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx=\frac{a^2x}{c^2}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))}$$

output
```
a^2*x/c^2-4/3*a^2*tan(f*x+e)/c^2/f/(1-sec(f*x+e))^2-4/3*a^2*tan(f*x+e)/c^2
/f/(1-sec(f*x+e))
```

3.7.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = -\frac{2a^2\cot^3\left(\frac{e}{2}+\frac{fx}{2}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3c^2f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a^2*Cot[e/2 + (f*x)/2]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e/2 + (
f*x)/2]^2])/(3*c^2*f)
```

3.7.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^2(e+fx)a^2}{(1-\sec(e+fx))^2}+\frac{2\sec(e+fx)a^2}{(1-\sec(e+fx))^2}+\frac{a^2}{(1-\sec(e+fx))^2}\right)dx}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{4a^2\tan(e+fx)}{3f(1-\sec(e+fx))}-\frac{4a^2\tan(e+fx)}{3f(1-\sec(e+fx))^2}+a^2x}{c^2}$$

input
```
Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^2,x]
```

output
```
(a^2*x - (4*a^2*Tan[e + f*x])/(3*f*(1 - Sec[e + f*x])^2) - (4*a^2*Tan[e +
f*x])/(3*f*(1 - Sec[e + f*x])))/c^2
```

3.7.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.7.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.58

method	result	size
parallelrisch	$-\frac{a^{2}\left(2 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}-3 f x-6 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{3 c^{2} f}$	41
derivativedivides	$\frac{2 a^{2}\left(\arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)-\frac{1}{3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)}\right)}{f\, c^{2}}$	47
default	$\frac{2 a^{2}\left(\arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)-\frac{1}{3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)}\right)}{f\, c^{2}}$	47
risch	$\frac{a^{2} x}{c^{2}}+\frac{8 i a^{2}\left(3 \,{\mathrm e}^{2 i \left(f x +e \right)}-3 \,{\mathrm e}^{i \left(f x +e \right)}+2\right)}{3 f \,c^{2} \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{3}}$	59
norman	$\frac{\frac{a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{c}+\frac{2 a^{2}}{3 c f}-\frac{8 a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{3 c f}+\frac{2 a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{c f}-\frac{a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{c}}{c \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right) \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}$	126

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2*(2*cot(1/2*f*x+1/2*e)^3-3*f*x-6*cot(1/2*f*x+1/2*e))/c^2/f
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{8\,a^{2}\cos\left(fx+e\right)^{2}+4\,a^{2}\cos\left(fx+e\right)-4\,a^{2}+3\,\left(a^{2}fx\cos\left(fx+e\right)-a^{2}fx\right)\sin\left(fx+e\right)}{3\,\left(c^{2}f\cos\left(fx+e\right)-c^{2}f\right)\sin\left(fx+e\right)}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(8*a^2*cos(f*x + e)^2 + 4*a^2*cos(f*x + e) - 4*a^2 + 3*(a^2*f*x*cos(f*
x + e) - a^2*f*x)*sin(f*x + e))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)
)
```

3.7.6 Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^2}\,dx$$
$$= \frac{a^2\left(\int \frac{2\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{\sec^2(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{1}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx\right)}{c^2}$$

input
```
integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

output
```
a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) +
 Integral(sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Int
egral(1/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

3.7.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(63) = 126$.

Time = 0.30 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.45

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^2}\,dx$$
$$= \frac{a^2\left(\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2} + \frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right) - \frac{a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3} + \frac{2a^2\left(\frac{3\sin(fx+e)}{(\cos(fx+e)+1}\right.}{c^2\,\mathrm{si}}}{6f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/6*(a^2*(12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2 + (9*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) -
a^2*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*
sin(f*x + e)^3) + 2*a^2*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f
*x + e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{\frac{3\,(fx+e)a^2}{c^2} + \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a^2\right)}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{3\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(3*(f*x + e)*a^2/c^2 + 2*(3*a^2*tan(1/2*f*x + 1/2*e)^2 - a^2)/(c^2*tan
(1/2*f*x + 1/2*e)^3))/f
```

3.7.9 Mupad [B] (verification not implemented)

Time = 14.56 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{a^2\left(-2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^3+6\cot\left(\frac{e}{2}+\frac{fx}{2}\right)+3\,f\,x\right)}{3\,c^2\,f}$$

input
```
int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^2,x)
```

output
```
(a^2*(6*cot(e/2 + (f*x)/2) - 2*cot(e/2 + (f*x)/2)^3 + 3*f*x))/(3*c^2*f)
```

3.8 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$

3.8.1 Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{a^2x}{c^3} - \frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} - \frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2} - \frac{23a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))}$$

output
```
a^2*x/c^3-4/5*a^2*tan(f*x+e)/c^3/f/(1-sec(f*x+e))^3-8/15*a^2*tan(f*x+e)/c^
3/f/(1-sec(f*x+e))^2-23/15*a^2*tan(f*x+e)/c^3/f/(1-sec(f*x+e))
```

3.8.2 Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.48

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$$
$$= \frac{a^{3/2}\tan(e+fx)\left(\sqrt{a}\sqrt{c}(43-11\sec(e+fx)-31\sec^2(e+fx)+23\sec^3(e+fx))-60\operatorname{arctanh}\left(\frac{\sqrt{-ac\tan}}{\sqrt{a}\sqrt{}}\right.\right.}{15c^{7/2}f(-1+\sec(e+fx))^3(1+\sec(e+fx))}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^3,x]
```

output
```
(a^(3/2)*Tan[e + f*x]*(Sqrt[a]*Sqrt[c]*(43 - 11*Sec[e + f*x] - 31*Sec[e +
f*x]^2 + 23*Sec[e + f*x]^3) - 60*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt
[a]*Sqrt[c])]*Sec[e + f*x]^2*Sin[(e + f*x)/2]^4*Sqrt[-(a*c*Tan[e + f*x]^2)
]))/(15*c^(7/2)*f*(-1 + Sec[e + f*x])^3*(1 + Sec[e + f*x]))
```

3.8.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^2(e+fx)a^2}{(1-\sec(e+fx))^3}+\frac{2\sec(e+fx)a^2}{(1-\sec(e+fx))^3}+\frac{a^2}{(1-\sec(e+fx))^3}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{23a^2\tan(e+fx)}{15f(1-\sec(e+fx))}-\frac{8a^2\tan(e+fx)}{15f(1-\sec(e+fx))^2}-\frac{4a^2\tan(e+fx)}{5f(1-\sec(e+fx))^3}+a^2x}{c^3}$$

input
```
Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^3,x]
```

output
```
(a^2*x - (4*a^2*Tan[e + f*x])/(5*f*(1 - Sec[e + f*x])^3) - (8*a^2*Tan[e +
f*x])/(15*f*(1 - Sec[e + f*x])^2) - (23*a^2*Tan[e + f*x])/(15*f*(1 - Sec[e
 + f*x])))/c^3
```

3.8.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.8.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.53

method	result	size
parallelrisch	$\frac{a^{2} \left(3 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}-10 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+15 f x +30 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{15 c^{3} f}$	54
derivativedivides	$\frac{a^{2} \left(\frac{1}{5 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{2}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)}+2 \arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f \,c^{3}}$	63
default	$\frac{a^{2} \left(\frac{1}{5 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{2}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)}+2 \arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f \,c^{3}}$	63
risch	$\frac{a^{2} x}{c^{3}}+\frac{2 i a^{2} \left(75 \,{\mathrm e}^{4 i \left(f x +e \right)}-180 \,{\mathrm e}^{3 i \left(f x +e \right)}+250 \,{\mathrm e}^{2 i \left(f x +e \right)}-140 \,{\mathrm e}^{i \left(f x +e \right)}+43\right)}{15 f \,c^{3} \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{5}}$	81
norman	$\frac{\frac{a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}{c}-\frac{a^{2}}{5 c f}+\frac{13 a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{15 c f}-\frac{8 a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{3 c f}+\frac{2 a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{c f}-\frac{a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{c}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right) c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}$	148

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*a^2*(3*cot(1/2*f*x+1/2*e)^5-10*cot(1/2*f*x+1/2*e)^3+15*f*x+30*cot(1/2
*f*x+1/2*e))/c^3/f
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^3}\,dx$$
$$= \frac{43\,a^2\cos\left(fx+e\right)^3 - 11\,a^2\cos\left(fx+e\right)^2 - 31\,a^2\cos\left(fx+e\right) + 23\,a^2 + 15\left(a^2fx\cos\left(fx+e\right)^2 - 2\,a^2fx\cos \cdots\right.}{15\left(c^3f\cos\left(fx+e\right)^2 - 2\,c^3f\cos\left(fx+e\right) + c^3f\right)\sin\left(fx+e\right)}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(43*a^2*cos(f*x + e)^3 - 11*a^2*cos(f*x + e)^2 - 31*a^2*cos(f*x + e)
+ 23*a^2 + 15*(a^2*f*x*cos(f*x + e)^2 - 2*a^2*f*x*cos(f*x + e) + a^2*f*x)*
sin(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f
*x + e))
```

3.8.6 Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^3}\,dx =$$
$$-\frac{a^2\left(\int \frac{2\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx + \int \frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx + \int \frac{\cdots}{\sec^3\left(e+fx\right)-3\sec^2\left(\cdots\right.}\cdots\right.}{c^3}$$

input
```
integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

output
```
-a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*se
c(e + f*x) - 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**3 - 3*sec(
e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(90) = 180$.

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.11

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$$
$$= \frac{a^2\left(\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^3} - \frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right) - \frac{2a^2\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 3\right)(\mathrm{co}}{c^3\sin(fx+e)^5}}{60f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/60*(a^2*(120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3 - (20*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*
(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5)) - 2*a^2*(10*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x +
e) + 1)^5/(c^3*sin(f*x + e)^5) - 3*a^2*(5*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{\frac{15(fx+e)a^2}{c^3} + \frac{30a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4 - 10a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 3a^2}{c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5}}{15f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/15*(15*(f*x + e)*a^2/c^3 + (30*a^2*tan(1/2*f*x + 1/2*e)^4 - 10*a^2*tan(1
/2*f*x + 1/2*e)^2 + 3*a^2)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

3.8.9 Mupad [B] (verification not implemented)

Time = 14.43 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$$
$$=\frac{a^2\,x}{c^3}+\frac{\frac{a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}-\frac{2\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+2\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{c^3\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

input
```
int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^3,x)
```

output
```
(a^2*x)/c^3 + ((a^2*cos(e/2 + (f*x)/2)^5)/5 + 2*a^2*cos(e/2 + (f*x)/2)*sin
(e/2 + (f*x)/2)^4 - (2*a^2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^2)/3)/(
c^3*f*sin(e/2 + (f*x)/2)^5)
```

3.9 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx$

3.9.1 Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{a^2x}{c^4} - \frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4} - \frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3} - \frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2} - \frac{164a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))}$$

output
```
a^2*x/c^4-4/7*a^2*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^4-12/35*a^2*tan(f*x+e)/c
^4/f/(1-sec(f*x+e))^3-59/105*a^2*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^2-164/105
*a^2*tan(f*x+e)/c^4/f/(1-sec(f*x+e))
```

3.9.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.83 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = -\frac{a^2\csc^7(e+fx)\,(7032+18165\cos(e+fx)+19348\cos(2(e+fx))+9303\cos(3(e+fx))+3080\cos(4}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^4,x]
```

output
```
-1/6720*(a^2*Csc[e + f*x]^7*(7032 + 18165*Cos[e + f*x] + 19348*Cos[2*(e +
f*x)] + 9303*Cos[3*(e + f*x)] + 3080*Cos[4*(e + f*x)] + 2149*Cos[5*(e + f*
x)] + 1260*Cos[6*(e + f*x)] + 143*Cos[7*(e + f*x)] + 960*Cos[e + f*x]^7*Hy
pergeometric2F1[-7/2, 1, -5/2, -Tan[e + f*x]^2]))/(c^4*f)
```

3.9.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^2(e+fx)a^2}{(1-\sec(e+fx))^4}+\frac{2\sec(e+fx)a^2}{(1-\sec(e+fx))^4}+\frac{a^2}{(1-\sec(e+fx))^4}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{164a^2\tan(e+fx)}{105f(1-\sec(e+fx))}-\frac{59a^2\tan(e+fx)}{105f(1-\sec(e+fx))^2}-\frac{12a^2\tan(e+fx)}{35f(1-\sec(e+fx))^3}-\frac{4a^2\tan(e+fx)}{7f(1-\sec(e+fx))^4}+a^2x}{c^4}$$

input
```
Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^4,x]
```

output
```
(a^2*x - (4*a^2*Tan[e + f*x])/(7*f*(1 - Sec[e + f*x])^4) - (12*a^2*Tan[e +
 f*x])/(35*f*(1 - Sec[e + f*x])^3) - (59*a^2*Tan[e + f*x])/(105*f*(1 - Sec
[e + f*x])^2) - (164*a^2*Tan[e + f*x])/(105*f*(1 - Sec[e + f*x])))/c^4
```

3.9.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.9.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50

method	result
parallelrisch	$-\frac{a^{2}\left(15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}-63\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+140\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-210fx-420\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{210c^{4}f}$
derivativedivides	$\frac{a^{2}\left(4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2f\,c^{4}}$
default	$\frac{a^{2}\left(4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2f\,c^{4}}$
risch	$\frac{a^{2}x}{c^{4}}+\frac{2ia^{2}\left(630\,e^{6i(fx+e)}-2415\,e^{5i(fx+e)}+5215\,e^{4i(fx+e)}-5950\,e^{3i(fx+e)}+4284\,e^{2i(fx+e)}-1603\,e^{i(fx+e)}+319\right)}{105f\,c^{4}\left(e^{i(fx+e)}-1\right)^{7}}$
norman	$\frac{\frac{a^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{c}+\frac{a^{2}}{14cf}-\frac{13a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{35cf}+\frac{29a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{30cf}-\frac{8a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{3cf}+\frac{2a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{cf}-\frac{a^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/210*a^2*(15*cot(1/2*f*x+1/2*e)^7-63*cot(1/2*f*x+1/2*e)^5+140*cot(1/2*f*
x+1/2*e)^3-210*f*x-420*cot(1/2*f*x+1/2*e))/c^4/f
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{319\,a^2\cos\left(fx+e\right)^4-327\,a^2\cos\left(fx+e\right)^3-95\,a^2\cos\left(fx+e\right)^2+387\,a^2\cos\left(fx+e\right)-164\,a^2+105\,\left(a^2 \ldots\right.}{105\,\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos \ldots\right.}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
1/105*(319*a^2*cos(f*x + e)^4 - 327*a^2*cos(f*x + e)^3 - 95*a^2*cos(f*x +
e)^2 + 387*a^2*cos(f*x + e) - 164*a^2 + 105*(a^2*f*x*cos(f*x + e)^3 - 3*a^
2*f*x*cos(f*x + e)^2 + 3*a^2*f*x*cos(f*x + e) - a^2*f*x)*sin(f*x + e))/((c
^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*
f)*sin(f*x + e))
```

3.9.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{a^2\left(\int \frac{2\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx \ldots\right.}{c^4}$$

input
```
integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

output
```
a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec
(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e +
 f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)
 + Integral(1/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4
*sec(e + f*x) + 1), x))/c**4
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(117) = 234$.

Time = 0.30 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.21

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^4}\,dx$$
$$= \frac{5\,a^2\left(\frac{336\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^4} + \frac{\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{77\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{315\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - 3\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}\right) + \frac{a^2\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{35\,\sin}{(\cos(}\right.}{}}{840\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
1/840*(5*a^2*(336*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^4 + (21*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 77*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3
15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(cos(f*x + e) + 1)^7/(c^4*sin(
f*x + e)^7)) + a^2*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*
(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + 6*a^2*(21*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^4}\,dx$$
$$= \frac{\frac{210\,(fx+e)a^2}{c^4} + \frac{420\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6 - 140\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 63\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 15\,a^2}{c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}}{210\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/210*(210*(f*x + e)*a^2/c^4 + (420*a^2*tan(1/2*f*x + 1/2*e)^6 - 140*a^2*t
an(1/2*f*x + 1/2*e)^4 + 63*a^2*tan(1/2*f*x + 1/2*e)^2 - 15*a^2)/(c^4*tan(1
/2*f*x + 1/2*e)^7))/f
```

3.9.9 Mupad [B] (verification not implemented)

Time = 14.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{a^2\,x}{c^4} - \frac{\frac{a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{14} - \frac{3\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{10} + \frac{2\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{3} - 2\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{c^4\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}$$

input
```
int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^4,x)
```

output
```
(a^2*x)/c^4 - ((a^2*cos(e/2 + (f*x)/2)^7)/14 - 2*a^2*cos(e/2 + (f*x)/2)*si
n(e/2 + (f*x)/2)^6 + (2*a^2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^4)/3 -
 (3*a^2*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^2)/10)/(c^4*f*sin(e/2 + (f
*x)/2)^7)
```

3.10 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$

3.10.1 Optimal result

Integrand size = 26, antiderivative size = 164

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{a^2x}{c^5} - \frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} - \frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} - \frac{179a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2} - \frac{494a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))}$$

output
```
a^2*x/c^5-4/9*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^5-16/63*a^2*tan(f*x+e)/c
^5/f/(1-sec(f*x+e))^4-37/105*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^3-179/315
*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^2-494/315*a^2*tan(f*x+e)/c^5/f/(1-sec
(f*x+e))
```

3.10.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.87 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.68

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{a^2\cot^9(e+fx)\left(441+35\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(e+fx)\right)+2205\sec(e+fx)+1323\sec\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^5,x]
```

output
```
(a^2*Cot[e + f*x]^9*(441 + 35*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*
x]^2] + 2205*Sec[e + f*x] + 1323*Sec[e + f*x]^2 - 2205*Sec[e + f*x]^3 + 44
1*Sec[e + f*x]^4 + 3969*Sec[e + f*x]^5 - 2223*Sec[e + f*x]^7 + 494*Sec[e +
 f*x]^9))/(315*c^5*f)
```

3.10.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^2(e+fx)a^2}{(1-\sec(e+fx))^5}+\frac{2\sec(e+fx)a^2}{(1-\sec(e+fx))^5}+\frac{a^2}{(1-\sec(e+fx))^5}\right)dx}{c^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{494a^2\tan(e+fx)}{315f(1-\sec(e+fx))}-\frac{179a^2\tan(e+fx)}{315f(1-\sec(e+fx))^2}-\frac{37a^2\tan(e+fx)}{105f(1-\sec(e+fx))^3}-\frac{16a^2\tan(e+fx)}{63f(1-\sec(e+fx))^4}-\frac{4a^2\tan(e+fx)}{9f(1-\sec(e+fx))^5}+a^2x}{c^5}$$

input
```
Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^5,x]
```

output
```
(a^2*x - (4*a^2*Tan[e + f*x])/(9*f*(1 - Sec[e + f*x])^5) - (16*a^2*Tan[e +
 f*x])/(63*f*(1 - Sec[e + f*x])^4) - (37*a^2*Tan[e + f*x])/(105*f*(1 - Sec
[e + f*x])^3) - (179*a^2*Tan[e + f*x])/(315*f*(1 - Sec[e + f*x])^2) - (494
*a^2*Tan[e + f*x])/(315*f*(1 - Sec[e + f*x])))/c^5
```

3.10.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.10.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

method	result
parallelrisch	$\frac{a^2\left(35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^9-180\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^7+441\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5-840\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+1260fx+2520\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{1260c^5f}$
derivativedivides	$\frac{a^2\left(8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{4}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{8}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{4f\,c^5}$
default	$\frac{a^2\left(8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{4}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{8}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{4f\,c^5}$
risch	$\frac{a^2x}{c^5}+\frac{2ia^2\left(2205\,e^{8i(fx+e)}-11655\,e^{7i(fx+e)}+34335\,e^{6i(fx+e)}-58905\,e^{5i(fx+e)}+67599\,e^{4i(fx+e)}-50001\,e^{3i(fx+e)}+2\right.}{315f\,c^5\left(e^{i(fx+e)}-1\right)^9}$
norman	$\frac{\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{c}-\frac{a^2}{36cf}+\frac{43a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{252cf}-\frac{69a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{140cf}+\frac{61a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{60cf}-\frac{8a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{3cf}+\frac{2a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output
```
1/1260*a^2*(35*cot(1/2*f*x+1/2*e)^9-180*cot(1/2*f*x+1/2*e)^7+441*cot(1/2*f
*x+1/2*e)^5-840*cot(1/2*f*x+1/2*e)^3+1260*f*x+2520*cot(1/2*f*x+1/2*e))/c^5
/f
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.29

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{1004\,a^2\cos(fx+e)^5 - 1811\,a^2\cos(fx+e)^4 + 797\,a^2\cos(fx+e)^3 + 1457\,a^2\cos(fx+e)^2 - 1661\,a^2\cos}{315\left(c^5 f\cos(fx+e)^4 - 4\,c^5 f\cos(fx\right.}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
1/315*(1004*a^2*cos(f*x + e)^5 - 1811*a^2*cos(f*x + e)^4 + 797*a^2*cos(f*x
 + e)^3 + 1457*a^2*cos(f*x + e)^2 - 1661*a^2*cos(f*x + e) + 494*a^2 + 315*
(a^2*f*x*cos(f*x + e)^4 - 4*a^2*f*x*cos(f*x + e)^3 + 6*a^2*f*x*cos(f*x + e
)^2 - 4*a^2*f*x*cos(f*x + e) + a^2*f*x)*sin(f*x + e))/((c^5*f*cos(f*x + e)
^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e
) + c^5*f)*sin(f*x + e))
```

3.10.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx =$$
$$-\frac{a^2\left(\int \frac{2\sec(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx + \int \frac{\sec^2(e+fx}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)}\right.}{c^5}$$

input
```
integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

output
```
-a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*s
ec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(s
ec(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 -
 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**
5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e
+ f*x) - 1), x))/c**5
```

3.10.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(144) = 288.

Time = 0.30 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.04

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{a^2\left(\frac{10080\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^5} - \frac{\left(\frac{270\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{1008\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{2730\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{9765\sin(fx+e)^8}{(\cos(fx+e)+1)^8} - 35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}\right) - \frac{2a^2\left(\right.}{}}{}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
1/5040*(a^2*(10080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^5 - (270*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 1008*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + 2730*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9765*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)) - 2*a^2*(180
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x + e)^8/(cos(f
*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 5*a^2*(18
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(c^
5*sin(f*x + e)^9))/f
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.63

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{\frac{1260(fx+e)a^2}{c^5} + \frac{2520a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8 - 840a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6 + 441a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4 - 180a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 35a^2}{c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9}}{1260f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
1/1260*(1260*(f*x + e)*a^2/c^5 + (2520*a^2*tan(1/2*f*x + 1/2*e)^8 - 840*a^
2*tan(1/2*f*x + 1/2*e)^6 + 441*a^2*tan(1/2*f*x + 1/2*e)^4 - 180*a^2*tan(1/
2*f*x + 1/2*e)^2 + 35*a^2)/(c^5*tan(1/2*f*x + 1/2*e)^9))/f
```

3.10.9 Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.89

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{a^2\left(\frac{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{36}-\frac{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{7}+\frac{7\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{20}-\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{3}+2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right.\right.}{c^5\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}$$

input
```
int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^5,x)
```

output
```
(a^2*(cos(e/2 + (f*x)/2)^9/36 + 2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^8
+ sin(e/2 + (f*x)/2)^9*(e + f*x) - (2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)
/2)^6)/3 + (7*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^4)/20 - (cos(e/2 + (
f*x)/2)^7*sin(e/2 + (f*x)/2)^2)/7))/(c^5*f*sin(e/2 + (f*x)/2)^9)
```

3.11 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$

3.11.1 Optimal result

Integrand size = 26, antiderivative size = 188

$$\begin{aligned}\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx &= a^3c^5x-\frac{5a^3c^5\operatorname{arctanh}(\sin(e+fx))}{8f}-\frac{a^3c^5\tan(e+fx)}{f}\\ &+\frac{5a^3c^5\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^3c^5\tan^3(e+fx)}{3f}-\frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{12f}\\ &-\frac{a^3c^5\tan^5(e+fx)}{5f}+\frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{3f}-\frac{a^3c^5\tan^7(e+fx)}{7f}\end{aligned}$$

output
```
a^3*c^5*x-5/8*a^3*c^5*arctanh(sin(f*x+e))/f-a^3*c^5*tan(f*x+e)/f+5/8*a^3*c
^5*sec(f*x+e)*tan(f*x+e)/f+1/3*a^3*c^5*tan(f*x+e)^3/f-5/12*a^3*c^5*sec(f*x
+e)*tan(f*x+e)^3/f-1/5*a^3*c^5*tan(f*x+e)^5/f+1/3*a^3*c^5*sec(f*x+e)*tan(f
*x+e)^5/f-1/7*a^3*c^5*tan(f*x+e)^7/f
```

3.11.2 Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.01

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx = \frac{a^3c^5\sec^7(e+fx)\left(14700(e+fx)\cos(e+fx)-16800\operatorname{arctanh}(\sin(e+fx))\cos^7(e+fx)+8820e\cos(3(e\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

output
```
(a^3*c^5*Sec[e + f*x]^7*(14700*(e + f*x)*Cos[e + f*x] - 16800*ArcTanh[Sin[
e + f*x]]*Cos[e + f*x]^7 + 8820*e*Cos[3*(e + f*x)] + 8820*f*x*Cos[3*(e + f
*x)] + 2940*e*Cos[5*(e + f*x)] + 2940*f*x*Cos[5*(e + f*x)] + 420*e*Cos[7*(
e + f*x)] + 420*f*x*Cos[7*(e + f*x)] - 4200*Sin[e + f*x] + 2975*Sin[2*(e +
 f*x)] - 2184*Sin[3*(e + f*x)] + 980*Sin[4*(e + f*x)] - 2408*Sin[5*(e + f*
x)] + 1155*Sin[6*(e + f*x)] - 584*Sin[7*(e + f*x)]))/(26880*f)
```

3.11.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3(c-c\sec(e+fx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5dx$$

$$\Big\downarrow \text{4392}$$

$$-a^3c^3\int (c-c\sec(e+fx))^2\tan^6(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \cot\left(e+fx+\frac{\pi}{2}\right)^6\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4374}$$

$$-a^3c^3\int \left(c^2\tan^6(e+fx)+c^2\sec^2(e+fx)\tan^6(e+fx)-2c^2\sec(e+fx)\tan^6(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-a^3c^3\left(\frac{5c^2\text{arctanh}(\sin(e+fx))}{8f}+\frac{c^2\tan^7(e+fx)}{7f}+\frac{c^2\tan^5(e+fx)}{5f}-\frac{c^2\tan^3(e+fx)}{3f}+\frac{c^2\tan(e+fx)}{f}-\frac{c^2 \text{t}}{}\right.$$

input
```
Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

output
```
-(a^3*c^3*(-(c^2*x) + (5*c^2*ArcTanh[Sin[e + f*x]])/(8*f) + (c^2*Tan[e + f
*x])/f - (5*c^2*Sec[e + f*x]*Tan[e + f*x])/(8*f) - (c^2*Tan[e + f*x]^3)/(3
*f) + (5*c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(12*f) + (c^2*Tan[e + f*x]^5)/(5
*f) - (c^2*Sec[e + f*x]*Tan[e + f*x]^5)/(3*f) + (c^2*Tan[e + f*x]^7)/(7*f)
))
```

3.11.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.11.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.15

method	result
risch	$a^3c^5x - \frac{ic^5a^3(1155\,e^{13i(fx+e)}+1680\,e^{12i(fx+e)}+980\,e^{11i(fx+e)}+10080\,e^{10i(fx+e)}+2975\,e^{9i(fx+e)}+16240\,e^{8i(fx+e}}{420f(1+e^{2}}$
parts	$a^3c^5x + \frac{c^5a^3\ln(\sec(fx+e)+\tan(fx+e))}{f} - \frac{2a^3c^5\tan(fx+e)}{f} + \frac{3a^3c^5\sec(fx+e)\tan(fx+e)}{f} - \frac{6c^5a^3\left(-\left(-\frac{\sec(}{}\right.\right.}{}$
parallelrisch	$-\frac{10a^3c^5\left(\frac{(-\cos(7fx+7e)-7\cos(5fx+5e)-21\cos(3fx+3e)-35\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16}+\frac{(\cos(7fx+7e)+7\cos(5fx+5e)+2}{}\right.}{}$
derivativedivides	$\frac{c^5a^3\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)+2c^5a^3\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\right.}{}$
default	$\frac{c^5a^3\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)+2c^5a^3\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\right.}{}$
norman	$\frac{a^3c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{14}-a^3c^5x+7a^3c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-21a^3c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+35a^3c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-35a^3c^5x\tan\left(\frac{fx}{2}\right.}{}$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output
```
a^3*c^5*x-1/420*I*c^5*a^3*(1155*exp(13*I*(f*x+e))+1680*exp(12*I*(f*x+e))+9
80*exp(11*I*(f*x+e))+10080*exp(10*I*(f*x+e))+2975*exp(9*I*(f*x+e))+16240*e
xp(8*I*(f*x+e))+24640*exp(6*I*(f*x+e))-2975*exp(5*I*(f*x+e))+14448*exp(4*I
*(f*x+e))-980*exp(3*I*(f*x+e))+6496*exp(2*I*(f*x+e))-1155*exp(I*(f*x+e))+1
168)/f/(1+exp(2*I*(f*x+e)))^7+5/8*c^5*a^3/f*ln(exp(I*(f*x+e))-I)-5/8*c^5*a
^3/f*ln(exp(I*(f*x+e))+I)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.04

$$\int(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$
$$=\frac{1680\,a^3c^5fx\cos(fx+e)^7-525\,a^3c^5\cos(fx+e)^7\log(\sin(fx+e)+1)+525\,a^3c^5\cos(fx+e)^7\log(-\text{si}}{}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
1/1680*(1680*a^3*c^5*f*x*cos(f*x + e)^7 - 525*a^3*c^5*cos(f*x + e)^7*log(s
in(f*x + e) + 1) + 525*a^3*c^5*cos(f*x + e)^7*log(-sin(f*x + e) + 1) - 2*(
1168*a^3*c^5*cos(f*x + e)^6 - 1155*a^3*c^5*cos(f*x + e)^5 - 256*a^3*c^5*co
s(f*x + e)^4 + 910*a^3*c^5*cos(f*x + e)^3 - 192*a^3*c^5*cos(f*x + e)^2 - 2
80*a^3*c^5*cos(f*x + e) + 120*a^3*c^5)*sin(f*x + e))/(f*cos(f*x + e)^7)
```

3.11.6 Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx = -a^3c^5\left(\int(-1)\,dx+\int 2\sec(e+fx)\,dx+\int 2\sec^2(e+fx)\,dx + \int\left(-6\sec^3(e+fx)\right)dx+\int 6\sec^5(e+fx)\,dx+\int\left(-2\sec^6(e+fx)\right)dx + \int\left(-2\sec^7(e+fx)\right)dx+\int\sec^8(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**5,x)
```

output
```
-a**3*c**5*(Integral(-1, x) + Integral(2*sec(e + f*x), x) + Integral(2*sec
(e + f*x)**2, x) + Integral(-6*sec(e + f*x)**3, x) + Integral(6*sec(e + f*
x)**5, x) + Integral(-2*sec(e + f*x)**6, x) + Integral(-2*sec(e + f*x)**7,
 x) + Integral(sec(e + f*x)**8, x))
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(174) = 348$.

Time = 0.20 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.89

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx = -\frac{48\left(5\tan(fx+e)^7+21\tan(fx+e)^5+35\tan(fx+e)^3+35\tan(fx+e)\right)a^3c^5-224\left(3\tan(fx+\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
-1/1680*(48*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35
*tan(f*x + e))*a^3*c^5 - 224*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*ta
n(f*x + e))*a^3*c^5 - 1680*(f*x + e)*a^3*c^5 + 35*a^3*c^5*(2*(15*sin(f*x +
 e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x +
 e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x
+ e) - 1)) - 630*a^3*c^5*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x +
 e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x +
e) - 1)) + 2520*a^3*c^5*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x
 + e) + 1) + log(sin(f*x + e) - 1)) + 3360*a^3*c^5*log(sec(f*x + e) + tan(
f*x + e)) + 3360*a^3*c^5*tan(f*x + e))/f
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.12

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$

$$= \frac{840\,(fx+e)a^3c^5 - 525\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) + 525\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(1365\,a^{3}\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
1/840*(840*(f*x + e)*a^3*c^5 - 525*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) +
1)) + 525*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(1365*a^3*c^5*tan
(1/2*f*x + 1/2*e)^13 - 9660*a^3*c^5*tan(1/2*f*x + 1/2*e)^11 + 29673*a^3*c^
5*tan(1/2*f*x + 1/2*e)^9 - 21216*a^3*c^5*tan(1/2*f*x + 1/2*e)^7 + 9863*a^3
*c^5*tan(1/2*f*x + 1/2*e)^5 - 2660*a^3*c^5*tan(1/2*f*x + 1/2*e)^3 + 315*a^
3*c^5*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^7)/f
```

3.11.9 Mupad [B] (verification not implemented)

Time = 15.85 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.38

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$
$$= \frac{\frac{13\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{4}-23\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}+\frac{1413\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{20}-\frac{1768\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{35}+\frac{1409\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{60}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-7\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+21\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}-21\right.}$$
$$+a^3\,c^5\,x-\frac{5\,a^3\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

input
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5,x)
```

output
```
((1409*a^3*c^5*tan(e/2 + (f*x)/2)^5)/60 - (19*a^3*c^5*tan(e/2 + (f*x)/2)^3
)/3 - (1768*a^3*c^5*tan(e/2 + (f*x)/2)^7)/35 + (1413*a^3*c^5*tan(e/2 + (f*
x)/2)^9)/20 - 23*a^3*c^5*tan(e/2 + (f*x)/2)^11 + (13*a^3*c^5*tan(e/2 + (f*
x)/2)^13)/4 + (3*a^3*c^5*tan(e/2 + (f*x)/2))/4)/(f*(7*tan(e/2 + (f*x)/2)^2
 - 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/
2)^8 + 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x
)/2)^14 - 1)) + a^3*c^5*x - (5*a^3*c^5*atanh(tan(e/2 + (f*x)/2)))/(4*f)
```

3.12 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$

3.12.1 Optimal result

Integrand size = 26, antiderivative size = 132

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = a^3c^4x-\frac{5a^3c^4\operatorname{arctanh}(\sin(e+fx))}{16f}-\frac{a^3(16c^4-5c^4\sec(e+fx))\tan(e+fx)}{16f}+\frac{a^3(8c^4-5c^4\sec(e+fx))\tan^3(e+fx)}{24f}-\frac{a^3(6c^4-5c^4\sec(e+fx))\tan^5(e+fx)}{30f}$$

output
```
a^3*c^4*x-5/16*a^3*c^4*arctanh(sin(f*x+e))/f-1/16*a^3*(16*c^4-5*c^4*sec(f*
x+e))*tan(f*x+e)/f+1/24*a^3*(8*c^4-5*c^4*sec(f*x+e))*tan(f*x+e)^3/f-1/30*a
^3*(6*c^4-5*c^4*sec(f*x+e))*tan(f*x+e)^5/f
```

3.12.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.25

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$
$$=\frac{a^3c^4\sec^6(e+fx)\left(1200e+1200fx-1200\operatorname{arctanh}(\sin(e+fx))\cos^6(e+fx)+1800(e+fx)\cos(2(e+f\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

output
```
(a^3*c^4*Sec[e + f*x]^6*(1200*e + 1200*f*x - 1200*ArcTanh[Sin[e + f*x]]*Co
s[e + f*x]^6 + 1800*(e + f*x)*Cos[2*(e + f*x)] + 720*e*Cos[4*(e + f*x)] +
720*f*x*Cos[4*(e + f*x)] + 120*e*Cos[6*(e + f*x)] + 120*f*x*Cos[6*(e + f*x
)] + 450*Sin[e + f*x] - 600*Sin[2*(e + f*x)] - 25*Sin[3*(e + f*x)] - 384*S
in[4*(e + f*x)] + 165*Sin[5*(e + f*x)] - 184*Sin[6*(e + f*x)]))/(3840*f)
```

3.12.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 4392, 3042, 4369, 3042, 4369, 27, 3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3(c-c\sec(e+fx))^4\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \ 4392$$

$$-a^3c^3\int (c-c\sec(e+fx))\tan^6(e+fx)dx$$

$$\Big\downarrow \ 3042$$

$$-a^3c^3\int \cot\left(e+fx+\frac{\pi}{2}\right)^6\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \ 4369$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)(6c-5c\sec(e+fx))}{30f}-\frac{1}{6}\int (6c-5c\sec(e+fx))\tan^4(e+fx)dx\right)$$

$$\Big\downarrow \ 3042$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)(6c-5c\sec(e+fx))}{30f}-\frac{1}{6}\int \cot\left(e+fx+\frac{\pi}{2}\right)^4\left(6c-5c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx\right)$$

$$\Big\downarrow \ 4369$$

$$-a^3c^3\left(\frac{1}{6}\left(\frac{1}{4}\int 3(8c-5c\sec(e+fx))\tan^2(e+fx)dx-\frac{\tan^3(e+fx)(8c-5c\sec(e+fx))}{4f}\right)+\frac{\tan^5(e+fx)(6c-}{30}\right.$$

$\downarrow$ 27

$$-a^3c^3\left(\frac{1}{6}\left(\frac{3}{4}\int (8c-5c\sec(e+fx))\tan^2(e+fx)dx-\frac{\tan^3(e+fx)(8c-5c\sec(e+fx))}{4f}\right)+\frac{\tan^5(e+fx)(6c-}{30f}\right.$$

$\downarrow$ 3042

$$-a^3c^3\left(\frac{1}{6}\left(\frac{3}{4}\int \cot\left(e+fx+\frac{\pi}{2}\right)^2\left(8c-5c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx-\frac{\tan^3(e+fx)(8c-5c\sec(e+fx))}{4f}\right)+\frac{\tan^5(}{}\right.$$

$\downarrow$ 4369

$$-a^3c^3\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{\tan(e+fx)(16c-5c\sec(e+fx))}{2f}-\frac{1}{2}\int(16c-5c\sec(e+fx))dx\right)-\frac{\tan^3(e+fx)(8c-5c\sec(e}{4f}\right.\right.$$

$\downarrow$ 2009

$$-a^3c^3\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{5c\text{arctanh}(\sin(e+fx))}{f}-16cx\right)+\frac{\tan(e+fx)(16c-5c\sec(e+fx))}{2f}\right)-\frac{\tan^3(e+fx)(8c-5}{4f}\right.\right.$$

input
```
Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

output
```
-(a^3*c^3*(((6*c - 5*c*Sec[e + f*x])*Tan[e + f*x]^5)/(30*f) + (-1/4*((8*c
- 5*c*Sec[e + f*x])*Tan[e + f*x]^3)/f + (3*((-16*c*x + (5*c*ArcTanh[Sin[e
+ f*x]])/f)/2 + ((16*c - 5*c*Sec[e + f*x])*Tan[e + f*x])/(2*f)))/4)/6))
```

3.12.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.12.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.83 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.56

method	result
risch	$a^3c^4x - \frac{ic^4a^3\left(165\,\mathrm{e}^{11i(fx+e)}+720\,\mathrm{e}^{10i(fx+e)}-25\,\mathrm{e}^{9i(fx+e)}+2160\,\mathrm{e}^{8i(fx+e)}+450\,\mathrm{e}^{7i(fx+e)}+3680\,\mathrm{e}^{6i(fx+e)}-450\,\mathrm{e}^{5i}\right.}{120f\left(1+\mathrm{e}^{2i(fx+e)}\right)^6}$
parallelrisch	$-\frac{5a^3\left(\frac{\left(-5-\frac{15\cos(2fx+2e)}{2}-3\cos(4fx+4e)-\frac{\cos(6fx+6e)}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8}+\frac{\left(5+\frac{\cos(6fx+6e)}{2}+3\cos(4fx+4e)+\frac{15\cos(2f}{2}\right.}{8}\right.}{}$
derivativedivides	$\frac{c^4a^3\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)+c^4a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\right.}{}$
default	$\frac{c^4a^3\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)+c^4a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\right.}{}$
parts	$a^3c^4x+\frac{c^4a^3\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)}{f}-\frac{c^4a^3\ln(\sec}{}$
norman	$\frac{a^3c^4x+a^3c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}-6a^3c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+15a^3c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-20a^3c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+15a^3c^4x\tan\left(\frac{fx}{2}\right.}{}$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*c^4*x-1/120*I*c^4*a^3*(165*exp(11*I*(f*x+e))+720*exp(10*I*(f*x+e))-25*
exp(9*I*(f*x+e))+2160*exp(8*I*(f*x+e))+450*exp(7*I*(f*x+e))+3680*exp(6*I*(
f*x+e))-450*exp(5*I*(f*x+e))+3360*exp(4*I*(f*x+e))+25*exp(3*I*(f*x+e))+148
8*exp(2*I*(f*x+e))-165*exp(I*(f*x+e))+368)/f/(1+exp(2*I*(f*x+e)))^6+5/16*c
^4*a^3/f*ln(exp(I*(f*x+e))-I)-5/16*c^4*a^3/f*ln(exp(I*(f*x+e))+I)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$
$$= \frac{480\,a^3c^4fx\cos(fx+e)^6 - 75\,a^3c^4\cos(fx+e)^6\log(\sin(fx+e)+1) + 75\,a^3c^4\cos(fx+e)^6\log(-\sin(f$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
1/480*(480*a^3*c^4*f*x*cos(f*x + e)^6 - 75*a^3*c^4*cos(f*x + e)^6*log(sin(
f*x + e) + 1) + 75*a^3*c^4*cos(f*x + e)^6*log(-sin(f*x + e) + 1) - 2*(368*
a^3*c^4*cos(f*x + e)^5 - 165*a^3*c^4*cos(f*x + e)^4 - 176*a^3*c^4*cos(f*x
+ e)^3 + 130*a^3*c^4*cos(f*x + e)^2 + 48*a^3*c^4*cos(f*x + e) - 40*a^3*c^4
)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

3.12.6 Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$
$$= a^3c^4\left(\int 1\,dx + \int(-\sec(e+fx))\,dx + \int\left(-3\sec^2(e+fx)\right)dx + \int 3\sec^3(e+fx)\,dx + \int 3\sec^4(e+fx)\,dx + \int\left(-3\sec^5(e+fx)\right)dx + \int\left(-\sec^6(e+fx)\right)dx + \int \sec^7(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**4,x)
```

output
```
a**3*c**4*(Integral(1, x) + Integral(-sec(e + f*x), x) + Integral(-3*sec(e
 + f*x)**2, x) + Integral(3*sec(e + f*x)**3, x) + Integral(3*sec(e + f*x)*
*4, x) + Integral(-3*sec(e + f*x)**5, x) + Integral(-sec(e + f*x)**6, x) +
 Integral(sec(e + f*x)**7, x))
```

3.12.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(124) = 248$.

Time = 0.20 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.53

$$\int (a + a\sec(e + fx))^3 (c - c\sec(e + fx))^4 \, dx =$$

$$-\frac{32 \left(3 \tan\left(fx+e\right)^5 + 10 \tan\left(fx+e\right)^3 + 15 \tan\left(fx+e\right)\right)a^3c^4 - 480 \left(\tan\left(fx+e\right)^3 + 3 \tan\left(fx+e\right)\right)\ldots}{\ldots}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
-1/480*(32*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^
4 - 480*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^4 - 480*(f*x + e)*a^3*c^4
+ 5*a^3*c^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(
sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x
 + e) + 1) + 15*log(sin(f*x + e) - 1)) - 90*a^3*c^4*(2*(3*sin(f*x + e)^3 -
 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x +
 e) + 1) + 3*log(sin(f*x + e) - 1)) + 360*a^3*c^4*(2*sin(f*x + e)/(sin(f*x
 + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 480*a^3*c^
4*log(sec(f*x + e) + tan(f*x + e)) + 1440*a^3*c^4*tan(f*x + e))/f
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

$$\int (a + a\sec(e + fx))^3 (c - c\sec(e + fx))^4 \, dx$$

$$= \frac{240 \left(fx+e\right)a^3c^4 - 75 \, a^3c^4 \log\left(\left|\tan\left(\frac{1}{2} \, fx + \frac{1}{2} \, e\right) + 1\right|\right) + 75 \, a^3c^4 \log\left(\left|\tan\left(\frac{1}{2} \, fx + \frac{1}{2} \, e\right) - 1\right|\right) + \frac{2 \left(315 \, a^3c^4 \ldots\right.}{\ldots}}{\ldots}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/240*(240*(f*x + e)*a^3*c^4 - 75*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1
)) + 75*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(315*a^3*c^4*tan(1/
2*f*x + 1/2*e)^11 - 1945*a^3*c^4*tan(1/2*f*x + 1/2*e)^9 + 5118*a^3*c^4*tan
(1/2*f*x + 1/2*e)^7 - 3138*a^3*c^4*tan(1/2*f*x + 1/2*e)^5 + 1095*a^3*c^4*t
an(1/2*f*x + 1/2*e)^3 - 165*a^3*c^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1
/2*e)^2 - 1)^6)/f
```

3.12.9 Mupad [B] (verification not implemented)

Time = 15.34 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.72

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = a^3\,c^4\,x$$
$$+\frac{\frac{21\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}-\frac{389\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24}+\frac{853\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{20}-\frac{523\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{20}+\frac{73\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{8}-}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-6\,\mathrm{ta}\right.}$$
$$-\frac{5\,a^3\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

input
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4,x)
```

output
```
a^3*c^4*x + ((73*a^3*c^4*tan(e/2 + (f*x)/2)^3)/8 - (523*a^3*c^4*tan(e/2 +
(f*x)/2)^5)/20 + (853*a^3*c^4*tan(e/2 + (f*x)/2)^7)/20 - (389*a^3*c^4*tan(
e/2 + (f*x)/2)^9)/24 + (21*a^3*c^4*tan(e/2 + (f*x)/2)^11)/8 - (11*a^3*c^4*
tan(e/2 + (f*x)/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^
2 - 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/
2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) - (5*a^3*c^4*atanh(tan(e/2 + (f*x)/2))
)/(8*f)
```

3.13 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$

3.13.1 Optimal result

Integrand size = 26, antiderivative size = 68

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = a^3c^3x-\frac{a^3c^3\tan(e+fx)}{f}+\frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan^5(e+fx)}{5f}$$

output
```
a^3*c^3*x-a^3*c^3*tan(f*x+e)/f+1/3*a^3*c^3*tan(f*x+e)^3/f-1/5*a^3*c^3*tan(
f*x+e)^5/f
```

3.13.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -a^3c^3\left(-\frac{\arctan(\tan(e+fx))}{f}+\frac{\tan(e+fx)}{f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan^5(e+fx)}{5f}\right)$$

input
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

output
```
-(a^3*c^3*(-(ArcTan[Tan[e + f*x]]/f) + Tan[e + f*x]/f - Tan[e + f*x]^3/(3*
f) + Tan[e + f*x]^5/(5*f)))
```

3.13.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3042, 4392, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3(c-c\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4392}$$

$$-a^3c^3\int \tan^6(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \tan(e+fx)^6dx$$

$$\Big\downarrow \text{3954}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)}{5f}-\int \tan^4(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)}{5f}-\int \tan(e+fx)^4dx\right)$$

$$\Big\downarrow \text{3954}$$

$$-a^3c^3\left(\int \tan^2(e+fx)dx+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\left(\int \tan(e+fx)^2dx+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}\right)$$

$$\Big\downarrow \text{3954}$$

$$-a^3c^3\left(-\int 1dx+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f}\right)$$

$$\Big\downarrow \text{24}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f}-x\right)$$

input
```
Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

output
```
-(a^3*c^3*(-x + Tan[e + f*x]/f - Tan[e + f*x]^3/(3*f) + Tan[e + f*x]^5/(5*
f)))
```

3.13.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.13.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

method	result
risch	$a^3c^3x - \frac{2ic^3a^3\left(45\,\mathrm{e}^{8i(fx+e)}+90\,\mathrm{e}^{6i(fx+e)}+140\,\mathrm{e}^{4i(fx+e)}+70\,\mathrm{e}^{2i(fx+e)}+23\right)}{15f\left(1+\mathrm{e}^{2i(fx+e)}\right)^5}$
derivativedivides	$\frac{c^3a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-3c^3a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-3c^3a^3\tan(fx+e)+c^3a^3(fx-}{f}$
default	$\frac{c^3a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-3c^3a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-3c^3a^3\tan(fx+e)+c^3a^3(fx-}{f}$
parts	$a^3c^3x - \frac{3a^3c^3\tan(fx+e)}{f} - \frac{3c^3a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f} + \frac{c^3a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(}{f}$
parallelrisch	$\frac{a^3c^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}xf-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8xf+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6xf-\frac{32\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{3}-10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4\right.}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$
norman	$\frac{a^3c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-a^3c^3x+5a^3c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-10a^3c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+10a^3c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-5a^3c^3x\tan\left(\frac{fx}{2}+\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*c^3*x-2/15*I*c^3*a^3*(45*exp(8*I*(f*x+e))+90*exp(6*I*(f*x+e))+140*exp(
4*I*(f*x+e))+70*exp(2*I*(f*x+e))+23)/f/(1+exp(2*I*(f*x+e)))^5
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$$
$$=\frac{15\,a^3c^3fx\cos\left(fx+e\right)^5-\left(23\,a^3c^3\cos\left(fx+e\right)^4-11\,a^3c^3\cos\left(fx+e\right)^2+3\,a^3c^3\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(15*a^3*c^3*f*x*cos(f*x + e)^5 - (23*a^3*c^3*cos(f*x + e)^4 - 11*a^3*
c^3*cos(f*x + e)^2 + 3*a^3*c^3)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

3.13.6 Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -a^3c^3\left(\int (-1)\,dx + \int 3\sec^2(e+fx)\,dx + \int \left(-3\sec^4(e+fx)\right)\,dx + \int \sec^6(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**3,x)
```

output
```
-a**3*c**3*(Integral(-1, x) + Integral(3*sec(e + f*x)**2, x) + Integral(-3
*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**6, x))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -\frac{\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^3c^3-15\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c^3}{15f}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/15*((3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^3 -
15*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^3 - 15*(f*x + e)*a^3*c^3 + 45*a
^3*c^3*tan(f*x + e))/f
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$$
$$= -\frac{3\,a^3c^3\tan\left(fx+e\right)^5-5\,a^3c^3\tan\left(fx+e\right)^3-15\left(fx+e\right)a^3c^3+15\,a^3c^3\tan\left(fx+e\right)}{15\,f}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/15*(3*a^3*c^3*tan(f*x + e)^5 - 5*a^3*c^3*tan(f*x + e)^3 - 15*(f*x + e)*
a^3*c^3 + 15*a^3*c^3*tan(f*x + e))/f
```

3.13.9 Mupad [B] (verification not implemented)

Time = 17.86 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.79

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = a^3\,c^3\,x$$
$$+\frac{2\,a^3\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9-\frac{32\,a^3\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{3}+\frac{356\,a^3\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15}-\frac{32\,a^3\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+2\,a^3\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^5}$$

input
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3,x)
```

output
```
a^3*c^3*x + ((356*a^3*c^3*tan(e/2 + (f*x)/2)^5)/15 - (32*a^3*c^3*tan(e/2 +
 (f*x)/2)^3)/3 - (32*a^3*c^3*tan(e/2 + (f*x)/2)^7)/3 + 2*a^3*c^3*tan(e/2 +
 (f*x)/2)^9 + 2*a^3*c^3*tan(e/2 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^2 - 1)^
5)
```

3.14 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$

3.14.1 Optimal result

Integrand size = 26, antiderivative size = 97

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = a^3c^2x+\frac{3a^3c^2\text{arctanh}(\sin(e+fx))}{8f}-\frac{c^2(8a^3+3a^3\sec(e+fx))\tan(e+fx)}{8f}+\frac{c^2(4a^3+3a^3\sec(e+fx))\tan^3(e+fx)}{12f}$$

output
```
a^3*c^2*x+3/8*a^3*c^2*arctanh(sin(f*x+e))/f-1/8*c^2*(8*a^3+3*a^3*sec(f*x+e
))*tan(f*x+e)/f+1/12*c^2*(4*a^3+3*a^3*sec(f*x+e))*tan(f*x+e)^3/f
```

3.14.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = \frac{a^3c^2\sec^4(e+fx)\left(72e+72fx+72\text{arctanh}(\sin(e+fx))\cos^4(e+fx)+96(e+fx)\cos(2(e+fx))+24e\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

output
```
(a^3*c^2*Sec[e + f*x]^4*(72*e + 72*f*x + 72*ArcTanh[Sin[e + f*x]]*Cos[e +
f*x]^4 + 96*(e + f*x)*Cos[2*(e + f*x)] + 24*e*Cos[4*(e + f*x)] + 24*f*x*Co
s[4*(e + f*x)] + 18*Sin[e + f*x] - 32*Sin[2*(e + f*x)] - 30*Sin[3*(e + f*x
)] - 32*Sin[4*(e + f*x)]))/(192*f)
```

3.14.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 4392, 3042, 4369, 3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3(c-c\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int (\sec(e+fx)a+a)\tan^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cot\left(e+fx+\frac{\pi}{2}\right)^4\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)dx$$

$$\Big\downarrow \text{4369}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)(3a\sec(e+fx)+4a)}{12f}-\frac{1}{4}\int (3\sec(e+fx)a+4a)\tan^2(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)(3a\sec(e+fx)+4a)}{12f}-\frac{1}{4}\int \cot\left(e+fx+\frac{\pi}{2}\right)^2\left(3\csc\left(e+fx+\frac{\pi}{2}\right)a+4a\right)dx\right)$$

$$\Big\downarrow \text{4369}$$

$$a^2c^2\left(\frac{1}{4}\left(\frac{1}{2}\int (3\sec(e+fx)a+8a)dx-\frac{\tan(e+fx)(3a\sec(e+fx)+8a)}{2f}\right)+\frac{\tan^3(e+fx)(3a\sec(e+fx)+4a)}{12f}\right.$$

$$\Big\downarrow \text{2009}$$

$$a^2c^2\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{3a\text{arctanh}(\sin(e+fx))}{f}+8ax\right)-\frac{\tan(e+fx)(3a\sec(e+fx)+8a)}{2f}\right)+\frac{\tan^3(e+fx)(3a\sec(e+fx)}{12f}\right.$$

input
```
Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

output
```
a^2*c^2*(((4*a + 3*a*Sec[e + f*x])*Tan[e + f*x]^3)/(12*f) + ((8*a*x + (3*a
*ArcTanh[Sin[e + f*x]])/f)/2 - ((8*a + 3*a*Sec[e + f*x])*Tan[e + f*x])/(2*
f))/4)
```

3.14.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.14.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.37

method	result
parts	$a^3c^2x-\frac{c^2a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{c^2a^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\mathrm{ta}}{8}\right.}{f}$
risch	$a^3c^2x+\frac{ic^2a^3\left(15\,\mathrm{e}^{7i(fx+e)}-48\,\mathrm{e}^{6i(fx+e)}-9\,\mathrm{e}^{5i(fx+e)}-96\,\mathrm{e}^{4i(fx+e)}+9\,\mathrm{e}^{3i(fx+e)}-80\,\mathrm{e}^{2i(fx+e)}-15\,\mathrm{e}^{i(fx+e)}-32\right)}{12f\left(1+\mathrm{e}^{2i(fx+e)}\right)^4}+$
derivativedivides	$\frac{c^2a^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-c^2a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2c}{f}$
default	$\frac{c^2a^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-c^2a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2c}{f}$
parallelrisch	$\frac{a^3c^2\left(-9(3+\cos(4fx+4e)+4\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-9(-\cos(4fx+4e)-4\cos(2fx+2e)-3)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.\right.}{24f(3+\cos(4fx+4}$
norman	$\frac{a^3c^2x+a^3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8-4a^3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+6a^3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-4a^3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-\frac{11c^2a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
a^3*c^2*x-c^2*a^3/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+c^2*a^3/f*(-(-1/4*s
ec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-2*c^
2*a^3/f*tan(f*x+e)-c^2*a^3/f*tan(f*x+e)*sec(f*x+e)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.52

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$=\frac{48\,a^3c^2fx\cos(fx+e)^4+9\,a^3c^2\cos(fx+e)^4\log(\sin(fx+e)+1)-9\,a^3c^2\cos(fx+e)^4\log(-\sin(fx+}{48\,f\cos(f}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/48*(48*a^3*c^2*f*x*cos(f*x + e)^4 + 9*a^3*c^2*cos(f*x + e)^4*log(sin(f*x
 + e) + 1) - 9*a^3*c^2*cos(f*x + e)^4*log(-sin(f*x + e) + 1) - 2*(32*a^3*c
^2*cos(f*x + e)^3 + 15*a^3*c^2*cos(f*x + e)^2 - 8*a^3*c^2*cos(f*x + e) - 6
*a^3*c^2)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

3.14.6 Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = a^3c^2\left(\int 1\,dx + \int \sec(e+fx)\,dx + \int \left(-2\sec^2(e+fx)\right)dx + \int \left(-2\sec^3(e+fx)\right)dx + \int \sec^4(e+fx)\,dx + \int \sec^5(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**2,x)
```

output
```
a**3*c**2*(Integral(1, x) + Integral(sec(e + f*x), x) + Integral(-2*sec(e
+ f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4
, x) + Integral(sec(e + f*x)**5, x))
```

3.14.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(91) = 182.

Time = 0.20 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.09

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = \frac{16\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c^2+48(fx+e)a^3c^2-3a^3c^2\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 + 48*(f*x + e)*a^3*c^2
- 3*a^3*c^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin
(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 24
*a^3*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) + 48*a^3*c^2*log(sec(f*x + e) + tan(f*x + e)) - 96*a^
3*c^2*tan(f*x + e))/f
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$=\frac{24\,(fx+e)a^3c^2+9\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-9\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(15\,a^3c^2\tan\left(\frac{1}{2}\right.\right.}{}}{24\,f}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/24*(24*(f*x + e)*a^3*c^2 + 9*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))
- 9*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(15*a^3*c^2*tan(1/2*f*x
 + 1/2*e)^7 - 71*a^3*c^2*tan(1/2*f*x + 1/2*e)^5 + 137*a^3*c^2*tan(1/2*f*x
+ 1/2*e)^3 - 33*a^3*c^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)
^4)/f
```

3.14.9 Mupad [B] (verification not implemented)

Time = 15.66 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.68

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$=\frac{\frac{5\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{71\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}+\frac{137\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}-\frac{11\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$
$$+a^3\,c^2\,x+\frac{3\,a^3\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

input
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2,x)
```

output
```
((137*a^3*c^2*tan(e/2 + (f*x)/2)^3)/12 - (71*a^3*c^2*tan(e/2 + (f*x)/2)^5)
/12 + (5*a^3*c^2*tan(e/2 + (f*x)/2)^7)/4 - (11*a^3*c^2*tan(e/2 + (f*x)/2))
/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x
)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) + a^3*c^2*x + (3*a^3*c^2*atanh(tan(e/2
 + (f*x)/2)))/(4*f)
```

3.14. $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$

3.15 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

3.15.1 Optimal result

Integrand size = 24, antiderivative size = 77

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = a^3cx+\frac{a^3c\,\text{arctanh}(\sin(e+fx))}{f}-\frac{a^3c\tan(e+fx)}{f}-\frac{a^3c\sec(e+fx)\tan(e+fx)}{f}-\frac{a^3c\tan^3(e+fx)}{3f}$$

output
```
a^3*c*x+a^3*c*arctanh(sin(f*x+e))/f-a^3*c*tan(f*x+e)/f-a^3*c*sec(f*x+e)*ta
n(f*x+e)/f-1/3*a^3*c*tan(f*x+e)^3/f
```

3.15.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = \frac{a^3c\sec^3(e+fx)\left(9(e+fx)\cos(e+fx)+12\text{arctanh}(\sin(e+fx))\cos^3(e+fx)+3e\cos(3(e+fx))+3f\right.}{12f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

output
```
(a^3*c*Sec[e + f*x]^3*(9*(e + f*x)*Cos[e + f*x] + 12*ArcTanh[Sin[e + f*x]]
*Cos[e + f*x]^3 + 3*e*Cos[3*(e + f*x)] + 3*f*x*Cos[3*(e + f*x)] - 6*Sin[e
+ f*x] - 6*Sin[2*(e + f*x)] - 2*Sin[3*(e + f*x)]))/(12*f)
```

3.15.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 4392$$

$$-ac\int (\sec(e+fx)a+a)^2\tan^2(e+fx)dx$$

$$\Big\downarrow \; 3042$$

$$-ac\int \cot\left(e+fx+\frac{\pi}{2}\right)^2\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2dx$$

$$\Big\downarrow \; 4374$$

$$-ac\int \left(a^2\tan^2(e+fx)+a^2\sec^2(e+fx)\tan^2(e+fx)+2a^2\sec(e+fx)\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-ac\left(-\frac{a^2\text{arctanh}(\sin(e+fx))}{f}+\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan(e+fx)}{f}+\frac{a^2\tan(e+fx)\sec(e+fx)}{f}-a^2x\right)$$

input
```
Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

output
```
-(a*c*(-(a^2*x) - (a^2*ArcTanh[Sin[e + f*x]])/f + (a^2*Tan[e + f*x])/f + (
a^2*Sec[e + f*x]*Tan[e + f*x])/f + (a^2*Tan[e + f*x]^3)/(3*f)))
```

3.15.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.15.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result
parts	$a^3cx+\frac{a^3c\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{a^3c\sec(fx+e)\tan(fx+e)}{f}+\frac{a^3c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
derivativedivides	$\frac{a^3c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2a^3c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2a^3c\ln(\sec(fx+e)+\tan(fx-}{f}$
default	$\frac{a^3c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2a^3c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2a^3c\ln(\sec(fx+e)+\tan(fx-}{f}$
risch	$a^3cx+\frac{2ia^3c(3\,e^{5i(fx+e)}-6\,e^{2i(fx+e)}-3\,e^{i(fx+e)}-2)}{3f\left(1+e^{2i(fx+e)}\right)^3}+\frac{a^3c\ln(e^{i(fx+e)}+i)}{f}-\frac{a^3c\ln(e^{i(fx+e)}-i)}{f}$
parallelrisch	$-\frac{2\left(\frac{3(\cos(3fx+3e)+3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{3(-\cos(3fx+3e)-3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}-\frac{9fx\cos(fx+e)}{2}-\frac{3j}{}\right.}{3f(\cos(3fx+3e)+3\cos(fx+e))}$
norman	$\frac{a^3cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-a^3cx+\frac{4a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{4a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+3a^3cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3a^3cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}+\frac{a^3c\ln(}{}$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
a^3*c*x+a^3*c/f*ln(sec(f*x+e)+tan(f*x+e))-a^3*c*sec(f*x+e)*tan(f*x+e)/f+a^
3*c/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.53

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$=\frac{6\,a^3cfx\cos(fx+e)^3+3\,a^3c\cos(fx+e)^3\log(\sin(fx+e)+1)-3\,a^3c\cos(fx+e)^3\log(-\sin(fx+e)\cdots}{6\,f\cos(fx+e)^3}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
1/6*(6*a^3*c*f*x*cos(f*x + e)^3 + 3*a^3*c*cos(f*x + e)^3*log(sin(f*x + e)
+ 1) - 3*a^3*c*cos(f*x + e)^3*log(-sin(f*x + e) + 1) - 2*(2*a^3*c*cos(f*x
+ e)^2 + 3*a^3*c*cos(f*x + e) + a^3*c)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

3.15.6 Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -a^3c\left(\int(-1)\,dx + \int(-2\sec(e+fx))\,dx + \int 2\sec^3(e+fx)\,dx + \int \sec^4(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
```

output
```
-a**3*c*(Integral(-1, x) + Integral(-2*sec(e + f*x), x) + Integral(2*sec(e
 + f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -\frac{2\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c-6(fx+e)a^3c-3a^3c\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\cdots\right.}{6f}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-1/6*(2*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c - 6*(f*x + e)*a^3*c - 3*a^
3*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin
(f*x + e) - 1)) - 12*a^3*c*log(sec(f*x + e) + tan(f*x + e)))/f
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))\,dx$$
$$= \frac{3\,(fx + e)a^3c + 3\,a^3c\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + 1\right|\right) - 3\,a^3c\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - 1\right|\right) - \frac{4\left(a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 \cdots\right.}{\left(\tan\left(\frac{1}{2}\,fx \cdots\right.\right.}}{3\,f}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
1/3*(3*(f*x + e)*a^3*c + 3*a^3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a^
3*c*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 4*(a^3*c*tan(1/2*f*x + 1/2*e)^3 -
 3*a^3*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

3.15.9 Mupad [B] (verification not implemented)

Time = 15.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))\,dx$$
$$= \frac{4\,a^3\,c\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right) - \frac{4\,a^3\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}}{f\,\left(\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^6 - 3\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^4 + 3\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^2 - 1\right)}$$
$$+ a^3\,c\,x + \frac{2\,a^3\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)\right)}{f}$$

input
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x)),x)
```

output
```
(4*a^3*c*tan(e/2 + (f*x)/2) - (4*a^3*c*tan(e/2 + (f*x)/2)^3)/3)/(f*(3*tan(
e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1)) + a
^3*c*x + (2*a^3*c*atanh(tan(e/2 + (f*x)/2)))/f
```

3.16 $\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$

3.16.1 Optimal result

Integrand size = 26, antiderivative size = 78

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = \frac{a^3x}{c} - \frac{4a^3\text{arctanh}(\sin(e+fx))}{cf} + \frac{8a^3\cot(e+fx)}{cf} + \frac{8a^3\csc(e+fx)}{cf} - \frac{a^3\tan(e+fx)}{cf}$$

output
```
a^3*x/c-4*a^3*arctanh(sin(f*x+e))/c/f+8*a^3*cot(f*x+e)/c/f+8*a^3*csc(f*x+e
)/c/f-a^3*tan(f*x+e)/c/f
```

3.16.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.39 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.53

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{\tan(e+fx)\left(-8\sqrt{2}a^3\sqrt{c}\cos^6\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sec^4(e+f\ldots}{\ldots}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x]),x]
```

output
```
-1/5*(Tan[e + f*x]*(-8*Sqrt[2]*a^3*Sqrt[c]*Cos[(e + f*x)/2]^6*Hypergeometr
ic2F1[3/2, 5/2, 7/2, (1 + Sec[e + f*x])/2]*Sec[e + f*x]^4*Sin[(e + f*x)/2]
^2 + 5*a^(5/2)*(4*Sqrt[c]*(Sqrt[a]*Sqrt[1 - Sec[e + f*x]]*(1 + Sec[e + f*x
]) + ArcSin[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqrt[a])]*Sec[e + f*x]*Sqr
t[a*(1 + Sec[e + f*x])]*Sin[(e + f*x)/2]^2) - ArcTanh[Sqrt[-(a*c*Tan[e + f
*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[1 - Sec[e + f*x]]*Sqrt[-(a*c*Tan[e + f*x]^
2)])))/(c^(3/2)*f*(1 - Sec[e + f*x])^(3/2)*(1 + Sec[e + f*x]))
```

3.16.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^3}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^2(e+fx)(\sec(e+fx)a+a)^4dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^4}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx}{ac}$$

$$\Big\downarrow \text{4374}$$

$$-\frac{\int \left(\cot^2(e+fx)a^4+6\csc^2(e+fx)a^4+\csc^2(e+fx)\sec^2(e+fx)a^4+4\cot(e+fx)\csc(e+fx)a^4+4\csc^2(e+\right.}{ac}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{4a^4\text{arctanh}(\sin(e+fx))}{f}+\frac{a^4\tan(e+fx)}{f}-\frac{8a^4\cot(e+fx)}{f}-\frac{8a^4\csc(e+fx)}{f}+a^4(-x)}{ac}$$

input
```
Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x]),x]
```

output
```
-((-(a^4*x) + (4*a^4*ArcTanh[Sin[e + f*x]])/f - (8*a^4*Cot[e + f*x])/f - (
8*a^4*Csc[e + f*x])/f + (a^4*Tan[e + f*x])/f)/(a*c))
```

3.16.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.16.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{8a^3\left(\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-8}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}\right)}{fc}$
default	$\frac{8a^3\left(\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-8}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}\right)}{fc}$
parallelrisch	$\frac{a^3\left(fx\cos(fx+e)+4\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)-4\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)+9\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\cos(fx+e)-\cot}{cf\cos(fx+e)}$
risch	$\frac{a^3x}{c}+\frac{2ia^3\left(8\,e^{2i(fx+e)}-e^{i(fx+e)}+9\right)}{fc\left(1+e^{2i(fx+e)}\right)\left(e^{i(fx+e)}-1\right)}-\frac{4a^3\ln\left(e^{i(fx+e)}+i\right)}{cf}+\frac{4a^3\ln\left(e^{i(fx+e)}-i\right)}{cf}$
norman	$\frac{\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}+\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{c}+\frac{8a^3}{cf}-\frac{18a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{cf}+\frac{10a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{cf}-\frac{2a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{4a^3\ln\left(\tan\left(\frac{f}{2}\right.\right.}{cf}$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
8/f*a^3/c*(1/8/(tan(1/2*f*x+1/2*e)+1)-1/2*ln(tan(1/2*f*x+1/2*e)+1)+1/8/(ta
n(1/2*f*x+1/2*e)-1)+1/2*ln(tan(1/2*f*x+1/2*e)-1)+1/tan(1/2*f*x+1/2*e)+1/4*
arctan(tan(1/2*f*x+1/2*e)))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.60

$$\int\frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$$
$$=\frac{a^3fx\cos(fx+e)\sin(fx+e)-2\,a^3\cos(fx+e)\log(\sin(fx+e)+1)\sin(fx+e)+2\,a^3\cos(fx+e)\log}{cf\cos(fx+e)\sin(fx+e)}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
(a^3*f*x*cos(f*x + e)*sin(f*x + e) - 2*a^3*cos(f*x + e)*log(sin(f*x + e) +
 1)*sin(f*x + e) + 2*a^3*cos(f*x + e)*log(-sin(f*x + e) + 1)*sin(f*x + e)
+ 9*a^3*cos(f*x + e)^2 + 8*a^3*cos(f*x + e) - a^3)/(c*f*cos(f*x + e)*sin(f
*x + e))
```

3.16.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{a^3\left(\int \frac{3\sec(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{3\sec^2(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{\sec^3(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{1}{\sec(e+fx)-1}\,dx\right)}{c}$$

input
```
integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

output
```
-a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(3*sec(e +
 f*x)**2/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) -
 1), x) + Integral(1/(sec(e + f*x) - 1), x))/c
```

3.16.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(78) = 156$.

Time = 0.29 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.51

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{a^3\left(\frac{\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right) - a^3\left(\frac{2\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c} + \frac{\cos(fx+e)+}{c\sin(fx+e)}\right.}{f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-(a^3*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x + e)/(cos(f*
x + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f*x + e)/(c
os(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c) - a^
3*(2*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c + (cos(f*x + e) + 1)/(c*sin
(f*x + e))) + 3*a^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e)))
- 3*a^3*(cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = \frac{\frac{(fx+e)a^3}{c} - \frac{4\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c} + \frac{4\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c} + \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-4\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)c}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
((f*x + e)*a^3/c - 4*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c + 4*a^3*log(
abs(tan(1/2*f*x + 1/2*e) - 1))/c + 2*(5*a^3*tan(1/2*f*x + 1/2*e)^2 - 4*a^3
)/((tan(1/2*f*x + 1/2*e)^3 - tan(1/2*f*x + 1/2*e))*c))/f
```

3.16.9 Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = \frac{a^3\,x}{c} - \frac{10\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-8\,a^3}{f\,\left(c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\right)} - \frac{8\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

input
```
int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x)),x)
```

output
```
(a^3*x)/c - (10*a^3*tan(e/2 + (f*x)/2)^2 - 8*a^3)/(f*(c*tan(e/2 + (f*x)/2)
 - c*tan(e/2 + (f*x)/2)^3)) - (8*a^3*atanh(tan(e/2 + (f*x)/2)))/(c*f)
```

3.17 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx$

3.17.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{a^3x}{c^2}+\frac{a^3\text{arctanh}(\sin(e+fx))}{c^2f} - \frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{4a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))}$$

output
```
a^3*x/c^2+a^3*arctanh(sin(f*x+e))/c^2/f-8/3*a^3*tan(f*x+e)/c^2/f/(1-sec(f*
x+e))^2+4/3*a^3*tan(f*x+e)/c^2/f/(1-sec(f*x+e))
```

3.17.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(88) = 176.

Time = 2.68 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.57

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = -\frac{a^{5/2}\tan(e+fx)\left(4\sqrt{c}\left(\sqrt{a}\sqrt{1-\sec(e+fx)}(1+\sec(e+fx))^2+6\arcsin\left(\frac{\sqrt{a(1+\sec(e+fx))}}{\sqrt{2}\sqrt{a}}\right)\sec^2(e+f}{3c^{5/}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^2,x]
```

output
```
-1/3*(a^(5/2)*Tan[e + f*x]*(4*Sqrt[c]*(Sqrt[a]*Sqrt[1 - Sec[e + f*x]]*(1 +
 Sec[e + f*x])^2 + 6*ArcSin[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqrt[a])]*
Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sin[(e + f*x)/2]^4) + 6*ArcTanh[
Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[1 - Sec[e + f*x]]*Sec[
e + f*x]*Sin[(e + f*x)/2]^2*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(c^(5/2)*f*(1 -
Sec[e + f*x])^(5/2)*(1 + Sec[e + f*x]))
```

3.17.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^3(e+fx)a^3}{(1-\sec(e+fx))^2}+\frac{3\sec^2(e+fx)a^3}{(1-\sec(e+fx))^2}+\frac{3\sec(e+fx)a^3}{(1-\sec(e+fx))^2}+\frac{a^3}{(1-\sec(e+fx))^2}\right)dx}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^3\text{arctanh}(\sin(e+fx))}{f}+\frac{4a^3\tan(e+fx)}{3f(1-\sec(e+fx))}-\frac{8a^3\tan(e+fx)}{3f(1-\sec(e+fx))^2}+a^3x}{c^2}$$

input
```
Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^2,x]
```

output
```
(a^3*x + (a^3*ArcTanh[Sin[e + f*x]])/f - (8*a^3*Tan[e + f*x])/(3*f*(1 - Se
c[e + f*x])^2) + (4*a^3*Tan[e + f*x])/(3*f*(1 - Sec[e + f*x])))/c^2
```

3.17.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.17.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{4a^{3}\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{f\,c^{2}}$
default	$\frac{4a^{3}\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{f\,c^{2}}$
parallelrisch	$\frac{a^{3}\left(3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}fx+3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-4\right)}{3f\,c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}$
risch	$\frac{a^{3}x}{c^{2}}+\frac{8ia^{3}\left(3\,e^{2i(fx+e)}+1\right)}{3f\,c^{2}\left(e^{i(fx+e)}-1\right)^{3}}+\frac{a^{3}\ln\left(e^{i(fx+e)}+i\right)}{c^{2}f}-\frac{a^{3}\ln\left(e^{i(fx+e)}-i\right)}{c^{2}f}$
norman	$\frac{\frac{a^{3}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{c}+\frac{a^{3}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{c}-\frac{4a^{3}}{3cf}+\frac{8a^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{3cf}-\frac{4a^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{3cf}-\frac{2a^{3}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{a^{3}\ln\left(\tan\left(\frac{fx}{2}+\right.\right.}{c^{2}f}$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
4/f*a^3/c^2*(-1/3/tan(1/2*f*x+1/2*e)^3+1/2*arctan(tan(1/2*f*x+1/2*e))-1/4*
ln(tan(1/2*f*x+1/2*e)-1)+1/4*ln(tan(1/2*f*x+1/2*e)+1))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx$$
$$= \frac{8\,a^3\cos\left(fx+e\right)^2+16\,a^3\cos\left(fx+e\right)+8\,a^3+3\left(a^3\cos\left(fx+e\right)-a^3\right)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{6\left(c^2f\cos\left(fx+e\right)\right.}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/6*(8*a^3*cos(f*x + e)^2 + 16*a^3*cos(f*x + e) + 8*a^3 + 3*(a^3*cos(f*x +
 e) - a^3)*log(sin(f*x + e) + 1)*sin(f*x + e) - 3*(a^3*cos(f*x + e) - a^3)
*log(-sin(f*x + e) + 1)*sin(f*x + e) + 6*(a^3*f*x*cos(f*x + e) - a^3*f*x)*
sin(f*x + e))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
```

3.17.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx$$
$$= \frac{a^3\left(\int \frac{3\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{}{\sec^2\left(e+fx\right)-}\right.}{c^2}$$

input
```
integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

output
```
a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) +
 Integral(3*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + I
ntegral(sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(1/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(80) = 160$.

Time = 0.29 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.11

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^2}\, dx$$
$$= \frac{a^3\left(\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2} + \frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right) + a^3\left(\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2} - \frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2} - \left(\right.\right.}{6f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/6*(a^3*(12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2 + (9*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) +
a^3*(6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(
cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) - 3*a^3*(3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) + 3*a^3*(3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x
+ e)^3))/f
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^2}\, dx$$
$$= \frac{\frac{3(fx+e)a^3}{c^2} + \frac{3a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{c^2} - \frac{3a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{c^2} - \frac{4a^3}{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}}{3f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(3*(f*x + e)*a^3/c^2 + 3*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^2 -
3*a^3*log(abs(tan(1/2*f*x + 1/2*e) - 1))/c^2 - 4*a^3/(c^2*tan(1/2*f*x + 1/
2*e)^3))/f
```

3.17.9 Mupad [B] (verification not implemented)

Time = 14.70 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx=\frac{a^3\,x}{c^2}+\frac{a^3\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{4\,{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{3}\right)}{c^2\,f}$$

input
```
int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^2,x)
```

output
```
(a^3*x)/c^2 + (a^3*(2*atanh(tan(e/2 + (f*x)/2)) - (4*cot(e/2 + (f*x)/2)^3)
/3))/(c^2*f)
```

3.18 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$

3.18.1 Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{a^3x}{c^3} - \frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} + \frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2} - \frac{26a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))}$$

output
```
a^3*x/c^3-8/5*a^3*tan(f*x+e)/c^3/f/(1-sec(f*x+e))^3+4/15*a^3*tan(f*x+e)/c^
3/f/(1-sec(f*x+e))^2-26/15*a^3*tan(f*x+e)/c^3/f/(1-sec(f*x+e))
```

3.18.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{2a^3\cot^5\left(\frac{e}{2}+\frac{fx}{2}\right)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5c^3f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a^3*Cot[e/2 + (f*x)/2]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e/2 + (f
*x)/2]^2])/(5*c^3*f)
```

3.18.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \ 4391$$

$$\frac{\int \left(\frac{\sec^3(e+fx)a^3}{(1-\sec(e+fx))^3}+\frac{3\sec^2(e+fx)a^3}{(1-\sec(e+fx))^3}+\frac{3\sec(e+fx)a^3}{(1-\sec(e+fx))^3}+\frac{a^3}{(1-\sec(e+fx))^3}\right)dx}{c^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{26a^3\tan(e+fx)}{15f(1-\sec(e+fx))}+\frac{4a^3\tan(e+fx)}{15f(1-\sec(e+fx))^2}-\frac{8a^3\tan(e+fx)}{5f(1-\sec(e+fx))^3}+a^3x}{c^3}$$

input
```
Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^3,x]
```

output
```
(a^3*x - (8*a^3*Tan[e + f*x])/(5*f*(1 - Sec[e + f*x])^3) + (4*a^3*Tan[e +
f*x])/(15*f*(1 - Sec[e + f*x])^2) - (26*a^3*Tan[e + f*x])/(15*f*(1 - Sec[e
 + f*x])))/c^3
```

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.18.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.53

method	result
parallelrisch	$\frac{a^{3}\left(6\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-10\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+15fx+30\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15c^{3}f}$
derivativedivides	$\frac{2a^{3}\left(\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^{3}}$
default	$\frac{2a^{3}\left(\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^{3}}$
risch	$\frac{a^{3}x}{c^{3}}+\frac{4ia^{3}\left(45\,e^{4i(fx+e)}-90\,e^{3i(fx+e)}+140\,e^{2i(fx+e)}-70\,e^{i(fx+e)}+23\right)}{15f\,c^{3}\left(e^{i(fx+e)}-1\right)^{5}}$
norman	$\frac{\frac{a^{3}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{c}+\frac{a^{3}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{c}+\frac{2a^{3}}{5cf}-\frac{22a^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{15cf}+\frac{56a^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{15cf}-\frac{14a^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{3cf}+\frac{2a^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*a^3*(6*cot(1/2*f*x+1/2*e)^5-10*cot(1/2*f*x+1/2*e)^3+15*f*x+30*cot(1/2
*f*x+1/2*e))/c^3/f
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int\frac{(a+a\sec(e+fx))^{3}}{(c-c\sec(e+fx))^{3}}\,dx$$
$$=\frac{46\,a^{3}\cos\left(fx+e\right)^{3}-2\,a^{3}\cos\left(fx+e\right)^{2}-22\,a^{3}\cos\left(fx+e\right)+26\,a^{3}+15\,\left(a^{3}fx\cos\left(fx+e\right)^{2}-2\,a^{3}fx\cos\right.}{15\,\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(46*a^3*cos(f*x + e)^3 - 2*a^3*cos(f*x + e)^2 - 22*a^3*cos(f*x + e) +
 26*a^3 + 15*(a^3*f*x*cos(f*x + e)^2 - 2*a^3*f*x*cos(f*x + e) + a^3*f*x)*s
in(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*
x + e))
```

3.18.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = -\frac{a^3\left(\int \frac{3\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int \frac{3\sec^2(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int \frac{\sec^3}{\sec^3(e+fx)-3\sec^2(}\right.}{c^3}$$

input
```
integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

output
```
-a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*se
c(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(
e + f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e +
f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(1/(sec(e
+ f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

3.18.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(90) = 180.

Time = 0.29 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.76

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{a^3\left(\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^3}-\frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right)+\frac{a^3\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+3\right)(\cos}{c^3\sin(fx+e)^5}}{60f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/60*(a^3*(120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3 - (20*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*
(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5)) + a^3*(10*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e)
 + 1)^5/(c^3*sin(f*x + e)^5) - 3*a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3
*sin(f*x + e)^5) - 9*a^3*(5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(
f*x + e) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)a^3}{c^3}+\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-5\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^3\right)}{c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{15\,f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/15*(15*(f*x + e)*a^3/c^3 + 2*(15*a^3*tan(1/2*f*x + 1/2*e)^4 - 5*a^3*tan(
1/2*f*x + 1/2*e)^2 + 3*a^3)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

3.18.9 Mupad [B] (verification not implemented)

Time = 14.50 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$$
$$= \frac{a^3\,x}{c^3}+\frac{\frac{2\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}-\frac{2\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+2\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{c^3\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

input
```
int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^3,x)
```

output
```
(a^3*x)/c^3 + ((2*a^3*cos(e/2 + (f*x)/2)^5)/5 + 2*a^3*cos(e/2 + (f*x)/2)*s
in(e/2 + (f*x)/2)^4 - (2*a^3*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^2)/3)
/(c^3*f*sin(e/2 + (f*x)/2)^5)
```

3.19 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$

3.19.1 Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{a^3x}{c^4} - \frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4} + \frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3} - \frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2} - \frac{167a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))}$$

output
```
a^3*x/c^4-8/7*a^3*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^4+4/35*a^3*tan(f*x+e)/c^
4/f/(1-sec(f*x+e))^3-62/105*a^3*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^2-167/105*
a^3*tan(f*x+e)/c^4/f/(1-sec(f*x+e))
```

3.19.2 Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.21

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{a^{5/2}\tan(e+fx)\left(\sqrt{a}\sqrt{c}(-337+276\sec(e+fx)+50\sec^2(e+fx)-396\sec^3(e+fx)+167\sec^4(e+fx\ldots\right.}{105c^{9/2}f(-1+\sec(e+fx))^4(1+\ldots}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^4,x]
```

output
```
(a^(5/2)*Tan[e + f*x]*(Sqrt[a]*Sqrt[c]*(-337 + 276*Sec[e + f*x] + 50*Sec[e
 + f*x]^2 - 396*Sec[e + f*x]^3 + 167*Sec[e + f*x]^4) - 840*ArcTanh[Sqrt[-(
a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sec[e + f*x]^3*Sin[(e + f*x)/2]^6*
Sqrt[-(a*c*Tan[e + f*x]^2)]))/(105*c^(9/2)*f*(-1 + Sec[e + f*x])^4*(1 + Se
c[e + f*x]))
```

3.19.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^3(e+fx)a^3}{(1-\sec(e+fx))^4}+\frac{3\sec^2(e+fx)a^3}{(1-\sec(e+fx))^4}+\frac{3\sec(e+fx)a^3}{(1-\sec(e+fx))^4}+\frac{a^3}{(1-\sec(e+fx))^4}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{167a^3\tan(e+fx)}{105f(1-\sec(e+fx))}-\frac{62a^3\tan(e+fx)}{105f(1-\sec(e+fx))^2}+\frac{4a^3\tan(e+fx)}{35f(1-\sec(e+fx))^3}-\frac{8a^3\tan(e+fx)}{7f(1-\sec(e+fx))^4}+a^3x}{c^4}$$

input
```
Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^4,x]
```

output
```
(a^3*x - (8*a^3*Tan[e + f*x])/(7*f*(1 - Sec[e + f*x])^4) + (4*a^3*Tan[e +
f*x])/(35*f*(1 - Sec[e + f*x])^3) - (62*a^3*Tan[e + f*x])/(105*f*(1 - Sec[
e + f*x])^2) - (167*a^3*Tan[e + f*x])/(105*f*(1 - Sec[e + f*x])))/c^4
```

3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.19.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50

method	result
parallelrisch	$-\frac{a^3\left(15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^7-42\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5+70\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3-105fx-210\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{105c^4f}$
derivativedivides	$\frac{a^3\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{2}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,c^4}$
default	$\frac{a^3\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{2}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,c^4}$
risch	$\frac{a^3x}{c^4}+\frac{2ia^3\left(735\,\mathrm{e}^{6i(fx+e)}-2520\,\mathrm{e}^{5i(fx+e)}+5635\,\mathrm{e}^{4i(fx+e)}-6160\,\mathrm{e}^{3i(fx+e)}+4557\,\mathrm{e}^{2i(fx+e)}-1624\,\mathrm{e}^{i(fx+e)}+337\right)}{105f\,c^4\left(\mathrm{e}^{i(fx+e)}-1\right)^7}$
norman	$\frac{\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{c}+\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{c}-\frac{a^3}{7cf}+\frac{24a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{35cf}-\frac{169a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{105cf}+\frac{56a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{15cf}-\frac{14a^3\tan\left(\frac{fx}{2}+\dots\right.}{3cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/105*a^3*(15*cot(1/2*f*x+1/2*e)^7-42*cot(1/2*f*x+1/2*e)^5+70*cot(1/2*f*x
+1/2*e)^3-105*f*x-210*cot(1/2*f*x+1/2*e))/c^4/f
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^4}\,dx$$
$$= \frac{337\,a^3\cos\left(fx + e\right)^4 - 276\,a^3\cos\left(fx + e\right)^3 - 50\,a^3\cos\left(fx + e\right)^2 + 396\,a^3\cos\left(fx + e\right) - 167\,a^3 + 105\,(a^3 \ldots}{105\,\left(c^4 f\cos\left(fx + e\right)^3 - 3\,c^4 f\cos\left(fx + e\right)^2 + 3\,c^4 f\cos \ldots\right.}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
1/105*(337*a^3*cos(f*x + e)^4 - 276*a^3*cos(f*x + e)^3 - 50*a^3*cos(f*x +
e)^2 + 396*a^3*cos(f*x + e) - 167*a^3 + 105*(a^3*f*x*cos(f*x + e)^3 - 3*a^
3*f*x*cos(f*x + e)^2 + 3*a^3*f*x*cos(f*x + e) - a^3*f*x)*sin(f*x + e))/((c
^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*
f)*sin(f*x + e))
```

3.19.6 Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^4}\,dx$$
$$= \frac{a^3\left(\int \frac{3\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx + \int \frac{3\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx \ldots\right.}{c \ldots}$$

input
```
integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

output
```
a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec
(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e
 + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1),
x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec
(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(1/(sec(e + f*x)**4 - 4*s
ec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(117) = 234$.

Time = 0.34 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.88

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^4}\, dx$$
$$= \frac{5\,a^3\left(\frac{336\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^4} + \frac{\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{77\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{315\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - 3\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}\right) + \frac{3\,a^3\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{35}{(\cos}\right.}{}}{}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
1/840*(5*a^3*(336*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^4 + (21*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 77*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3
15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(cos(f*x + e) + 1)^7/(c^4*sin(
f*x + e)^7)) + 3*a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15
)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + 9*a^3*(21*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) -
 a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*x + e)
+ 1)^7/(c^4*sin(f*x + e)^7))/f
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^4}\, dx$$
$$= \frac{\frac{105\,(fx+e)a^3}{c^4} + \frac{210\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6 - 70\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 42\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 15\,a^3}{c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}}{105\,f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/105*(105*(f*x + e)*a^3/c^4 + (210*a^3*tan(1/2*f*x + 1/2*e)^6 - 70*a^3*ta
n(1/2*f*x + 1/2*e)^4 + 42*a^3*tan(1/2*f*x + 1/2*e)^2 - 15*a^3)/(c^4*tan(1/
2*f*x + 1/2*e)^7))/f
```

3.19.9 Mupad [B] (verification not implemented)

Time = 14.80 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{a^3\left(-\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^7}{7}+\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{5}-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{3}+2\,\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\,\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6+(e-\right.}{c^4\,f\,\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^7}$$

input
```
int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^4,x)
```

output
```
(a^3*(2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^6 - cos(e/2 + (f*x)/2)^7/7 +
 sin(e/2 + (f*x)/2)^7*(e + f*x) - (2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/
2)^4)/3 + (2*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^2)/5))/(c^4*f*sin(e/2
 + (f*x)/2)^7)
```

3.20 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$

3.20.1 Optimal result

Integrand size = 26, antiderivative size = 164

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx &= \frac{a^3x}{c^5}-\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}\\&\quad+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}\\&\quad-\frac{181a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2}-\frac{496a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))}\end{aligned}$$

output
```
a^3*x/c^5-8/9*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^5+4/63*a^3*tan(f*x+e)/c^
5/f/(1-sec(f*x+e))^4-38/105*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^3-181/315*
a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^2-496/315*a^3*tan(f*x+e)/c^5/f/(1-sec(
f*x+e))
```

3.20.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.85

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{a^3\csc^9(e+fx)\left(48242+81711\cos(e+fx)+59544\cos(2(e+fx))+45591\cos(3(e+fx))+30744\cos(\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^5,x]
```

output
```
(a^3*Csc[e + f*x]^9*(48242 + 81711*Cos[e + f*x] + 59544*Cos[2*(e + f*x)] +
 45591*Cos[3*(e + f*x)] + 30744*Cos[4*(e + f*x)] + 13221*Cos[5*(e + f*x)]
+ 4200*Cos[6*(e + f*x)] + 1656*Cos[7*(e + f*x)] + 630*Cos[8*(e + f*x)] + 6
1*Cos[9*(e + f*x)] + 1120*Cos[e + f*x]^9*Hypergeometric2F1[-9/2, 1, -7/2,
-Tan[e + f*x]^2]))/(10080*c^5*f)
```

3.20.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^3(e+fx)a^3}{(1-\sec(e+fx))^5}+\frac{3\sec^2(e+fx)a^3}{(1-\sec(e+fx))^5}+\frac{3\sec(e+fx)a^3}{(1-\sec(e+fx))^5}+\frac{a^3}{(1-\sec(e+fx))^5}\right)dx}{c^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{496a^3\tan(e+fx)}{315f(1-\sec(e+fx))}-\frac{181a^3\tan(e+fx)}{315f(1-\sec(e+fx))^2}-\frac{38a^3\tan(e+fx)}{105f(1-\sec(e+fx))^3}+\frac{4a^3\tan(e+fx)}{63f(1-\sec(e+fx))^4}-\frac{8a^3\tan(e+fx)}{9f(1-\sec(e+fx))^5}+a^3x}{c^5}$$

input
```
Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^5,x]
```

output
```
(a^3*x - (8*a^3*Tan[e + f*x])/(9*f*(1 - Sec[e + f*x])^5) + (4*a^3*Tan[e +
f*x])/(63*f*(1 - Sec[e + f*x])^4) - (38*a^3*Tan[e + f*x])/(105*f*(1 - Sec[
e + f*x])^3) - (181*a^3*Tan[e + f*x])/(315*f*(1 - Sec[e + f*x])^2) - (496*
a^3*Tan[e + f*x])/(315*f*(1 - Sec[e + f*x])))/c^5
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.20.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

method	result
parallelrisch	$\frac{a^3\left(35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^9-135\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^7+252\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5-420\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+630fx+1260\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{630c^5f}$
derivativedivides	$\frac{a^3\left(4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2f\,c^5}$
default	$\frac{a^3\left(4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2f\,c^5}$
risch	$\frac{a^3x}{c^5}+\frac{2ia^3\left(2520\,e^{8i(fx+e)}-12285\,e^{7i(fx+e)}+36645\,e^{6i(fx+e)}-61425\,e^{5i(fx+e)}+71001\,e^{4i(fx+e)}-51639\,e^{3i(fx+e)}+2\right.}{315f\,c^5\left(e^{i(fx+e)}-1\right)^9}$
norman	$\frac{\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{c}+\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{c}+\frac{a^3}{18cf}-\frac{41a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{126cf}+\frac{557a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{630cf}-\frac{353a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{210cf}+\frac{56a^3\tan\left(\frac{fx}{2}\right.}{15cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output
```
1/630*a^3*(35*cot(1/2*f*x+1/2*e)^9-135*cot(1/2*f*x+1/2*e)^7+252*cot(1/2*f*
x+1/2*e)^5-420*cot(1/2*f*x+1/2*e)^3+630*f*x+1260*cot(1/2*f*x+1/2*e))/c^5/f
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.29

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^5}\,dx$$
$$= \frac{1051\,a^3\cos\left(fx+e\right)^5 - 1684\,a^3\cos\left(fx+e\right)^4 + 898\,a^3\cos\left(fx+e\right)^3 + 1468\,a^3\cos\left(fx+e\right)^2 - 1669\,a^3\cos}{315\left(c^5 f\cos\left(fx+e\right)^4 - 4\,c^5 f\cos\left(fx\right.\right.}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
1/315*(1051*a^3*cos(f*x + e)^5 - 1684*a^3*cos(f*x + e)^4 + 898*a^3*cos(f*x
 + e)^3 + 1468*a^3*cos(f*x + e)^2 - 1669*a^3*cos(f*x + e) + 496*a^3 + 315*
(a^3*f*x*cos(f*x + e)^4 - 4*a^3*f*x*cos(f*x + e)^3 + 6*a^3*f*x*cos(f*x + e
)^2 - 4*a^3*f*x*cos(f*x + e) + a^3*f*x)*sin(f*x + e))/((c^5*f*cos(f*x + e)
^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e
) + c^5*f)*sin(f*x + e))
```

3.20.6 Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^5}\,dx =$$
$$-\frac{a^3\left(\int \frac{3\sec\left(e+fx\right)}{\sec^5\left(e+fx\right) - 5\sec^4\left(e+fx\right) + 10\sec^3\left(e+fx\right) - 10\sec^2\left(e+fx\right) + 5\sec\left(e+fx\right) - 1}\,dx + \int \frac{3\sec^2\left(e+f\right.}{\sec^5\left(e+fx\right) - 5\sec^4\left(e+fx\right) + 10\sec^3\left(e+fx\right)}\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

output
```
-a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*s
ec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3
*sec(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3
 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3
/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*
x)**2 + 5*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**5 - 5*sec(e +
f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x
))/c**5
```

3.20.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(144) = 288$.

Time = 0.36 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.46

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{a^3\left(\frac{10080\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^5} - \frac{\left(\frac{270\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{1008\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{2730\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{9765\sin(fx+e)^8}{(\cos(fx+e)+1)^8} - 35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}\right) - \frac{3a^3\left(\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
1/5040*(a^3*(10080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^5 - (270*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 1008*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + 2730*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9765*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)) - 3*a^3*(180
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x + e)^8/(cos(f
*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 15*a^3*(1
8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) +
1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(c
^5*sin(f*x + e)^9) - 7*a^3*(18*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 45*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x +
 e)^9))/f
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.63

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{\frac{630(fx+e)a^3}{c^5} + \frac{1260a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8 - 420a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6 + 252a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4 - 135a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 35a^3}{c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9}}{630f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
1/630*(630*(f*x + e)*a^3/c^5 + (1260*a^3*tan(1/2*f*x + 1/2*e)^8 - 420*a^3*
tan(1/2*f*x + 1/2*e)^6 + 252*a^3*tan(1/2*f*x + 1/2*e)^4 - 135*a^3*tan(1/2*
f*x + 1/2*e)^2 + 35*a^3)/(c^5*tan(1/2*f*x + 1/2*e)^9))/f
```

3.20.9 Mupad [B] (verification not implemented)

Time = 16.94 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.89

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^5}\,dx$$
$$= \frac{a^3\left(\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^9}{18} - \frac{3\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^7\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{14} + \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{5} - \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6}{3} + 2\cos\left(\frac{e}{2}+\frac{f}{2}\right.\right.}{c^5 f \sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9}$$

input
```
int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^5,x)
```

output
```
(a^3*(cos(e/2 + (f*x)/2)^9/18 + 2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^8
+ sin(e/2 + (f*x)/2)^9*(e + f*x) - (2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)
/2)^6)/3 + (2*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^4)/5 - (3*cos(e/2 +
(f*x)/2)^7*sin(e/2 + (f*x)/2)^2)/14))/(c^5*f*sin(e/2 + (f*x)/2)^9)
```

3.21 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

3.21.1 Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{c^5x}{a^2} - \frac{47c^5\operatorname{arctanh}(\sin(e+fx))}{2a^2f} + \frac{13c^5\tan(e+fx)}{2a^2f} + \frac{112c^5\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{32c^5\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c^5-c^5\sec(e+fx))\tan(e+fx)}{2a^2f}$$

output
```
c^5*x/a^2-47/2*c^5*arctanh(sin(f*x+e))/a^2/f+13/2*c^5*tan(f*x+e)/a^2/f+112
/3*c^5*tan(f*x+e)/a^2/f/(1+sec(f*x+e))-32/3*c^5*tan(f*x+e)/f/(a+a*sec(f*x+
e))^2+1/2*(c^5-c^5*sec(f*x+e))*tan(f*x+e)/a^2/f
```

3.21.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.18 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.58

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$$

$$= \frac{c^{9/2}\tan(e+fx)\left(8\sqrt{a}\sqrt{c}+16\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1-\sec}\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^(9/2)*Tan[e + f*x]*(8*Sqrt[a]*Sqrt[c] + 16*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hype
rgeometric2F1[-7/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x
]] + 8*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-5/2, -3/2, -1/2, (1 + Se
c[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeo
metric2F1[-3/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] -
 4*Sqrt[a]*Sqrt[c]*Sec[e + f*x] - 4*Sqrt[a]*Sqrt[c]*Sec[e + f*x]^2 - 3*Arc
Tanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[-(a*c*Tan[e + f*x
]^2)] - 3*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sec[e + f
*x]*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(3*a^(5/2)*f*(-1 + Sec[e + f*x])*(1 + Se
c[e + f*x])^2)
```

3.21.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^5}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^4(e+fx)(c-c\sec(e+fx))^7dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^7}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{4374}$$

$$\int \left(\cot^4(e+fx)c^7+35\csc^4(e+fx)c^7-35\cot(e+fx)\csc^3(e+fx)c^7-\csc^4(e+fx)\sec^3(e+fx)c^7+21\cot^2(e\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{47c^7\text{arctanh}(\sin(e+fx))}{2f}+\frac{7c^7\tan(e+fx)}{f}-\frac{64c^7\cot^3(e+fx)}{3f}-\frac{48c^7\cot(e+fx)}{f}+\frac{131c^7\csc^3(e+fx)}{6f}+\frac{33c^7\csc(e+fx)}{2f}-\frac{c^7\csc^3(e+}{}}{a^2c^2}$$

input
```
Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^7*x - (47*c^7*ArcTanh[Sin[e + f*x]])/(2*f) - (48*c^7*Cot[e + f*x])/f -
(64*c^7*Cot[e + f*x]^3)/(3*f) + (33*c^7*Csc[e + f*x])/(2*f) + (131*c^7*Csc
[e + f*x]^3)/(6*f) - (c^7*Csc[e + f*x]^3*Sec[e + f*x]^2)/(2*f) + (7*c^7*Ta
n[e + f*x])/f)/(a^2*c^2)
```

3.21.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.21.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{16c^5\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8}+\frac{1}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{15}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{47\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{32}\right.}{f\,a^2}$
default	$\frac{16c^5\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8}+\frac{1}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{15}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{47\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{32}\right.}{f\,a^2}$
parallelrisch	$\frac{125\left(\frac{94(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{125}+\frac{94(-1-\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{125}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\cos(fx+e)+\frac{61\cos(2f}{75}\right.\right.}{4f\,a^2(1+\cos(2fx+2e))}$
risch	$\frac{c^5x}{a^2}+\frac{ic^5\left(99\,e^{6i(fx+e)}+435\,e^{5i(fx+e)}+484\,e^{4i(fx+e)}+930\,e^{3i(fx+e)}+575\,e^{2i(fx+e)}+507\,e^{i(fx+e)}+202\right)}{3f\,a^2\left(e^{i(fx+e)}+1\right)^3\left(1+e^{2i(fx+e)}\right)^2}-\frac{47c^5\ln(e}{2}$
norman	$\frac{\frac{c^5x}{a}+\frac{c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{a}-\frac{4c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{a}+\frac{6c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{a}-\frac{4c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{a}+\frac{45c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{491c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4a}$

input
```
int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
16/f*c^5/a^2*(1/3*tan(1/2*f*x+1/2*e)^3+2*tan(1/2*f*x+1/2*e)+1/8*arctan(tan
(1/2*f*x+1/2*e))+1/32/(tan(1/2*f*x+1/2*e)+1)^2-15/32/(tan(1/2*f*x+1/2*e)+1
)-47/32*ln(tan(1/2*f*x+1/2*e)+1)-1/32/(tan(1/2*f*x+1/2*e)-1)^2-15/32/(tan(
1/2*f*x+1/2*e)-1)+47/32*ln(tan(1/2*f*x+1/2*e)-1))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.78

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{12\,c^5fx\cos(fx+e)^4+24\,c^5fx\cos(fx+e)^3+12\,c^5fx\cos(fx+e)^2-141\left(c^5\cos(fx+e)^4+2\,c^5\cos(f\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/12*(12*c^5*f*x*cos(f*x + e)^4 + 24*c^5*f*x*cos(f*x + e)^3 + 12*c^5*f*x*c
os(f*x + e)^2 - 141*(c^5*cos(f*x + e)^4 + 2*c^5*cos(f*x + e)^3 + c^5*cos(f
*x + e)^2)*log(sin(f*x + e) + 1) + 141*(c^5*cos(f*x + e)^4 + 2*c^5*cos(f*x
 + e)^3 + c^5*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) + 2*(202*c^5*cos(f*x
+ e)^3 + 305*c^5*cos(f*x + e)^2 + 36*c^5*cos(f*x + e) - 3*c^5)*sin(f*x + e
))/(a^2*f*cos(f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e)^2)
```

3.21.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^5\left(\int \frac{5\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int\left(-\frac{10\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx + \int \frac{10\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int\left(-\right.\right.}{a^2}$$

input
```
integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

output
```
-c**5*(Integral(5*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(-10*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(10*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(-5*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(127) = 254$.

Time = 0.35 (sec) , antiderivative size = 603, normalized size of antiderivative = 4.43

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{c^5\left(\frac{6\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2-\frac{2a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}} + \frac{\frac{21\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2} + \frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right) + 5\,c}{}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/6*(c^5*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + si
n(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e)
 + 1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 5*c^5*
((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3
)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x +
 e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 10*c^5*((9*sin(f*x + e)/(c
os(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*
x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1)
 - 1)/a^2) - c^5*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos
(f*x + e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) +
10*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - 5*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)/a^2)/f
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$$

$$= \frac{\frac{6\,(fx+e)c^5}{a^2} - \frac{141\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2} + \frac{141\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2} - \frac{6\left(15\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-13\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a^2} + 32}{6\,f}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/6*(6*(f*x + e)*c^5/a^2 - 141*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2
+ 141*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 - 6*(15*c^5*tan(1/2*f*x +
 1/2*e)^3 - 13*c^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a
^2) + 32*(a^4*c^5*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c^5*tan(1/2*f*x + 1/2*e))
/a^6)/f
```

3.21.9 Mupad [B] (verification not implemented)

Time = 13.64 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{c^5\,x}{a^2} - \frac{15\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3 - 13\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 - 2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + a^2\right)} + \frac{32\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f} + \frac{16\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f} - \frac{47\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}$$

input
```
int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^2,x)
```

output
```
(c^5*x)/a^2 - (15*c^5*tan(e/2 + (f*x)/2)^3 - 13*c^5*tan(e/2 + (f*x)/2))/(f
*(a^2*tan(e/2 + (f*x)/2)^4 - 2*a^2*tan(e/2 + (f*x)/2)^2 + a^2)) + (32*c^5*
tan(e/2 + (f*x)/2))/(a^2*f) + (16*c^5*tan(e/2 + (f*x)/2)^3)/(3*a^2*f) - (4
7*c^5*atanh(tan(e/2 + (f*x)/2)))/(a^2*f)
```

3.22 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

3.22.1 Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{c^4x}{a^2}-\frac{6c^4\text{arctanh}(\sin(e+fx))}{a^2f}-\frac{16c^4\cot(e+fx)}{a^2f} -\frac{32c^4\cot^3(e+fx)}{3a^2f}+\frac{32c^4\csc^3(e+fx)}{3a^2f}+\frac{c^4\tan(e+fx)}{a^2f}$$

output
```
c^4*x/a^2-6*c^4*arctanh(sin(f*x+e))/a^2/f-16*c^4*cot(f*x+e)/a^2/f-32/3*c^4
*cot(f*x+e)^3/a^2/f+32/3*c^4*csc(f*x+e)^3/a^2/f+c^4*tan(f*x+e)/a^2/f
```

3.22.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.44 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.92

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^{7/2}\tan(e+fx)\left(-8\sqrt{a}\sqrt{c}-8\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1-}\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^2,x]
```

output
```
-1/3*(c^(7/2)*Tan[e + f*x]*(-8*Sqrt[a]*Sqrt[c] - 8*Sqrt[2]*Sqrt[a]*Sqrt[c]
*Hypergeometric2F1[-5/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e
+ f*x]] - 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1
 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] + 4*Sqrt[a]*Sqrt[c]*Sec[e + f*x
] + 4*Sqrt[a]*Sqrt[c]*Sec[e + f*x]^2 + 3*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2
)]/(Sqrt[a]*Sqrt[c])]*Sqrt[-(a*c*Tan[e + f*x]^2)] + 3*ArcTanh[Sqrt[-(a*c*T
an[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(a*c*Tan[e + f*x]^2)
]))/(a^(5/2)*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^2)
```

3.22.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^4}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^4(e+fx)(c-c\sec(e+fx))^6dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^6}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{4374}$$

$$\frac{\int \left(\cot^4(e+fx)c^6+15\csc^4(e+fx)c^6-20\cot(e+fx)\csc^3(e+fx)c^6+15\cot^2(e+fx)\csc^2(e+fx)c^6+\csc^4(e\right.}{a^2c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{6c^6\text{arctanh}(\sin(e+fx))}{f}+\frac{c^6\tan(e+fx)}{f}-\frac{32c^6\cot^3(e+fx)}{3f}-\frac{16c^6\cot(e+fx)}{f}+\frac{32c^6\csc^3(e+fx)}{3f}+c^6x}{a^2c^2}$$

input
```
Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^6*x - (6*c^6*ArcTanh[Sin[e + f*x]])/f - (16*c^6*Cot[e + f*x])/f - (32*c
^6*Cot[e + f*x]^3)/(3*f) + (32*c^6*Csc[e + f*x]^3)/(3*f) + (c^6*Tan[e + f*
x])/f)/(a^2*c^2)
```

3.22.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.22.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{8c^4\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\right.}{f\,a^2}$
default	$\frac{8c^4\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\right.}{f\,a^2}$
parallelrisch	$\frac{6\left(\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)+\frac{19\left(\cos(fx+e)+\frac{\cos(2fx+2e)}{4}+\frac{25}{76}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\right.}{18}\right.}{f\,a^2\cos(fx+e)}$
risch	$\frac{c^4x}{a^2}+\frac{2ic^4\left(51\,\mathrm{e}^{3i(fx+e)}+25\,\mathrm{e}^{2i(fx+e)}+57\,\mathrm{e}^{i(fx+e)}+19\right)}{3f\,a^2\left(1+\mathrm{e}^{2i(fx+e)}\right)\left(\mathrm{e}^{i(fx+e)}+1\right)^3}+\frac{6c^4\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{a^2f}-\frac{6c^4\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{a^2f}$
norman	$\frac{\frac{c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{a}-\frac{c^4x}{a}-\frac{10c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{76c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}-\frac{18c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{af}+\frac{8c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{3af}+\frac{3c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{a}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3a}$

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
8/f*c^4/a^2*(1/3*tan(1/2*f*x+1/2*e)^3+tan(1/2*f*x+1/2*e)-1/8/(tan(1/2*f*x+
1/2*e)+1)-3/4*ln(tan(1/2*f*x+1/2*e)+1)-1/8/(tan(1/2*f*x+1/2*e)-1)+3/4*ln(t
an(1/2*f*x+1/2*e)-1)+1/4*arctan(tan(1/2*f*x+1/2*e)))
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(98) = 196$.

Time = 0.30 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.16

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{3\,c^4fx\cos\left(fx+e\right)^3+6\,c^4fx\cos\left(fx+e\right)^2+3\,c^4fx\cos\left(fx+e\right)-9\,\left(c^4\cos\left(fx+e\right)^3+2\,c^4\cos\left(fx+e\right)^2\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(3*c^4*f*x*cos(f*x + e)^3 + 6*c^4*f*x*cos(f*x + e)^2 + 3*c^4*f*x*cos(f
*x + e) - 9*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))
*log(sin(f*x + e) + 1) + 9*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^
4*cos(f*x + e))*log(-sin(f*x + e) + 1) + (19*c^4*cos(f*x + e)^2 + 38*c^4*c
os(f*x + e) + 3*c^4)*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x
 + e)^2 + a^2*f*cos(f*x + e))
```

3.22.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{c^4\left(\int\left(-\frac{4\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{6\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\left(-\frac{4\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\right.}{a^2}$$

input
```
integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

output
```
c**4*(Integral(-4*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(6*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(-4*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integ
ral(1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(98) = 196$.

Time = 0.32 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.05

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{c^4\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)+4}{}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/6*(c^4*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 4*c^4*((9*sin(f*x
 + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x +
 e) + 1) - 1)/a^2) - c^4*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))
/a^2) + 6*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/a^2 - 4*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.31

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^2}\, dx$$
$$= \frac{\frac{3\,(fx+e)c^4}{a^2} - \frac{18\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2} + \frac{18\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2} - \frac{6\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^2} + \frac{8\left(a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3\,a\right.}{a^6}}{3\,f}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(3*(f*x + e)*c^4/a^2 - 18*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 +
 18*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 - 6*c^4*tan(1/2*f*x + 1/2*e
)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^2) + 8*(a^4*c^4*tan(1/2*f*x + 1/2*e)^3 +
 3*a^4*c^4*tan(1/2*f*x + 1/2*e))/a^6)/f
```

3.22.9 Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^2}\, dx = \frac{c^4\,x}{a^2} + \frac{8\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f} + \frac{8\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f} - \frac{12\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f} - \frac{2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)}$$

input
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^2,x)
```

output
```
(c^4*x)/a^2 + (8*c^4*tan(e/2 + (f*x)/2))/(a^2*f) + (8*c^4*tan(e/2 + (f*x)/
2)^3)/(3*a^2*f) - (12*c^4*atanh(tan(e/2 + (f*x)/2)))/(a^2*f) - (2*c^4*tan(
e/2 + (f*x)/2))/(f*(a^2*tan(e/2 + (f*x)/2)^2 - a^2))
```

3.22. $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

3.23 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

3.23.1 Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{c^3x}{a^2}-\frac{c^3\text{arctanh}(\sin(e+fx))}{a^2f} - \frac{8c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}+\frac{4c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

output
```
c^3*x/a^2-c^3*arctanh(sin(f*x+e))/a^2/f-8/3*c^3*tan(f*x+e)/a^2/f/(1+sec(f*
x+e))^2+4/3*c^3*tan(f*x+e)/a^2/f/(1+sec(f*x+e))
```

3.23.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.22

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{c^{5/2}\tan(e+fx)\left(4\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1-\sec(e+fx)}-\right.}{3a^{5/2}f(-1+\text{s}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^(5/2)*Tan[e + f*x]*(4*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-3/2, -
3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] - 4*Sqrt[a]*Sqrt[c
]*(-2 + Sec[e + f*x] + Sec[e + f*x]^2) - 6*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]
^2)]/(Sqrt[a]*Sqrt[c])]*Cos[(e + f*x)/2]^2*Sec[e + f*x]*Sqrt[-(a*c*Tan[e +
 f*x]^2)]))/(3*a^(5/2)*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^2)
```

3.23.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^3}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(-\frac{\sec^3(e+fx)c^3}{(\sec(e+fx)+1)^2}+\frac{3\sec^2(e+fx)c^3}{(\sec(e+fx)+1)^2}-\frac{3\sec(e+fx)c^3}{(\sec(e+fx)+1)^2}+\frac{c^3}{(\sec(e+fx)+1)^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{c^3\text{arctanh}(\sin(e+fx))}{f}+\frac{4c^3\tan(e+fx)}{3f(\sec(e+fx)+1)}-\frac{8c^3\tan(e+fx)}{3f(\sec(e+fx)+1)^2}+c^3x}{a^2}$$

input
```
Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^3*x - (c^3*ArcTanh[Sin[e + f*x]])/f - (8*c^3*Tan[e + f*x])/(3*f*(1 + Se
c[e + f*x])^2) + (4*c^3*Tan[e + f*x])/(3*f*(1 + Sec[e + f*x])))/a^2
```

3.23.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.23.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{c^{3} \left(4 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+3 f x+3 \ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)-3 \ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)\right)}{3 a^{2} f}$
derivativedivides	$\frac{4 c^{3} \left(\frac{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{\arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{4}\right)}{f \,a^{2}}$
default	$\frac{4 c^{3} \left(\frac{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{\arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{4}\right)}{f \,a^{2}}$
risch	$\frac{c^{3} x}{a^{2}}-\frac{8 i c^{3} \left(3 \,{\mathrm e}^{2 i \left(f x +e \right)}+1\right)}{3 f \,a^{2} \left({\mathrm e}^{i \left(f x +e \right)}+1\right)^{3}}+\frac{c^{3} \ln \left({\mathrm e}^{i \left(f x +e \right)}-i\right)}{a^{2} f}-\frac{c^{3} \ln \left({\mathrm e}^{i \left(f x +e \right)}+i\right)}{a^{2} f}$
norman	$\frac{\frac{c^{3} x}{a}+\frac{c^{3} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{a}+\frac{4 c^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 a f}-\frac{8 c^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{3 a f}+\frac{4 c^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}{3 a f}-\frac{2 c^{3} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{a}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{2} a}+\frac{c^{3} \ln \left(\tan \left(\frac{f x}{2}+\right.\right.}{a^{2} f}$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*c^3*(4*tan(1/2*f*x+1/2*e)^3+3*f*x+3*ln(tan(1/2*f*x+1/2*e)-1)-3*ln(tan(
1/2*f*x+1/2*e)+1))/a^2/f
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(81) = 162$.

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.04

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{6\,c^3fx\cos(fx+e)^2+12\,c^3fx\cos(fx+e)+6\,c^3fx-3\left(c^3\cos(fx+e)^2+2\,c^3\cos(fx+e)+c^3\right)\log(\sin}{6\left(a^2f\cos(fx\right.}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/6*(6*c^3*f*x*cos(f*x + e)^2 + 12*c^3*f*x*cos(f*x + e) + 6*c^3*f*x - 3*(c
^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*log(sin(f*x + e) + 1) + 3*(c
^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*log(-sin(f*x + e) + 1) - 8*(
c^3*cos(f*x + e) - c^3)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(
f*x + e) + a^2*f)
```

3.23.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx =$$
$$-\frac{c^3\left(\int \frac{3\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\left(-\frac{3\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\left(-\right.\right.}{a^2}$$

input
```
integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

output
```
-c**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(-3*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

3.23.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(81) = 162.

Time = 0.29 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.15

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{c^3\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)-c^3\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\,\mathrm{arct}}{}\right.}{6f}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/6*(c^3*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin
(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - c^3*((9*sin(f*x + e)/(cos(f*x + e
) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a^2) + 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin
(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 3*c^3*(3*sin(f*x + e)/(cos(f*x + e
) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{4\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}{a^2}+\frac{3\,(fx+e)c^3}{a^2}-\frac{3\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}+\frac{3\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}}{3\,f}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(4*c^3*tan(1/2*f*x + 1/2*e)^3/a^2 + 3*(f*x + e)*c^3/a^2 - 3*c^3*log(ab
s(tan(1/2*f*x + 1/2*e) + 1))/a^2 + 3*c^3*log(abs(tan(1/2*f*x + 1/2*e) - 1)
)/a^2)/f
```

3.23.9 Mupad [B] (verification not implemented)

Time = 13.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.54

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx=\frac{c^3\,x}{a^2}-\frac{c^3\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}\right)}{a^2\,f}$$

input
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^2,x)
```

output
```
(c^3*x)/a^2 - (c^3*(2*atanh(tan(e/2 + (f*x)/2)) - (4*tan(e/2 + (f*x)/2)^3)
/3))/(a^2*f)
```

3.24 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

3.24.1 Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx=\frac{c^2x}{a^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

output
```
c^2*x/a^2-4/3*c^2*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2-4/3*c^2*tan(f*x+e)/a^2
/f/(1+sec(f*x+e))
```

3.24.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx=\frac{c^2\left(\frac{2\arctan\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{2\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}\right)}{a^2}$$

input
```
Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^2*((2*ArcTan[Tan[e/2 + (f*x)/2]])/f - (2*Tan[e/2 + (f*x)/2])/f + (2*Tan
[e/2 + (f*x)/2]^3)/(3*f)))/a^2
```

3.24.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^2}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^2(e+fx)c^2}{(\sec(e+fx)+1)^2}-\frac{2\sec(e+fx)c^2}{(\sec(e+fx)+1)^2}+\frac{c^2}{(\sec(e+fx)+1)^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{4c^2\tan(e+fx)}{3f(\sec(e+fx)+1)}-\frac{4c^2\tan(e+fx)}{3f(\sec(e+fx)+1)^2}+c^2x}{a^2}$$

input
```
Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^2*x - (4*c^2*Tan[e + f*x])/(3*f*(1 + Sec[e + f*x])^2) - (4*c^2*Tan[e +
f*x])/(3*f*(1 + Sec[e + f*x])))/a^2
```

3.24.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.24.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

method	result	size
parallelrisch	$\frac{c^{2}\left(2 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+3 f x -6 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{3 a^{2} f}$	41
derivativedivides	$\frac{2 c^{2}\left(\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}-\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f \,a^{2}}$	47
default	$\frac{2 c^{2}\left(\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}-\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f \,a^{2}}$	47
risch	$\frac{c^{2} x}{a^{2}}-\frac{8 i c^{2} \left(3 \,{\mathrm e}^{2 i \left(f x +e \right)}+3 \,{\mathrm e}^{i \left(f x +e \right)}+2\right)}{3 f \,a^{2} \left({\mathrm e}^{i \left(f x +e \right)}+1\right)^{3}}$	59
norman	$\frac{\frac{c^{2} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{a}-\frac{c^{2} x}{a}+\frac{2 c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{a f}-\frac{8 c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 a f}+\frac{2 c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{3 a f}}{a \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)}$	113

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*c^2*(2*tan(1/2*f*x+1/2*e)^3+3*f*x-6*tan(1/2*f*x+1/2*e))/a^2/f
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{3\,c^2fx\cos(fx+e)^2+6\,c^2fx\cos(fx+e)+3\,c^2fx-4\,(2\,c^2\cos(fx+e)+c^2)\sin(fx+e)}{3\,(a^2f\cos(fx+e)^2+2\,a^2f\cos(fx+e)+a^2f)}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(3*c^2*f*x*cos(f*x + e)^2 + 6*c^2*f*x*cos(f*x + e) + 3*c^2*f*x - 4*(2*
c^2*cos(f*x + e) + c^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(
f*x + e) + a^2*f)
```

3.24.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{c^2\left(\int\left(-\frac{2\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\frac{1}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx\right)}{a^2}$$

input
```
integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
```

output
```
c**2*(Integral(-2*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

3.24.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(63) = 126$.

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.54

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-\frac{c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}+\frac{2c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+}{(\cos(fx+e}\right.}{a^2}}{6f}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/6*(c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^2*(3*
sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2
 + 2*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/a^2)/f
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{3\,(fx+e)c^2}{a^2}+\frac{2\left(a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{3\,f}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(3*(f*x + e)*c^2/a^2 + 2*(a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^2*t
an(1/2*f*x + 1/2*e))/a^6)/f
```

3.24.9 Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{2\,c^2\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{3\,f\,x}{2}\right)}{3\,a^2\,f}$$

input
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^2,x)
```

output
```
(2*c^2*(tan(e/2 + (f*x)/2)^3 - 3*tan(e/2 + (f*x)/2) + (3*f*x)/2))/(3*a^2*f
)
```

3.25 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx$

3.25.1 Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx=\frac{cx}{a^2}-\frac{2c\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{5c\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

output
```
c*x/a^2-2/3*c*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2-5/3*c*tan(f*x+e)/a^2/f/(1+
sec(f*x+e))
```

3.25.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 139 vs. 2(61) = 122.

Time = 0.67 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.28

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx=-\frac{\sqrt{c}\tan(e+fx)\left(\sqrt{a}\sqrt{c}(-7+2\sec(e+fx)+5\sec^2(e+fx))+6\text{arctanh}\left(\frac{\sqrt{-ac\tan^2(e+fx)}}{\sqrt{a}\sqrt{c}}\right)\cos^2\left(\frac{1}{2}(e+\right.\right.}{3a^{5/2}f(-1+\sec(e+fx))(1+\sec(e+fx))^2}$$

input
```
Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^2,x]
```

output
```
-1/3*(Sqrt[c]*Tan[e + f*x]*(Sqrt[a]*Sqrt[c]*(-7 + 2*Sec[e + f*x] + 5*Sec[e
 + f*x]^2) + 6*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Cos[
(e + f*x)/2]^2*Sec[e + f*x]*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(a^(5/2)*f*(-1 +
 Sec[e + f*x])*(1 + Sec[e + f*x])^2)
```

3.25.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-c\sec(e+fx)}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \; 4391$$

$$\frac{\int \left(\frac{c}{(\sec(e+fx)+1)^2}-\frac{c\sec(e+fx)}{(\sec(e+fx)+1)^2}\right)dx}{a^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{5c\tan(e+fx)}{3f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)}{3f(\sec(e+fx)+1)^2}+cx}{a^2}$$

input
```
Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c*x - (2*c*Tan[e + f*x])/(3*f*(1 + Sec[e + f*x])^2) - (5*c*Tan[e + f*x])/
(3*f*(1 + Sec[e + f*x])))/a^2
```

3.25.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.25.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

method	result	size
parallelrisch	$\frac{c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3fx-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3a^{2}f}$	37
derivativedivides	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^{2}}$	46
default	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^{2}}$	46
norman	$\frac{\frac{cx}{a}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3af}}{a}$	50
risch	$\frac{cx}{a^{2}}-\frac{2ic\left(9\,\mathrm{e}^{2i(fx+e)}+12\,\mathrm{e}^{i(fx+e)}+7\right)}{3f\,a^{2}\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}}$	55

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*c*(tan(1/2*f*x+1/2*e)^3+3*f*x-6*tan(1/2*f*x+1/2*e))/a^2/f
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.41

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{3\,cfx\cos\left(fx+e\right)^{2}+6\,cfx\cos\left(fx+e\right)+3\,cfx-\left(7\,c\cos\left(fx+e\right)+5\,c\right)\sin\left(fx+e\right)}{3\left(a^{2}f\cos\left(fx+e\right)^{2}+2\,a^{2}f\cos\left(fx+e\right)+a^{2}f\right)}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(3*c*f*x*cos(f*x + e)^2 + 6*c*f*x*cos(f*x + e) + 3*c*f*x - (7*c*cos(f*
x + e) + 5*c)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) +
 a^2*f)
```

3.25.6 Sympy [F]

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\left(\int \frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \left(-\frac{1}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx\right)}{a^2}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

output
```
-c*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(57) = 114$.

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right) + \frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/6*(c*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + c*(3*sin(
f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^2}\,dx = \frac{\frac{3\,(fx+e)c}{a^2} + \frac{a^4c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 6\,a^4c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^6}}{3\,f}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(3*(f*x + e)*c/a^2 + (a^4*c*tan(1/2*f*x + 1/2*e)^3 - 6*a^4*c*tan(1/2*f
*x + 1/2*e))/a^6)/f
```

3.25.9 Mupad [B] (verification not implemented)

Time = 15.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^2}\,dx = \frac{c\,x}{a^2} - \frac{c\left(6\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right) - \tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^3\right)}{3\,a^2\,f}$$

input
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^2,x)
```

output
```
(c*x)/a^2 - (c*(6*tan(e/2 + (f*x)/2) - tan(e/2 + (f*x)/2)^3))/(3*a^2*f)
```

3.26 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx$

3.26.1 Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx=\frac{x}{a^2c}+\frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf}-\frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf}$$

output
```
x/a^2/c+1/3*cot(f*x+e)*(3-2*sec(f*x+e))/a^2/c/f-1/3*cot(f*x+e)^3*(1-sec(f*
x+e))/a^2/c/f
```

3.26.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.76 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx=-\frac{\cot^3(e+fx)\left(\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)-3\sec(e+fx)+2\sec^3(e+fx)\right)}{3a^2cf}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

output
```
-1/3*(Cot[e + f*x]^3*(Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2] -
3*Sec[e + f*x] + 2*Sec[e + f*x]^3))/(a^2*c*f)
```

3.26.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 4392, 3042, 4370, 25, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^4(e+fx)(c-c\sec(e+fx))dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{4370}$$

$$\frac{\frac{1}{3}\int -\cot^2(e+fx)(3c-2c\sec(e+fx))dx-\frac{\cot^3(e+fx)(c-c\sec(e+fx))}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{1}{3}\int \cot^2(e+fx)(3c-2c\sec(e+fx))dx-\frac{\cot^3(e+fx)(c-c\sec(e+fx))}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{1}{3}\int \frac{3c-2c\csc\left(e+fx+\frac{\pi}{2}\right)}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx-\frac{\cot^3(e+fx)(c-c\sec(e+fx))}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{4370}$$

$$\frac{\frac{1}{3}\left(\frac{\cot(e+fx)(3c-2c\sec(e+fx))}{f}-\int -3cdx\right)-\frac{\cot^3(e+fx)(c-c\sec(e+fx))}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{1}{3}\left(\frac{\cot(e+fx)(3c-2c\sec(e+fx))}{f}+3cx\right)-\frac{\cot^3(e+fx)(c-c\sec(e+fx))}{3f}}{a^2c^2}$$

input
```
Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

output
```
(-1/3*(Cot[e + f*x]^3*(c - c*Sec[e + f*x]))/f + (3*c*x + (Cot[e + f*x]*(3*
c - 2*c*Sec[e + f*x]))/f)/3)/(a^2*c^2)
```

3.26.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.26.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+12fx-12\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{12a^{2}cf}$	50
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4f\,a^{2}c}$	60
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4f\,a^{2}c}$	60
risch	$\frac{x}{a^{2}c}-\frac{2i\left(3\,\mathrm{e}^{3i(fx+e)}-5\,\mathrm{e}^{i(fx+e)}-4\right)}{3f\,a^{2}c\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}\left(\mathrm{e}^{i(fx+e)}-1\right)}$	72
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{ca}+\frac{1}{4acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{12acf}}{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	89

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(tan(1/2*f*x+1/2*e)^3+12*f*x-12*tan(1/2*f*x+1/2*e)+3*cot(1/2*f*x+1/2*
e))/a^2/c/f
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{4\cos(fx+e)^2+3(fx\cos(fx+e)+fx)\sin(fx+e)+\cos(fx+e)-2}{3(a^2cf\cos(fx+e)+a^2cf)\sin(fx+e)}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*(4*cos(f*x + e)^2 + 3*(f*x*cos(f*x + e) + f*x)*sin(f*x + e) + cos(f*x
+ e) - 2)/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e))
```

3.26.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{\sec^3(e+fx)+\sec^2(e+fx)-\sec(e+fx)-1}\,dx}{a^2c}$$

input
```
integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(1/(sec(e + f*x)**3 + sec(e + f*x)**2 - sec(e + f*x) - 1), x)/(a*
*2*c)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{\frac{\frac{12\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2c}-\frac{24\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c}-\frac{3(\cos(fx+e)+1)}{a^2c\sin(fx+e)}}{12f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-1/12*((12*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3)/(a^2*c) - 24*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c) - 3*(
cos(f*x + e) + 1)/(a^2*c*sin(f*x + e)))/f
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{\frac{12(fx+e)}{a^2c}+\frac{3}{a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-12a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6c^3}}{12f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
1/12*(12*(f*x + e)/(a^2*c) + 3/(a^2*c*tan(1/2*f*x + 1/2*e)) + (a^4*c^2*tan
(1/2*f*x + 1/2*e)^3 - 12*a^4*c^2*tan(1/2*f*x + 1/2*e))/(a^6*c^3))/f
```

3.26.9 Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{x}{a^2\,c}+\frac{\frac{4\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{3}-\frac{7\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{6}+\frac{1}{12}}{a^2\,c\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}$$

input
```
int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))),x)
```

output
```
x/(a^2*c) + ((4*cos(e/2 + (f*x)/2)^4)/3 - (7*cos(e/2 + (f*x)/2)^2)/6 + 1/1
2)/(a^2*c*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2))
```

3.27 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$

3.27.1 Optimal result

Integrand size = 26, antiderivative size = 46

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{x}{a^2c^2}+\frac{\cot(e+fx)}{a^2c^2f}-\frac{\cot^3(e+fx)}{3a^2c^2f}$$

output
```
x/a^2/c^2+cot(f*x+e)/a^2/c^2/f-1/3*cot(f*x+e)^3/a^2/c^2/f
```

3.27.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = -\frac{\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)}{3a^2c^2f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

output
```
-1/3*(Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/(a
^2*c^2*f)
```

3.27.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 4392, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^4(e+fx)dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \tan\left(e+fx+\frac{\pi}{2}\right)^4dx}{a^2c^2}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-\int \cot^2(e+fx)dx-\frac{\cot^3(e+fx)}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \tan\left(e+fx+\frac{\pi}{2}\right)^2dx-\frac{\cot^3(e+fx)}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\int 1dx-\frac{\cot^3(e+fx)}{3f}+\frac{\cot(e+fx)}{f}}{a^2c^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{-\frac{\cot^3(e+fx)}{3f}+\frac{\cot(e+fx)}{f}+x}{a^2c^2}$$

input
```
Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

output
```
(x + Cot[e + f*x]/f - Cot[e + f*x]^3/(3*f))/(a^2*c^2)
```

3.27.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.27.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)-\frac{\pi}{2}+\operatorname{arccot}(\cot(fx+e))}{a^2c^2f}$	38
parallelrisch	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+24fx-15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{24f\,a^2c^2}$	63
risch	$\frac{x}{a^2c^2}+\frac{4i\left(3\,\mathrm{e}^{4i(fx+e)}-3\,\mathrm{e}^{2i(fx+e)}+2\right)}{3f\,a^2c^2\left(\mathrm{e}^{i(fx+e)}+1\right)^3\left(\mathrm{e}^{i(fx+e)}-1\right)^3}$	72
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{ca}-\frac{1}{24acf}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{8acf}-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{8acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{24acf}}{ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	116

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2/c^2/f*(-1/3*cot(f*x+e)^3+cot(f*x+e)-1/2*Pi+arccot(cot(f*x+e)))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$$
$$=\frac{4\cos(fx+e)^3+3\left(fx\cos(fx+e)^2-fx\right)\sin(fx+e)-3\cos(fx+e)}{3\left(a^2c^2f\cos(fx+e)^2-a^2c^2f\right)\sin(fx+e)}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(4*cos(f*x + e)^3 + 3*(f*x*cos(f*x + e)^2 - f*x)*sin(f*x + e) - 3*cos(
f*x + e))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))
```

3.27.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx=\frac{\int\frac{1}{\sec^4(e+fx)-2\sec^2(e+fx)+1}\,dx}{a^2c^2}$$

input
```
integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(1/(sec(e + f*x)**4 - 2*sec(e + f*x)**2 + 1), x)/(a**2*c**2)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx=\frac{\frac{3\,(fx+e)}{a^2c^2}+\frac{3\tan(fx+e)^2-1}{a^2c^2\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*(3*(f*x + e)/(a^2*c^2) + (3*tan(f*x + e)^2 - 1)/(a^2*c^2*tan(f*x + e)^
3))/f
```

3.27.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(44) = 88$.

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$$
$$= \frac{\frac{24\,(fx+e)}{a^2c^2} + \frac{15\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1}{a^2c^2\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3} + \frac{a^4c^4\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,a^4c^4\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^6c^6}}{24\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/24*(24*(f*x + e)/(a^2*c^2) + (15*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^2*c^2*ta
n(1/2*f*x + 1/2*e)^3) + (a^4*c^4*tan(1/2*f*x + 1/2*e)^3 - 15*a^4*c^4*tan(1
/2*f*x + 1/2*e))/(a^6*c^6))/f
```

3.27.9 Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$$
$$= -\frac{\cos\left(3\,e+3\,f\,x\right)+\frac{3\,\sin(3\,e+3\,f\,x)\,(e+f\,x)}{4}-\frac{9\,\sin(e+f\,x)\,(e+f\,x)}{4}}{3\,a^2\,c^2\,f\,\sin\left(e+f\,x\right)^3}$$

input
```
int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2),x)
```

output
```
-(cos(3*e + 3*f*x) + (3*sin(3*e + 3*f*x)*(e + f*x))/4 - (9*sin(e + f*x)*(e
 + f*x))/4)/(3*a^2*c^2*f*sin(e + f*x)^3)
```

3.28 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$

3.28.1 Optimal result

Integrand size = 26, antiderivative size = 98

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{x}{a^2c^3}+\frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+4\sec(e+fx))}{15a^2c^3f}+\frac{\cot(e+fx)(15+8\sec(e+fx))}{15a^2c^3f}$$

output
```
x/a^2/c^3+1/5*cot(f*x+e)^5*(1+sec(f*x+e))/a^2/c^3/f-1/15*cot(f*x+e)^3*(5+4
*sec(f*x+e))/a^2/c^3/f+1/15*cot(f*x+e)*(15+8*sec(f*x+e))/a^2/c^3/f
```

3.28.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{\cot^5(e+fx)\left(3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)+15\sec(e+fx)-20\sec^3(e+fx)+8\,s\right.}{15a^2c^3f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

output
```
(Cot[e + f*x]^5*(3*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2] + 15*
Sec[e + f*x] - 20*Sec[e + f*x]^3 + 8*Sec[e + f*x]^5))/(15*a^2*c^3*f)
```

3.28.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {3042, 4392, 3042, 4370, 25, 3042, 4370, 25, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(\sec(e+fx)a+a)dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4370}$$

$$-\frac{\frac{1}{5}\int -\cot^4(e+fx)(4\sec(e+fx)a+5a)dx-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{1}{5}\int \cot^4(e+fx)(4\sec(e+fx)a+5a)dx-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{5}\int \frac{4\csc\left(e+fx+\frac{\pi}{2}\right)a+5a}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{4370}$$

$$-\frac{\frac{1}{5}\left(\frac{\cot^3(e+fx)(4a\sec(e+fx)+5a)}{3f}-\frac{1}{3}\int -\cot^2(e+fx)(8\sec(e+fx)a+15a)dx\right)-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \cot^2(e+fx)(8\sec(e+fx)a+15a)dx+\frac{\cot^3(e+fx)(4a\sec(e+fx)+5a)}{3f}\right)-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{8\csc\left(e+fx+\frac{\pi}{2}\right)a+15a}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx+\frac{\cot^3(e+fx)(4a\sec(e+fx)+5a)}{3f}\right)-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{4370}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(\int -15adx-\frac{\cot(e+fx)(8a\sec(e+fx)+15a)}{f}\right)+\frac{\cot^3(e+fx)(4a\sec(e+fx)+5a)}{3f}\right)-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\frac{1}{5}\left(\frac{\cot^3(e+fx)(4a\sec(e+fx)+5a)}{3f}+\frac{1}{3}\left(-\frac{\cot(e+fx)(8a\sec(e+fx)+15a)}{f}-15ax\right)\right)-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

input
```
Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

output
```
-((-1/5*(Cot[e + f*x]^5*(a + a*Sec[e + f*x]))/f + ((Cot[e + f*x]^3*(5*a +
4*a*Sec[e + f*x]))/(3*f) + (-15*a*x - (Cot[e + f*x]*(15*a + 8*a*Sec[e + f*
x]))/f)/3)/5)/(a^3*c^3))
```

3.28.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.28.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

method	result	si
parallelrisch	$\frac{3 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}-30 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+5 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+240 f x+240 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)-90 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{240 f \,a^{2} c^{3}}$	78
derivativedivides	$\frac{\frac{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}-6 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)+\frac{1}{5 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{16}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)}+32 \arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{16 f \,a^{2} c^{3}}$	88
default	$\frac{\frac{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}-6 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)+\frac{1}{5 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{16}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)}+32 \arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{16 f \,a^{2} c^{3}}$	88
risch	$\frac{x}{a^{2} c^{3}}+\frac{2 i \left(15 \,{\mathrm e}^{7 i \left(f x +e \right)}+15 \,{\mathrm e}^{6 i \left(f x +e \right)}-65 \,{\mathrm e}^{5 i \left(f x +e \right)}+25 \,{\mathrm e}^{4 i \left(f x +e \right)}+73 \,{\mathrm e}^{3 i \left(f x +e \right)}-31 \,{\mathrm e}^{2 i \left(f x +e \right)}-31 \,{\mathrm e}^{i \left(f x +e \right)}+23\right)}{15 f \,a^{2} c^{3} \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{5} \left({\mathrm e}^{i \left(f x +e \right)}+1\right)^{3}}$	12
norman	$\frac{\frac{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{a c f}+\frac{x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{c a}+\frac{1}{80 a c f}-\frac{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{8 a c f}-\frac{3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{8 a c f}+\frac{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{8}}{48 a c f}}{a \,c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}$	13

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/240*(3*cot(1/2*f*x+1/2*e)^5-30*cot(1/2*f*x+1/2*e)^3+5*tan(1/2*f*x+1/2*e)
^3+240*f*x+240*cot(1/2*f*x+1/2*e)-90*tan(1/2*f*x+1/2*e))/f/a^2/c^3
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.57

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$$
$$= \frac{23\cos(fx+e)^4-8\cos(fx+e)^3-27\cos(fx+e)^2+15\left(fx\cos(fx+e)^3-fx\cos(fx+e)^2-fx\cos\right.}{15\left(a^2c^3f\cos(fx+e)^3-a^2c^3f\cos(fx+e)^2-a^2c^3f\cos(fx+e)+a^2c\right.}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(23*cos(f*x + e)^4 - 8*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + 15*(f*x*c
os(f*x + e)^3 - f*x*cos(f*x + e)^2 - f*x*cos(f*x + e) + f*x)*sin(f*x + e)
+ 7*cos(f*x + e) + 8)/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^
2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))
```

3.28.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$$
$$= -\frac{\int \frac{1}{\sec^5(e+fx)-\sec^4(e+fx)-2\sec^3(e+fx)+2\sec^2(e+fx)+\sec(e+fx)-1}\,dx}{a^2c^3}$$

input
```
integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

output
```
-Integral(1/(sec(e + f*x)**5 - sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec
(e + f*x)**2 + sec(e + f*x) - 1), x)/(a**2*c**3)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.50

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx =$$
$$-\frac{\frac{5\left(\frac{18\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^3}-\frac{480\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c^3}+\frac{3\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{80\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{a^2c^3\sin(fx+e)^5}}{240\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/240*(5*(18*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/(a^2*c^3) - 480*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c^
3) + 3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 80*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(a^2*c^3*sin(f*x + e)^5))/f
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{\frac{240\,(fx+e)}{a^2c^3} + \frac{3\left(80\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-10\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)}{a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5} + \frac{5\left(a^4c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-18\,a^4c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6c^9}}{240\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/240*(240*(f*x + e)/(a^2*c^3) + 3*(80*tan(1/2*f*x + 1/2*e)^4 - 10*tan(1/2
*f*x + 1/2*e)^2 + 1)/(a^2*c^3*tan(1/2*f*x + 1/2*e)^5) + 5*(a^4*c^6*tan(1/2
*f*x + 1/2*e)^3 - 18*a^4*c^6*tan(1/2*f*x + 1/2*e))/(a^6*c^9))/f
```

3.28.9 Mupad [B] (verification not implemented)

Time = 14.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.64

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{3\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^8+5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^8-90\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6+240\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4-3\cdots}{240\,a^2\,c^3\,f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\cdots\right.}$$

input
```
int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3),x)
```

output
```
(3*cos(e/2 + (f*x)/2)^8 + 5*sin(e/2 + (f*x)/2)^8 - 90*cos(e/2 + (f*x)/2)^2
*sin(e/2 + (f*x)/2)^6 + 240*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^4 - 30
*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^2 + 240*cos(e/2 + (f*x)/2)^3*sin(
e/2 + (f*x)/2)^5*(e + f*x))/(240*a^2*c^3*f*cos(e/2 + (f*x)/2)^3*sin(e/2 +
(f*x)/2)^5)
```

3.28. $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$

3.29 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$

3.29.1 Optimal result

Integrand size = 26, antiderivative size = 166

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{x}{a^2c^4}+\frac{\cot(e+fx)}{a^2c^4f}-\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{2\csc(e+fx)}{a^2c^4f}-\frac{2\csc^3(e+fx)}{a^2c^4f}+\frac{6\csc^5(e+fx)}{5a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}$$

output
```
x/a^2/c^4+cot(f*x+e)/a^2/c^4/f-1/3*cot(f*x+e)^3/a^2/c^4/f+1/5*cot(f*x+e)^5
/a^2/c^4/f-2/7*cot(f*x+e)^7/a^2/c^4/f+2*csc(f*x+e)/a^2/c^4/f-2*csc(f*x+e)^
3/a^2/c^4/f+6/5*csc(f*x+e)^5/a^2/c^4/f-2/7*csc(f*x+e)^7/a^2/c^4/f
```

3.29.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = -\frac{\cot^7(e+fx)\left(5+5\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(e+fx)\right)+70\sec(e+fx)-140\sec^3(e+}{35a^2c^4f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

output
```
-1/35*(Cot[e + f*x]^7*(5 + 5*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[e + f*x
]^2] + 70*Sec[e + f*x] - 140*Sec[e + f*x]^3 + 112*Sec[e + f*x]^5 - 32*Sec[
e + f*x]^7))/(a^2*c^4*f)
```

3.29.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^8(e+fx)(\sec(e+fx)a+a)^2dx}{a^4c^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\cot\left(e+fx+\frac{\pi}{2}\right)^8}dx}{a^4c^4}$$

$$\Big\downarrow \text{4374}$$

$$\frac{\int \left(a^2\cot^8(e+fx)+2a^2\csc(e+fx)\cot^7(e+fx)+a^2\csc^2(e+fx)\cot^6(e+fx)\right)dx}{a^4c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2a^2\cot^7(e+fx)}{7f}+\frac{a^2\cot^5(e+fx)}{5f}-\frac{a^2\cot^3(e+fx)}{3f}+\frac{a^2\cot(e+fx)}{f}-\frac{2a^2\csc^7(e+fx)}{7f}+\frac{6a^2\csc^5(e+fx)}{5f}-\frac{2a^2\csc^3(e+fx)}{f}+\frac{2a^2\csc(}{f}}{a^4c^4}$$

input
```
Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

output
```
(a^2*x + (a^2*Cot[e + f*x])/f - (a^2*Cot[e + f*x]^3)/(3*f) + (a^2*Cot[e +
f*x]^5)/(5*f) - (2*a^2*Cot[e + f*x]^7)/(7*f) + (2*a^2*Csc[e + f*x])/f - (2
*a^2*Csc[e + f*x]^3)/f + (6*a^2*Csc[e + f*x]^5)/(5*f) - (2*a^2*Csc[e + f*x
]^7)/(7*f))/(a^4*c^4)
```

3.29.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.29.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.55

method	result
parallelrisch	$\frac{-15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+147\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-770\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3360fx-735\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4410\cot\left(\frac{fx}{2}+\right.}{3360f\,a^{2}c^{4}}$
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{22}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{42}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+64\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32f\,c^{4}a^{2}}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{22}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{42}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+64\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32f\,c^{4}a^{2}}$
risch	$\frac{x}{a^{2}c^{4}}+\frac{2i\left(210\,e^{9i(fx+e)}-315\,e^{8i(fx+e)}-420\,e^{7i(fx+e)}+1470\,e^{6i(fx+e)}-504\,e^{5i(fx+e)}-1204\,e^{4i(fx+e)}+1108\,e^{3i(fx+e}\right.}{105f\,c^{4}a^{2}\left(e^{i(fx+e)}-1\right)^{7}\left(e^{i(fx+e)}+1\right)^{3}}$
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{ca}-\frac{1}{224acf}+\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{160acf}-\frac{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{48acf}+\frac{21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{16acf}-\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{32acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{96acf}}{a\,c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$

input

```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output

```
1/3360*(-15*cot(1/2*f*x+1/2*e)^7+147*cot(1/2*f*x+1/2*e)^5+35*tan(1/2*f*x+1
/2*e)^3-770*cot(1/2*f*x+1/2*e)^3+3360*f*x-735*tan(1/2*f*x+1/2*e)+4410*cot(
1/2*f*x+1/2*e))/f/a^2/c^4
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{191\cos(fx+e)^{5}-172\cos(fx+e)^{4}-253\cos(fx+e)^{3}+258\cos(fx+e)^{2}+105\left(fx\cos(fx+e)^{4}-\right.}{105\left(a^{2}c^{4}f\cos(fx+e)^{4}-2\,a^{2}c^{4}f\cos(fx+e)^{3}+2\,a^{2}c^{4}f\right.}$$

input

```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output

```
1/105*(191*cos(f*x + e)^5 - 172*cos(f*x + e)^4 - 253*cos(f*x + e)^3 + 258*
cos(f*x + e)^2 + 105*(f*x*cos(f*x + e)^4 - 2*f*x*cos(f*x + e)^3 + 2*f*x*co
s(f*x + e) - f*x)*sin(f*x + e) + 87*cos(f*x + e) - 96)/((a^2*c^4*f*cos(f*x
 + e)^4 - 2*a^2*c^4*f*cos(f*x + e)^3 + 2*a^2*c^4*f*cos(f*x + e) - a^2*c^4*
f)*sin(f*x + e))
```

3.29.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\int \frac{1}{\sec^6(e+fx)-2\sec^5(e+fx)-\sec^4(e+fx)+4\sec^3(e+fx)-\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{a^2c^4}$$

input
```
integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

output
```
Integral(1/(sec(e + f*x)**6 - 2*sec(e + f*x)**5 - sec(e + f*x)**4 + 4*sec(
e + f*x)**3 - sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x)/(a**2*c**4)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = -\frac{\frac{35\left(\frac{21\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^4}-\frac{6720\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c^4}-\frac{\left(\frac{147\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{770\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{4410\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)}{a^2c^4\sin(fx+e)^7}}{3360\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
-1/3360*(35*(21*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3)/(a^2*c^4) - 6720*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2
*c^4) - (147*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 770*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + 4410*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x
 + e) + 1)^7/(a^2*c^4*sin(f*x + e)^7))/f
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\frac{3360\,(fx+e)}{a^2c^4} + \frac{4410\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-770\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+147\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15}{a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7} + \frac{35\left(a^4c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-21\,a^4c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6c^{12}}}{3360\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/3360*(3360*(f*x + e)/(a^2*c^4) + (4410*tan(1/2*f*x + 1/2*e)^6 - 770*tan(
1/2*f*x + 1/2*e)^4 + 147*tan(1/2*f*x + 1/2*e)^2 - 15)/(a^2*c^4*tan(1/2*f*x
 + 1/2*e)^7) + 35*(a^4*c^8*tan(1/2*f*x + 1/2*e)^3 - 21*a^4*c^8*tan(1/2*f*x
 + 1/2*e))/(a^6*c^12))/f
```

3.29.9 Mupad [B] (verification not implemented)

Time = 14.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{35\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{10} - 15\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{10} - 735\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^8 + 4410\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\ldots}{3360\,a^2\,c^4\,f\,\mathrm{co}\ldots}$$

input
```
int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4),x)
```

output
```
(35*sin(e/2 + (f*x)/2)^10 - 15*cos(e/2 + (f*x)/2)^10 - 735*cos(e/2 + (f*x)
/2)^2*sin(e/2 + (f*x)/2)^8 + 4410*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^
6 - 770*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^4 + 147*cos(e/2 + (f*x)/2)
^8*sin(e/2 + (f*x)/2)^2 + 3360*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^7*(
e + f*x))/(3360*a^2*c^4*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^7)
```

3.30 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$

3.30.1 Optimal result

Integrand size = 26, antiderivative size = 210

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx &= \frac{x}{a^2c^5}+\frac{\cot(e+fx)}{a^2c^5f}-\frac{\cot^3(e+fx)}{3a^2c^5f}+\frac{\cot^5(e+fx)}{5a^2c^5f}-\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\cot^9(e+fx)}{9a^2c^5f}\\ &\quad+\frac{3\csc(e+fx)}{a^2c^5f}-\frac{13\csc^3(e+fx)}{3a^2c^5f}+\frac{21\csc^5(e+fx)}{5a^2c^5f}-\frac{15\csc^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}\end{aligned}$$

output
```
x/a^2/c^5+cot(f*x+e)/a^2/c^5/f-1/3*cot(f*x+e)^3/a^2/c^5/f+1/5*cot(f*x+e)^5
/a^2/c^5/f-1/7*cot(f*x+e)^7/a^2/c^5/f+4/9*cot(f*x+e)^9/a^2/c^5/f+3*csc(f*x
+e)/a^2/c^5/f-13/3*csc(f*x+e)^3/a^2/c^5/f+21/5*csc(f*x+e)^5/a^2/c^5/f-15/7
*csc(f*x+e)^7/a^2/c^5/f+4/9*csc(f*x+e)^9/a^2/c^5/f
```

3.30.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.91 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{\cot^9(e+fx)\left(105+35\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(e+fx)\right)+945\sec(e+fx)-2415\sec^3(e}{315a^2c^5f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

output
```
(Cot[e + f*x]^9*(105 + 35*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*x]^2
] + 945*Sec[e + f*x] - 2415*Sec[e + f*x]^3 + 2898*Sec[e + f*x]^5 - 1656*Se
c[e + f*x]^7 + 368*Sec[e + f*x]^9))/(315*a^2*c^5*f)
```

3.30.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^{10}(e+fx)(\sec(e+fx)a+a)^3dx}{a^5c^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3}{\cot\left(e+fx+\frac{\pi}{2}\right)^{10}}dx}{a^5c^5}$$

$$\Big\downarrow \text{4374}$$

$$-\frac{\int \left(a^3\cot^{10}(e+fx)+3a^3\csc(e+fx)\cot^9(e+fx)+3a^3\csc^2(e+fx)\cot^8(e+fx)+a^3\csc^3(e+fx)\cot^7(e+f\right.}{a^5c^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{4a^3\cot^9(e+fx)}{9f}+\frac{a^3\cot^7(e+fx)}{7f}-\frac{a^3\cot^5(e+fx)}{5f}+\frac{a^3\cot^3(e+fx)}{3f}-\frac{a^3\cot(e+fx)}{f}-\frac{4a^3\csc^9(e+fx)}{9f}+\frac{15a^3\csc^7(e+fx)}{7f}-\frac{21a^3}{}}{a^5c^5}$$

input
```
Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

output
```
-((-(a^3*x) - (a^3*Cot[e + f*x])/f + (a^3*Cot[e + f*x]^3)/(3*f) - (a^3*Cot
[e + f*x]^5)/(5*f) + (a^3*Cot[e + f*x]^7)/(7*f) - (4*a^3*Cot[e + f*x]^9)/(
9*f) - (3*a^3*Csc[e + f*x])/f + (13*a^3*Csc[e + f*x]^3)/(3*f) - (21*a^3*Cs
c[e + f*x]^5)/(5*f) + (15*a^3*Csc[e + f*x]^7)/(7*f) - (4*a^3*Csc[e + f*x]^
9)/(9*f))/(a^5*c^5))
```

3.30.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.30.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.50

method	result
parallelrisch	$\frac{35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}-360\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+1827\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-6720\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+20160fx-2520\tan\left(\right.}{20160f\,a^{2}c^{5}}$
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+128\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{8}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{29}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{64}{3\tan\left(\frac{fx}{2}+\right.}}{64f\,c^{5}a^{2}}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+128\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{8}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{29}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{64}{3\tan\left(\frac{fx}{2}+\right.}}{64f\,c^{5}a^{2}}$
risch	$\frac{x}{a^{2}c^{5}}+\frac{2i\left(945\,e^{11i(fx+e)}-3150\,e^{10i(fx+e)}+2625\,e^{9i(fx+e)}+6300\,e^{8i(fx+e)}-13482\,e^{7i(fx+e)}+5292\,e^{6i(fx+e)}+10566\right.}{315f\,c^{5}a^{2}\left(e^{i(fx+e)}-1\right)^{9}\left(e^{i(fx+e)}\right.}$
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{ca}+\frac{1}{576acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{56acf}+\frac{29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{320acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{3acf}+\frac{99\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{64acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{8acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{192acf}}{a\,c^{4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}$

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output
```
1/20160*(35*cot(1/2*f*x+1/2*e)^9-360*cot(1/2*f*x+1/2*e)^7+1827*cot(1/2*f*x
+1/2*e)^5+105*tan(1/2*f*x+1/2*e)^3-6720*cot(1/2*f*x+1/2*e)^3+20160*f*x-252
0*tan(1/2*f*x+1/2*e)+31185*cot(1/2*f*x+1/2*e))/f/a^2/c^5
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{668\cos(fx+e)^{6}-1059\cos(fx+e)^{5}-573\cos(fx+e)^{4}+1813\cos(fx+e)^{3}-393\cos(fx+e)^{2}+3}{315\left(a^{2}c^{5}f\cos(fx+e)^{5}-3\,a^{2}c^{5}f\cos(fx+e)^{4}\right.}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
1/315*(668*cos(f*x + e)^6 - 1059*cos(f*x + e)^5 - 573*cos(f*x + e)^4 + 181
3*cos(f*x + e)^3 - 393*cos(f*x + e)^2 + 315*(f*x*cos(f*x + e)^5 - 3*f*x*co
s(f*x + e)^4 + 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(f*x + e)^2 - 3*f*x*cos(f*x
 + e) + f*x)*sin(f*x + e) - 789*cos(f*x + e) + 368)/((a^2*c^5*f*cos(f*x +
e)^5 - 3*a^2*c^5*f*cos(f*x + e)^4 + 2*a^2*c^5*f*cos(f*x + e)^3 + 2*a^2*c^5
*f*cos(f*x + e)^2 - 3*a^2*c^5*f*cos(f*x + e) + a^2*c^5*f)*sin(f*x + e))
```

3.30.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$$
$$= -\frac{\int \frac{1}{\sec^7(e+fx)-3\sec^6(e+fx)+\sec^5(e+fx)+5\sec^4(e+fx)-5\sec^3(e+fx)-\sec^2(e+fx)+3\sec(e+fx)-1}\,dx}{a^2c^5}$$

input
```
integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

output
```
-Integral(1/(sec(e + f*x)**7 - 3*sec(e + f*x)**6 + sec(e + f*x)**5 + 5*sec
(e + f*x)**4 - 5*sec(e + f*x)**3 - sec(e + f*x)**2 + 3*sec(e + f*x) - 1),
x)/(a**2*c**5)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx =$$
$$-\frac{\frac{105\left(\frac{24\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^5}-\frac{40320\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c^5}+\frac{\left(\frac{360\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1827\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{6720\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{31185\sin(fx+}{(\cos(fx+e)+}\right.}{a^2c^5\sin(fx+e)^9}}{20160\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
-1/20160*(105*(24*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/(a^2*c^5) - 40320*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(
a^2*c^5) + (360*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1827*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 6720*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 31185*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(a^2*c^5*sin(
f*x + e)^9))/f
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{\frac{20160\,(fx+e)}{a^2c^5} + \frac{31185\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-6720\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+1827\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-360\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35}{a^2c^5\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9} + \frac{105\,\left(a^4c^{10}\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{}}{20160\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
1/20160*(20160*(f*x + e)/(a^2*c^5) + (31185*tan(1/2*f*x + 1/2*e)^8 - 6720*
tan(1/2*f*x + 1/2*e)^6 + 1827*tan(1/2*f*x + 1/2*e)^4 - 360*tan(1/2*f*x + 1
/2*e)^2 + 35)/(a^2*c^5*tan(1/2*f*x + 1/2*e)^9) + 105*(a^4*c^10*tan(1/2*f*x
 + 1/2*e)^3 - 24*a^4*c^10*tan(1/2*f*x + 1/2*e))/(a^6*c^15))/f
```

3.30.9 Mupad [B] (verification not implemented)

Time = 14.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{35\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+105\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}-2520\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+31185\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\right.}{}$$

input
```
int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5),x)
```

output
```
(35*cos(e/2 + (f*x)/2)^12 + 105*sin(e/2 + (f*x)/2)^12 - 2520*cos(e/2 + (f*
x)/2)^2*sin(e/2 + (f*x)/2)^10 + 31185*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)
/2)^8 - 6720*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^6 + 1827*cos(e/2 + (f
*x)/2)^8*sin(e/2 + (f*x)/2)^4 - 360*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)/
2)^2 + 20160*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^9*(e + f*x))/(20160*a
^2*c^5*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^9)
```

3.31 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

3.31.1 Optimal result

Integrand size = 26, antiderivative size = 162

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{c^5x}{a^3} + \frac{8c^5\text{arctanh}(\sin(e+fx))}{a^3f} + \frac{32c^5\cot(e+fx)}{a^3f} + \frac{128c^5\cot^3(e+fx)}{3a^3f} + \frac{128c^5\cot^5(e+fx)}{5a^3f} - \frac{16c^5\csc(e+fx)}{a^3f} + \frac{64c^5\csc^3(e+fx)}{3a^3f} - \frac{128c^5\csc^5(e+fx)}{5a^3f} - \frac{c^5\tan(e+fx)}{a^3f}$$

output
```
c^5*x/a^3+8*c^5*arctanh(sin(f*x+e))/a^3/f+32*c^5*cot(f*x+e)/a^3/f+128/3*c^
5*cot(f*x+e)^3/a^3/f+128/5*c^5*cot(f*x+e)^5/a^3/f-16*c^5*csc(f*x+e)/a^3/f+
64/3*c^5*csc(f*x+e)^3/a^3/f-128/5*c^5*csc(f*x+e)^5/a^3/f-c^5*tan(f*x+e)/a^
3/f
```

3.31.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.29 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.30

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^{9/2}\tan(e+fx)\left(-46\sqrt{a}\sqrt{c} - 48\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{5}{2},-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1}\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^3,x]
```

output
```
-1/15*(c^(9/2)*Tan[e + f*x]*(-46*Sqrt[a]*Sqrt[c] - 48*Sqrt[2]*Sqrt[a]*Sqrt
[c]*Hypergeometric2F1[-7/2, -5/2, -3/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec
[e + f*x]] - 24*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-5/2, -5/2, -3/2
, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] - 2*Sqrt[a]*Sqrt[c]*Sec[e +
 f*x] + 22*Sqrt[a]*Sqrt[c]*Sec[e + f*x]^2 + 26*Sqrt[a]*Sqrt[c]*Sec[e + f*x
]^3 + 15*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[-(a*c
*Tan[e + f*x]^2)] + 30*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c
])]*Sec[e + f*x]*Sqrt[-(a*c*Tan[e + f*x]^2)] + 15*ArcTanh[Sqrt[-(a*c*Tan[e
 + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sec[e + f*x]^2*Sqrt[-(a*c*Tan[e + f*x]^2)])
)/(a^(7/2)*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^3)
```

3.31.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^5}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))^8dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^8}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4374}$$

$$-\frac{\int \left(\cot^6(e+fx)c^8+28\csc^6(e+fx)c^8-56\cot(e+fx)\csc^5(e+fx)c^8+70\cot^2(e+fx)\csc^4(e+fx)c^8-56\,\text{co}\right.}{}$$

$\downarrow$ 2009

$$-\frac{-\frac{8c^8\text{arctanh}(\sin(e+fx))}{f}+\frac{c^8\tan(e+fx)}{f}-\frac{128c^8\cot^5(e+fx)}{5f}-\frac{128c^8\cot^3(e+fx)}{3f}-\frac{32c^8\cot(e+fx)}{f}+\frac{128c^8\csc^5(e+fx)}{5f}-\frac{64c^8\text{cs}}{}}{a^3c^3}$$

input
```
Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^3,x]
```

output
```
-((-(c^8*x) - (8*c^8*ArcTanh[Sin[e + f*x]])/f - (32*c^8*Cot[e + f*x])/f -
(128*c^8*Cot[e + f*x]^3)/(3*f) - (128*c^8*Cot[e + f*x]^5)/(5*f) + (16*c^8*
Csc[e + f*x])/f - (64*c^8*Csc[e + f*x]^3)/(3*f) + (128*c^8*Csc[e + f*x]^5)
/(5*f) + (c^8*Tan[e + f*x])/f)/(a^3*c^3))
```

3.31.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.31.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{8c^5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}+\frac{}{8\,\mathrm{ta}}\right.}{f\,a^3}$
default	$\frac{8c^5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}+\frac{}{8\,\mathrm{ta}}\right.}{f\,a^3}$
parallelrisch	$-\frac{2113\left(\frac{1920\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)}{2113}-\frac{1920\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)}{2113}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\cos(fx+e)+\frac{954\cos(2fx+2e}{2113}\right.\right.}{240\cos(fx+e)a^3f}$
risch	$\frac{c^5x}{a^3}-\frac{2ic^5\left(240\,\mathrm{e}^{6i(fx+e)}+735\,\mathrm{e}^{5i(fx+e)}+1835\,\mathrm{e}^{4i(fx+e)}+1750\,\mathrm{e}^{3i(fx+e)}+1894\,\mathrm{e}^{2i(fx+e)}+955\,\mathrm{e}^{i(fx+e)}+239\right)}{15f\,a^3\left(\mathrm{e}^{i(fx+e)}+1\right)^5\left(1+\mathrm{e}^{2i(fx+e)}\right)}+8c^5$
norman	$\frac{\frac{c^5x}{a}+\frac{c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{a}-\frac{4c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{a}+\frac{6c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{a}-\frac{4c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{a}-\frac{18c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{202c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right.}$

input
```
int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
8/f*c^5/a^3*(-1/5*tan(1/2*f*x+1/2*e)^5-1/3*tan(1/2*f*x+1/2*e)^3-2*tan(1/2*
f*x+1/2*e)+1/4*arctan(tan(1/2*f*x+1/2*e))+ln(tan(1/2*f*x+1/2*e)+1)+1/8/(ta
n(1/2*f*x+1/2*e)+1)+1/8/(tan(1/2*f*x+1/2*e)-1)-ln(tan(1/2*f*x+1/2*e)-1))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.78

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{15\,c^5fx\cos\left(fx+e\right)^4+45\,c^5fx\cos\left(fx+e\right)^3+45\,c^5fx\cos\left(fx+e\right)^2+15\,c^5fx\cos\left(fx+e\right)+60\left(c^5\cos\left(\right.\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(15*c^5*f*x*cos(f*x + e)^4 + 45*c^5*f*x*cos(f*x + e)^3 + 45*c^5*f*x*c
os(f*x + e)^2 + 15*c^5*f*x*cos(f*x + e) + 60*(c^5*cos(f*x + e)^4 + 3*c^5*c
os(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 + c^5*cos(f*x + e))*log(sin(f*x + e)
+ 1) - 60*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5*cos(f*x + e)^
2 + c^5*cos(f*x + e))*log(-sin(f*x + e) + 1) - (239*c^5*cos(f*x + e)^3 + 4
77*c^5*cos(f*x + e)^2 + 349*c^5*cos(f*x + e) + 15*c^5)*sin(f*x + e))/(a^3*
f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f
*cos(f*x + e))
```

3.31.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx =$$

$$-\frac{c^5\left(\int \frac{5\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\left(-\frac{10\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int \frac{}{\sec^3(e+fx)+}\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

output
```
-c**5*(Integral(5*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*se
c(e + f*x) + 1), x) + Integral(-10*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*se
c(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*sec(e + f*x)**3/(sec
(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-5*s
ec(e + f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),
 x) + Integral(sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*se
c(e + f*x) + 1), x) + Integral(-1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3
*sec(e + f*x) + 1), x))/a**3
```

3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(154) = 308$.

Time = 0.31 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.47

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx =$$

$$-\frac{3\,c^5\left(\frac{40\,\sin(fx+e)}{\left(a^3-\frac{a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}+\frac{\frac{85\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\mathrm{si}}{\mathrm{cos}}\right.}{}\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/60*(3*c^5*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 -
 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(co
s(f*x + e) + 1) - 1)/a^3) + 5*c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) +
20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) +
1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f
*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + c^5*((105*sin(f*x + e)/(cos(f*x + e
) + 1) - 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3 - 120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 10
*c^5*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 5*c^5*(15*sin(f*x +
 e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 30*c^5*(5*sin(f*x + e)/(cos(f*x + e)
+ 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.95

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\frac{15\,(fx+e)c^5}{a^3} + \frac{120\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3} - \frac{120\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3} + \frac{30\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^3} - \frac{8\left(3\,a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.}{}}{15\,f}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/15*(15*(f*x + e)*c^5/a^3 + 120*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^
3 - 120*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 + 30*c^5*tan(1/2*f*x +
1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^3) - 8*(3*a^12*c^5*tan(1/2*f*x + 1/
2*e)^5 + 5*a^12*c^5*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^5*tan(1/2*f*x + 1/2
*e))/a^15)/f
```

3.31.9 Mupad [B] (verification not implemented)

Time = 14.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{c^5\,x}{a^3}-\frac{16\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f}-\frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^3\,f} - \frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}+\frac{16\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f} + \frac{2\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^3\right)}$$

input
```
int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^3,x)
```

output
```
(c^5*x)/a^3 - (16*c^5*tan(e/2 + (f*x)/2))/(a^3*f) - (8*c^5*tan(e/2 + (f*x)
/2)^3)/(3*a^3*f) - (8*c^5*tan(e/2 + (f*x)/2)^5)/(5*a^3*f) + (16*c^5*atanh(
tan(e/2 + (f*x)/2)))/(a^3*f) + (2*c^5*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2
+ (f*x)/2)^2 - a^3))
```

3.32 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

3.32.1 Optimal result

Integrand size = 26, antiderivative size = 148

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{c^4x}{a^3} + \frac{c^4\text{arctanh}(\sin(e+fx))}{a^3f} - \frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3} - \frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} + \frac{14c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2} - \frac{23c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))}$$

output
```
c^4*x/a^3+c^4*arctanh(sin(f*x+e))/a^3/f-3*c^4*tan(f*x+e)/a^3/f/(1+sec(f*x+
e))^3-1/5*c^4*sec(f*x+e)^2*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3+14/5*c^4*tan(
f*x+e)/a^3/f/(1+sec(f*x+e))^2-23/5*c^4*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

3.32.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.66 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.36

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{2c^{7/2}\tan(e+fx)\left(12\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{2},-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1-\sec(e+fx)}\right.}{15\ldots}$$

input
```
Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^3,x]
```

output
```
(2*c^(7/2)*Tan[e + f*x]*(12*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-5/2
, -5/2, -3/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] + Sqrt[a]*Sqrt[
c]*(23 + Sec[e + f*x] - 11*Sec[e + f*x]^2 - 13*Sec[e + f*x]^3) - 30*ArcTan
h[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Cos[(e + f*x)/2]^4*Sec[e
+ f*x]^2*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(15*a^(7/2)*f*(-1 + Sec[e + f*x])*(
1 + Sec[e + f*x])^3)
```

3.32.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^4}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^4(e+fx)c^4}{(\sec(e+fx)+1)^3}-\frac{4\sec^3(e+fx)c^4}{(\sec(e+fx)+1)^3}+\frac{6\sec^2(e+fx)c^4}{(\sec(e+fx)+1)^3}-\frac{4\sec(e+fx)c^4}{(\sec(e+fx)+1)^3}+\frac{c^4}{(\sec(e+fx)+1)^3}\right)dx}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c^4\text{arctanh}(\sin(e+fx))}{f}-\frac{c^4\tan(e+fx)\sec^2(e+fx)}{5f(\sec(e+fx)+1)^3}-\frac{23c^4\tan(e+fx)}{5f(\sec(e+fx)+1)}+\frac{14c^4\tan(e+fx)}{5f(\sec(e+fx)+1)^2}-\frac{3c^4\tan(e+fx)}{f(\sec(e+fx)+1)^3}+c^4x}{a^3}$$

input
```
Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^3,x]
```

output
```
(c^4*x + (c^4*ArcTanh[Sin[e + f*x]])/f - (3*c^4*Tan[e + f*x])/(f*(1 + Sec[
e + f*x])^3) - (c^4*Sec[e + f*x]^2*Tan[e + f*x])/(5*f*(1 + Sec[e + f*x])^3
) + (14*c^4*Tan[e + f*x])/(5*f*(1 + Sec[e + f*x])^2) - (23*c^4*Tan[e + f*x
])/(5*f*(1 + Sec[e + f*x])))/a^3
```

3.32.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.32.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

method	result
parallelrisch	$\frac{c^{4}\left(-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+5fx+5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5a^{3}f}$
derivativedivides	$\frac{4c^{4}\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}\right)}{f\,a^{3}}$
default	$\frac{4c^{4}\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}\right)}{f\,a^{3}}$
risch	$\frac{c^{4}x}{a^{3}}-\frac{16ic^{4}\left(5\,{\mathrm e}^{4i(fx+e)}+10\,{\mathrm e}^{3i(fx+e)}+20\,{\mathrm e}^{2i(fx+e)}+10\,{\mathrm e}^{i(fx+e)}+3\right)}{5f\,a^{3}\left({\mathrm e}^{i(fx+e)}+1\right)^{5}}+\frac{c^{4}\ln\left({\mathrm e}^{i(fx+e)}+i\right)}{a^{3}f}-\frac{c^{4}\ln\left({\mathrm e}^{i(fx+e)}-i\right)}{a^{3}f}$
norman	$\frac{\frac{c^{4}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{a}-\frac{c^{4}x}{a}+\frac{4c^{4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{12c^{4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{af}+\frac{64c^{4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5af}-\frac{32c^{4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{5af}+\frac{12c^{4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{5af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}a^{2}}$

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*c^4*(-4*tan(1/2*f*x+1/2*e)^5+5*f*x+5*ln(tan(1/2*f*x+1/2*e)+1)-5*ln(tan
(1/2*f*x+1/2*e)-1)-20*tan(1/2*f*x+1/2*e))/a^3/f
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.64

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{10\,c^4fx\cos{(fx+e)}^3 + 30\,c^4fx\cos{(fx+e)}^2 + 30\,c^4fx\cos{(fx+e)} + 10\,c^4fx + 5\,\left(c^4\cos{(fx+e)}^3 + 3\,c\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/10*(10*c^4*f*x*cos(f*x + e)^3 + 30*c^4*f*x*cos(f*x + e)^2 + 30*c^4*f*x*c
os(f*x + e) + 10*c^4*f*x + 5*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 +
3*c^4*cos(f*x + e) + c^4)*log(sin(f*x + e) + 1) - 5*(c^4*cos(f*x + e)^3 +
3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*log(-sin(f*x + e) + 1) -
16*(3*c^4*cos(f*x + e)^2 + 4*c^4*cos(f*x + e) + 3*c^4)*sin(f*x + e))/(a^3*
f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

3.32.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{c^4\left(\int \left(-\frac{4\sec{(e+fx)}}{\sec^3{(e+fx)}+3\sec^2{(e+fx)}+3\sec{(e+fx)}+1}\right)dx + \int \frac{6\sec^2{(e+fx)}}{\sec^3{(e+fx)}+3\sec^2{(e+fx)}+3\sec{(e+fx)}+1}\,dx + \int \left(-\frac{}{\sec^3{(e+fx)}\right.\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

output
```
c**4*(Integral(-4*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*se
c(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(
e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-4*sec(e + f*x)**3/(sec(e
 + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e
+ f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) +
 Integral(1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x)
)/a**3
```

3.32.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(142) = 284$.

Time = 0.30 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.68

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^3}\, dx =$$

$$-\frac{c^4\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+c^4\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}-\frac{2}{(c}\cdots\right.}{\cdots}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/60*(c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(
f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) +
 1) - 1)/a^3) + c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 12
0*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 4*c^4*(15*sin(f*x + e)/(c
os(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5)/a^3 + 4*c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) -
 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5)/a^3 - 18*c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(c
os(f*x + e) + 1)^5)/a^3)/f
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.69

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{5(fx+e)c^4}{a^3}+\frac{5c^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^3}-\frac{5c^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^3}-\frac{4\left(a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+5a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{15}}}{5f}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/5*(5*(f*x + e)*c^4/a^3 + 5*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 -
5*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 - 4*(a^12*c^4*tan(1/2*f*x + 1
/2*e)^5 + 5*a^12*c^4*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.32.9 Mupad [B] (verification not implemented)

Time = 14.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.34

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{c^4\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}{5}+fx\right)}{a^3 f}$$

input
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^3,x)
```

output
```
(c^4*(2*atanh(tan(e/2 + (f*x)/2)) - 4*tan(e/2 + (f*x)/2) - (4*tan(e/2 + (f
*x)/2)^5)/5 + f*x))/(a^3*f)
```

3.33 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

3.33.1 Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{c^3x}{a^3}-\frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}+\frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2}-\frac{26c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))}$$

output
```
c^3*x/a^3-8/5*c^3*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3+4/15*c^3*tan(f*x+e)/a^
3/f/(1+sec(f*x+e))^2-26/15*c^3*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

3.33.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^3\left(-\frac{2\arctan\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{2\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}\right)}{a^3}$$

input
```
Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^3,x]
```

output
```
-((c^3*((-2*ArcTan[Tan[e/2 + (f*x)/2]])/f + (2*Tan[e/2 + (f*x)/2])/f - (2*
Tan[e/2 + (f*x)/2]^3)/(3*f) + (2*Tan[e/2 + (f*x)/2]^5)/(5*f)))/a^3)
```

3.33.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^3}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(-\frac{\sec^3(e+fx)c^3}{(\sec(e+fx)+1)^3}+\frac{3\sec^2(e+fx)c^3}{(\sec(e+fx)+1)^3}-\frac{3\sec(e+fx)c^3}{(\sec(e+fx)+1)^3}+\frac{c^3}{(\sec(e+fx)+1)^3}\right)dx}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{26c^3\tan(e+fx)}{15f(\sec(e+fx)+1)}+\frac{4c^3\tan(e+fx)}{15f(\sec(e+fx)+1)^2}-\frac{8c^3\tan(e+fx)}{5f(\sec(e+fx)+1)^3}+c^3x}{a^3}$$

input
```
Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^3,x]
```

output
```
(c^3*x - (8*c^3*Tan[e + f*x])/(5*f*(1 + Sec[e + f*x])^3) + (4*c^3*Tan[e +
f*x])/(15*f*(1 + Sec[e + f*x])^2) - (26*c^3*Tan[e + f*x])/(15*f*(1 + Sec[e
 + f*x])))/a^3
```

3.33.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.33.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

method	result
parallelrisch	$-\frac{c^{3}\left(6 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}-10 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}-15 f x+30 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{15 a^{3} f}$
derivativedivides	$\frac{2 c^{3}\left(-\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}-\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f\, a^{3}}$
default	$\frac{2 c^{3}\left(-\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}-\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f\, a^{3}}$
risch	$\frac{c^{3} x}{a^{3}}-\frac{4 i c^{3}\left(45\, e^{4 i (f x+e)}+90\, e^{3 i (f x+e)}+140\, e^{2 i (f x+e)}+70\, e^{i (f x+e)}+23\right)}{15 f\, a^{3}\left(e^{i (f x+e)}+1\right)^{5}}$
norman	$\frac{\frac{c^{3} x}{a}+\frac{c^{3} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{a}-\frac{2 c^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{a f}+\frac{14 c^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 a f}-\frac{56 c^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{15 a f}+\frac{22 c^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}{15 a f}-\frac{2 c^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}{5 a f} -}{a^{2}\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}}$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15*c^3*(6*tan(1/2*f*x+1/2*e)^5-10*tan(1/2*f*x+1/2*e)^3-15*f*x+30*tan(1/
2*f*x+1/2*e))/a^3/f
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.44

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{15\,c^3fx\cos(fx+e)^3+45\,c^3fx\cos(fx+e)^2+45\,c^3fx\cos(fx+e)+15\,c^3fx-2\,(23\,c^3\cos(fx+e)^2+}{15\,(a^3f\cos(fx+e)^3+3\,a^3f\cos(fx+e)^2+3\,a^3f\cos(fx+e)+a^3}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(15*c^3*f*x*cos(f*x + e)^3 + 45*c^3*f*x*cos(f*x + e)^2 + 45*c^3*f*x*c
os(f*x + e) + 15*c^3*f*x - 2*(23*c^3*cos(f*x + e)^2 + 24*c^3*cos(f*x + e)
+ 13*c^3)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3
*a^3*f*cos(f*x + e) + a^3*f)
```

3.33.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^3\left(\int \frac{3\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \left(-\frac{3\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx + \int \frac{}{\sec^3(e+fx)+}\right.}{a^3}$$

input
```
integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

output
```
-c**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*se
c(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec
(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e +
 f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-1/(sec(
e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(90) = 180$.

Time = 0.29 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.89

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^3\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+\frac{c^3\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right.}{a^3}}{60\,f}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/60*(c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + c^3*(15*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5)/a^3 + 3*c^3*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3
- 9*c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5)/a^3)/f
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)c^3}{a^3} - \frac{2\left(3\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 5\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 15\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}}}{15\,f}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/15*(15*(f*x + e)*c^3/a^3 - 2*(3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 - 5*a^12
*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^3*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.33.9 Mupad [B] (verification not implemented)

Time = 14.42 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{c^3\,x}{a^3} - \frac{\frac{46\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{15} - \frac{22\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{15} + \frac{2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3}{5}}{a^3\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

input
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^3,x)
```

output
```
(c^3*x)/a^3 - ((2*c^3*sin(e/2 + (f*x)/2))/5 - (22*c^3*cos(e/2 + (f*x)/2)^2
*sin(e/2 + (f*x)/2))/15 + (46*c^3*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))
/15)/(a^3*f*cos(e/2 + (f*x)/2)^5)
```

3.34 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$

3.34.1 Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{c^2x}{a^3} - \frac{4c^2\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{8c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2} - \frac{23c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))}$$

output
```
c^2*x/a^3-4/5*c^2*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3-8/15*c^2*tan(f*x+e)/a^
3/f/(1+sec(f*x+e))^2-23/15*c^2*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

3.34.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.57

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^{3/2}\tan(e+fx)\left(\sqrt{a}\sqrt{c}(-43-11\sec(e+fx)+31\sec^2(e+fx)+23\sec^3(e+fx))+60\text{arctanh}\left(\frac{\sqrt{-c}}{\cdots}\right.\right.}{15a^{7/2}f(-1+\sec(e+fx))(1+\sec(e+f\cdots}$$

input
```
Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^3,x]
```

output
```
-1/15*(c^(3/2)*Tan[e + f*x]*(Sqrt[a]*Sqrt[c]*(-43 - 11*Sec[e + f*x] + 31*S
ec[e + f*x]^2 + 23*Sec[e + f*x]^3) + 60*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)
]/(Sqrt[a]*Sqrt[c])]*Cos[(e + f*x)/2]^4*Sec[e + f*x]^2*Sqrt[-(a*c*Tan[e +
f*x]^2)]))/(a^(7/2)*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^3)
```

3.34.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^2}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^2(e+fx)c^2}{(\sec(e+fx)+1)^3}-\frac{2\sec(e+fx)c^2}{(\sec(e+fx)+1)^3}+\frac{c^2}{(\sec(e+fx)+1)^3}\right)dx}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{23c^2\tan(e+fx)}{15f(\sec(e+fx)+1)}-\frac{8c^2\tan(e+fx)}{15f(\sec(e+fx)+1)^2}-\frac{4c^2\tan(e+fx)}{5f(\sec(e+fx)+1)^3}+c^2x}{a^3}$$

input
```
Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^3,x]
```

output
```
(c^2*x - (4*c^2*Tan[e + f*x])/(5*f*(1 + Sec[e + f*x])^3) - (8*c^2*Tan[e +
f*x])/(15*f*(1 + Sec[e + f*x])^2) - (23*c^2*Tan[e + f*x])/(15*f*(1 + Sec[e
 + f*x])))/a^3
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.34.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

method	result	size
parallelrisch	$-\frac{c^{2}\left(3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-15fx+30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15a^{3}f}$	54
derivativedivides	$\frac{c^{2}\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^{3}}$	61
default	$\frac{c^{2}\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^{3}}$	61
risch	$\frac{c^{2}x}{a^{3}}-\frac{2ic^{2}\left(75\,{\mathrm e}^{4i(fx+e)}+180\,{\mathrm e}^{3i(fx+e)}+250\,{\mathrm e}^{2i(fx+e)}+140\,{\mathrm e}^{i(fx+e)}+43\right)}{15f\,a^{3}\left({\mathrm e}^{i(fx+e)}+1\right)^{5}}$	81
norman	$\frac{\frac{c^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{a}-\frac{c^{2}x}{a}+\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{8c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3af}+\frac{13c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{15af}-\frac{c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{5af}}{a^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}$	135

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15*c^2*(3*tan(1/2*f*x+1/2*e)^5-10*tan(1/2*f*x+1/2*e)^3-15*f*x+30*tan(1/
2*f*x+1/2*e))/a^3/f
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.44

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{15\,c^2fx\cos{(fx+e)}^3+45\,c^2fx\cos{(fx+e)}^2+45\,c^2fx\cos{(fx+e)}+15\,c^2fx-\left(43\,c^2\cos{(fx+e)}^2+54\right.}{15\,\left(a^3f\cos{(fx+e)}^3+3\,a^3f\cos{(fx+e)}^2+3\,a^3f\cos{(fx+e)}+a^3f\right.}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(15*c^2*f*x*cos(f*x + e)^3 + 45*c^2*f*x*cos(f*x + e)^2 + 45*c^2*f*x*c
os(f*x + e) + 15*c^2*f*x - (43*c^2*cos(f*x + e)^2 + 54*c^2*cos(f*x + e) +
23*c^2)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a
^3*f*cos(f*x + e) + a^3*f)
```

3.34.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{c^2\left(\int\left(-\frac{2\sec{(e+fx)}}{\sec^3{(e+fx)}+3\sec^2{(e+fx)}+3\sec{(e+fx)}+1}\right)dx+\int\frac{\sec^2{(e+fx)}}{\sec^3{(e+fx)}+3\sec^2{(e+fx)}+3\sec{(e+fx)}+1}\,dx+\int\frac{}{\sec^3{(e+fx)}+3\,\mathrm{se}}\right.}{a^3}$$

input
```
integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

output
```
c**2*(Integral(-2*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*se
c(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(1/(sec(e + f*x)**3 + 3*sec(
e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

3.34.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(90) = 180$.

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.20

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^3}\,dx =$$

$$-\frac{c^2\left(\frac{\frac{105\,\sin(fx+e)}{\cos(fx+e)+1} - \frac{20\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3} - \frac{120\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right) + \frac{2\,c^2\left(\frac{15\,\sin(fx+e)}{\cos(fx+e)+1} - \frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)}\right.}{a^3}}{60\,f}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/60*(c^2*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 2*c^2*(15*sin(f*x + e)/(cos(f*x +
e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f
*x + e) + 1)^5)/a^3 - 3*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^3}\,dx = \frac{\frac{15\,(fx+e)c^2}{a^3} - \frac{3\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 10\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}}}{15\,f}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/15*(15*(f*x + e)*c^2/a^3 - (3*a^12*c^2*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*
c^2*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^2*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.34.9 Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^3}\,dx$$
$$= \frac{c^2\,x}{a^3} - \frac{\frac{43\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{15} - \frac{16\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{15} + \frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2}{5}}{a^3\,f\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

input
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^3,x)
```

output
```
(c^2*x)/a^3 - ((c^2*sin(e/2 + (f*x)/2))/5 - (16*c^2*cos(e/2 + (f*x)/2)^2*s
in(e/2 + (f*x)/2))/15 + (43*c^2*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))/1
5)/(a^3*f*cos(e/2 + (f*x)/2)^5)
```

3.35 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx$

3.35.1 Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx = \frac{cx}{a^3}-\frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{3c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2}-\frac{8c\tan(e+fx)}{5a^3f(1+\sec(e+fx))}$$

output
```
c*x/a^3-2/5*c*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3-3/5*c*tan(f*x+e)/a^3/f/(1+
sec(f*x+e))^2-8/5*c*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

3.35.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx = \frac{c\cot^5(e+fx)\left(16+3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)-60\sec(e+fx)+5\sec^2(e+fx}{15a^3f}$$

input
```
Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^3,x]
```

output
```
(c*Cot[e + f*x]^5*(16 + 3*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2
] - 60*Sec[e + f*x] + 5*Sec[e + f*x]^2 + 60*Sec[e + f*x]^3 - 24*Sec[e + f*
x]^5))/(15*a^3*f)
```

3.35.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-c\sec(e+fx)}{(a\sec(e+fx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{c}{(\sec(e+fx)+1)^3}-\frac{c\sec(e+fx)}{(\sec(e+fx)+1)^3}\right)dx}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{8c\tan(e+fx)}{5f(\sec(e+fx)+1)}-\frac{3c\tan(e+fx)}{5f(\sec(e+fx)+1)^2}-\frac{2c\tan(e+fx)}{5f(\sec(e+fx)+1)^3}+cx}{a^3}$$

input
```
Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^3,x]
```

output
```
(c*x - (2*c*Tan[e + f*x])/(5*f*(1 + Sec[e + f*x])^3) - (3*c*Tan[e + f*x])/
(5*f*(1 + Sec[e + f*x])^2) - (8*c*Tan[e + f*x])/(5*f*(1 + Sec[e + f*x])))/
a^3
```

3.35.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

3.35.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

method	result	size
parallelrisch	$-\frac{c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-10fx+20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{10a^{3}f}$	50
derivativedivides	$\frac{c\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{2f\,a^{3}}$	58
default	$\frac{c\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{2f\,a^{3}}$	58
norman	$\frac{\frac{cx}{a}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{2af}-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{10af}}{a^{2}}$	70
risch	$\frac{cx}{a^{3}}-\frac{2ic\left(20\,e^{4i(fx+e)}+55\,e^{3i(fx+e)}+75\,e^{2i(fx+e)}+45\,e^{i(fx+e)}+13\right)}{5f\,a^{3}\left(e^{i(fx+e)}+1\right)^{5}}$	77

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/10*c*(tan(1/2*f*x+1/2*e)^5-5*tan(1/2*f*x+1/2*e)^3-10*f*x+20*tan(1/2*f*x
+1/2*e))/a^3/f
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.41

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{5\,cfx\cos\left(fx+e\right)^{3}+15\,cfx\cos\left(fx+e\right)^{2}+15\,cfx\cos\left(fx+e\right)+5\,cfx-\left(13\,c\cos\left(fx+e\right)^{2}+19\,c\cos\left(\right.\right.}{5\left(a^{3}f\cos\left(fx+e\right)^{3}+3\,a^{3}f\cos\left(fx+e\right)^{2}+3\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/5*(5*c*f*x*cos(f*x + e)^3 + 15*c*f*x*cos(f*x + e)^2 + 15*c*f*x*cos(f*x +
 e) + 5*c*f*x - (13*c*cos(f*x + e)^2 + 19*c*cos(f*x + e) + 8*c)*sin(f*x +
e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e)
+ a^3*f)
```

3.35.6 Sympy [F]

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx$$
$$=-\frac{c\left(\int\frac{\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{1}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx\right)}{a^{3}}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

output
```
-c*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
 f*x) + 1), x) + Integral(-1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x))/a**3
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.81

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx = -\frac{c\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+\frac{c\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60\,f}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/60*(c*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a^3) + c*(15*sin(f*x + e)/(cos(f*x + e) + 1
) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5)/a^3)/f
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{10\,(fx+e)c}{a^3}-\frac{a^{12}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-5\,a^{12}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+20\,a^{12}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}}}{10\,f}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/10*(10*(f*x + e)*c/a^3 - (a^12*c*tan(1/2*f*x + 1/2*e)^5 - 5*a^12*c*tan(1
/2*f*x + 1/2*e)^3 + 20*a^12*c*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.35.9 Mupad [B] (verification not implemented)

Time = 14.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{c\,x}{a^3}-\frac{\frac{13\,c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{5}-\frac{7\,c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{10}+\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{10}}{a^3\,f\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

input
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^3,x)
```

output
```
(c*x)/a^3 - ((c*sin(e/2 + (f*x)/2))/10 - (7*c*cos(e/2 + (f*x)/2)^2*sin(e/2
 + (f*x)/2))/10 + (13*c*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))/5)/(a^3*f
*cos(e/2 + (f*x)/2)^5)
```

3.36 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$

3.36.1 Optimal result

Integrand size = 26, antiderivative size = 126

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{x}{a^3c}+\frac{\cot(e+fx)}{a^3cf}-\frac{\cot^3(e+fx)}{3a^3cf}+\frac{2\cot^5(e+fx)}{5a^3cf}-\frac{2\csc(e+fx)}{a^3cf}+\frac{4\csc^3(e+fx)}{3a^3cf}-\frac{2\csc^5(e+fx)}{5a^3cf}$$

output
```
x/a^3/c+cot(f*x+e)/a^3/c/f-1/3*cot(f*x+e)^3/a^3/c/f+2/5*cot(f*x+e)^5/a^3/c
/f-2*csc(f*x+e)/a^3/c/f+4/3*csc(f*x+e)^3/a^3/c/f-2/5*csc(f*x+e)^5/a^3/c/f
```

3.36.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{\cot^5(e+fx)\left(3+3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)-30\sec(e+fx)+40\sec^3(e+fx)\right.}{15a^3cf}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

output
```
(Cot[e + f*x]^5*(3 + 3*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2] -
 30*Sec[e + f*x] + 40*Sec[e + f*x]^3 - 16*Sec[e + f*x]^5))/(15*a^3*c*f)
```

3.36.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))^2dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4374}$$

$$-\frac{\int \left(c^2\cot^6(e+fx)-2c^2\csc(e+fx)\cot^5(e+fx)+c^2\csc^2(e+fx)\cot^4(e+fx)\right)dx}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2c^2\cot^5(e+fx)}{5f}+\frac{c^2\cot^3(e+fx)}{3f}-\frac{c^2\cot(e+fx)}{f}+\frac{2c^2\csc^5(e+fx)}{5f}-\frac{4c^2\csc^3(e+fx)}{3f}+\frac{2c^2\csc(e+fx)}{f}-c^2x}{a^3c^3}$$

input
```
Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

output
```
-((-(c^2*x) - (c^2*Cot[e + f*x])/f + (c^2*Cot[e + f*x]^3)/(3*f) - (2*c^2*C
ot[e + f*x]^5)/(5*f) + (2*c^2*Csc[e + f*x])/f - (4*c^2*Csc[e + f*x]^3)/(3*
f) + (2*c^2*Csc[e + f*x]^5)/(5*f))/(a^3*c^3))
```

3.36.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.36.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52

method	result	size
parallelrisch	$\frac{-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+25\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+120fx+15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-165\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{120f\,a^{3}c}$	65
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+16\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8f\,a^{3}c}$	73
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+16\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8f\,a^{3}c}$	73
risch	$\frac{x}{a^{3}c}-\frac{4i\left(15\,{\mathrm e}^{5i(fx+e)}+30\,{\mathrm e}^{4i(fx+e)}+10\,{\mathrm e}^{3i(fx+e)}-35\,{\mathrm e}^{2i(fx+e)}-37\,{\mathrm e}^{i(fx+e)}-13\right)}{15f\,a^{3}c\left({\mathrm e}^{i(fx+e)}+1\right)^{5}\left({\mathrm e}^{i(fx+e)}-1\right)}$	105
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{ca}+\frac{1}{8acf}-\frac{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{8acf}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{24acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{40acf}}{a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	111

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/120*(-3*tan(1/2*f*x+1/2*e)^5+25*tan(1/2*f*x+1/2*e)^3+120*f*x+15*cot(1/2*
f*x+1/2*e)-165*tan(1/2*f*x+1/2*e))/f/a^3/c
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$$
$$= \frac{26\,\cos\left(fx+e\right)^3+22\,\cos\left(fx+e\right)^2+15\left(fx\cos\left(fx+e\right)^2+2\,fx\cos\left(fx+e\right)+fx\right)\sin\left(fx+e\right)-17\,\text{co}}{15\left(a^3cf\cos\left(fx+e\right)^2+2\,a^3cf\cos\left(fx+e\right)+a^3cf\right)\sin\left(fx+e\right)}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
1/15*(26*cos(f*x + e)^3 + 22*cos(f*x + e)^2 + 15*(f*x*cos(f*x + e)^2 + 2*f
*x*cos(f*x + e) + f*x)*sin(f*x + e) - 17*cos(f*x + e) - 16)/((a^3*c*f*cos(
f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e))
```

3.36.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{\sec^4(e+fx)+2\sec^3(e+fx)-2\sec(e+fx)-1}\,dx}{a^3c}$$

input
```
integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(1/(sec(e + f*x)**4 + 2*sec(e + f*x)**3 - 2*sec(e + f*x) - 1), x)
/(a**3*c)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$$
$$= -\frac{\frac{\frac{165\sin(fx+e)}{\cos(fx+e)+1}-\frac{25\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3c}-\frac{240\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c}-\frac{15(\cos(fx+e)+1)}{a^3c\sin(fx+e)}}{120f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-1/120*((165*sin(f*x + e)/(cos(f*x + e) + 1) - 25*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c) - 240*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c) - 15*(cos(f*x + e) + 1)/(a^3*c*si
n(f*x + e)))/f
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$$
$$= \frac{\frac{120(fx+e)}{a^3c}+\frac{15}{a^3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}-\frac{3a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-25a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+165a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{15}c^5}}{120f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
1/120*(120*(f*x + e)/(a^3*c) + 15/(a^3*c*tan(1/2*f*x + 1/2*e)) - (3*a^12*c
^4*tan(1/2*f*x + 1/2*e)^5 - 25*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 165*a^12*
c^4*tan(1/2*f*x + 1/2*e))/(a^15*c^5))/f
```

3.36.9 Mupad [B] (verification not implemented)

Time = 14.70 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$$
$$=\frac{x}{a^3\,c}+\frac{\frac{26\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{15}-\frac{28\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{15}+\frac{17\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{60}-\frac{1}{40}}{a^3\,c\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}$$

input
```
int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))),x)
```

output
```
x/(a^3*c) + ((17*cos(e/2 + (f*x)/2)^2)/60 - (28*cos(e/2 + (f*x)/2)^4)/15 +
 (26*cos(e/2 + (f*x)/2)^6)/15 - 1/40)/(a^3*c*f*cos(e/2 + (f*x)/2)^5*sin(e/
2 + (f*x)/2))
```

3.37 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$

3.37.1 Optimal result

Integrand size = 26, antiderivative size = 100

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx &= \frac{x}{a^3c^2}+\frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f} \\ &\quad -\frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f}+\frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f}\end{aligned}$$

output
```
x/a^3/c^2+1/15*cot(f*x+e)*(15-8*sec(f*x+e))/a^3/c^2/f-1/15*cot(f*x+e)^3*(5
-4*sec(f*x+e))/a^3/c^2/f+1/5*cot(f*x+e)^5*(1-sec(f*x+e))/a^3/c^2/f
```

3.37.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\cot^5(e+fx)\left(3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)-15\sec(e+fx)+20\sec^3(e+fx)-8}{15a^3c^2f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

output
```
(Cot[e + f*x]^5*(3*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2] - 15*
Sec[e + f*x] + 20*Sec[e + f*x]^3 - 8*Sec[e + f*x]^5))/(15*a^3*c^2*f)
```

3.37.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {3042, 4392, 3042, 4370, 25, 3042, 4370, 25, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4370}$$

$$-\frac{\frac{1}{5}\int -\cot^4(e+fx)(5c-4c\sec(e+fx))dx-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{1}{5}\int \cot^4(e+fx)(5c-4c\sec(e+fx))dx-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{5}\int \frac{5c-4c\csc\left(e+fx+\frac{\pi}{2}\right)}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{4370}$$

$$-\frac{\frac{1}{5}\left(\frac{\cot^3(e+fx)(5c-4c\sec(e+fx))}{3f}-\frac{1}{3}\int -\cot^2(e+fx)(15c-8c\sec(e+fx))dx\right)-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \cot^2(e+fx)(15c-8c\sec(e+fx))dx+\frac{\cot^3(e+fx)(5c-4c\sec(e+fx))}{3f}\right)-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{15c-8c\csc\left(e+fx+\frac{\pi}{2}\right)}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx+\frac{\cot^3(e+fx)(5c-4c\sec(e+fx))}{3f}\right)-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{ 4370}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(\int -15cdx-\frac{\cot(e+fx)(15c-8c\sec(e+fx))}{f}\right)+\frac{\cot^3(e+fx)(5c-4c\sec(e+fx))}{3f}\right)-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{\frac{1}{5}\left(\frac{\cot^3(e+fx)(5c-4c\sec(e+fx))}{3f}+\frac{1}{3}\left(-\frac{\cot(e+fx)(15c-8c\sec(e+fx))}{f}-15cx\right)\right)-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

input
```
Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

output
```
-((-1/5*(Cot[e + f*x]^5*(c - c*Sec[e + f*x]))/f + ((Cot[e + f*x]^3*(5*c -
4*c*Sec[e + f*x]))/(3*f) + (-15*c*x - (Cot[e + f*x]*(15*c - 8*c*Sec[e + f*
x]))/f)/3)/5)/(a^3*c^3))
```

3.37.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.37.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

method	result	si
parallelrisch	$\frac{-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-5\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+240fx+90\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{240f\,a^{3}c^{2}}$	78
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-16\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^{2}a^{3}}$	88
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-16\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^{2}a^{3}}$	88
risch	$\frac{x}{a^{3}c^{2}}-\frac{2i\left(15\,{\rm e}^{7i(fx+e)}-15\,{\rm e}^{6i(fx+e)}-65\,{\rm e}^{5i(fx+e)}-25\,{\rm e}^{4i(fx+e)}+73\,{\rm e}^{3i(fx+e)}+31\,{\rm e}^{2i(fx+e)}-31\,{\rm e}^{i(fx+e)}-23\right)}{15f\,c^{2}a^{3}\left({\rm e}^{i(fx+e)}+1\right)^{5}\left({\rm e}^{i(fx+e)}-1\right)^{3}}$	12
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{ca}-\frac{1}{48acf}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{8acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{8acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{80acf}}{a^{2}c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}$	13

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/240*(-3*tan(1/2*f*x+1/2*e)^5-5*cot(1/2*f*x+1/2*e)^3+30*tan(1/2*f*x+1/2*e
)^3+240*f*x+90*cot(1/2*f*x+1/2*e)-240*tan(1/2*f*x+1/2*e))/f/a^3/c^2
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.54

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{23\cos(fx+e)^4+8\cos(fx+e)^3-27\cos(fx+e)^2+15\left(fx\cos(fx+e)^3+fx\cos(fx+e)^2-fx\cos\right.}{15\left(a^3c^2f\cos(fx+e)^3+a^3c^2f\cos(fx+e)^2-a^3c^2f\cos(fx+e)-a^3c\right.}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/15*(23*cos(f*x + e)^4 + 8*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + 15*(f*x*c
os(f*x + e)^3 + f*x*cos(f*x + e)^2 - f*x*cos(f*x + e) - f*x)*sin(f*x + e)
- 7*cos(f*x + e) + 8)/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^
2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))
```

3.37.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{1}{\sec^5(e+fx)+\sec^4(e+fx)-2\sec^3(e+fx)-2\sec^2(e+fx)+\sec(e+fx)+1}\,dx}{a^3c^2}$$

input
```
integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(1/(sec(e + f*x)**5 + sec(e + f*x)**4 - 2*sec(e + f*x)**3 - 2*sec(
e + f*x)**2 + sec(e + f*x) + 1), x)/(a**3*c**2)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.46

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{\frac{3\left(\frac{80\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^2}-\frac{480\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^2}-\frac{5\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{a^3c^2\sin(fx+e)^3}}{240f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/240*(3*(80*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^2) - 480*arctan
(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^2) - 5*(18*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a^3*c^2*sin(f*x + e)^3))/f
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\frac{240\,(fx+e)}{a^3c^2}+\frac{5\left(18\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)}{a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}-\frac{3\left(a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10\,a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+80\,a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}c^{10}}}{240\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/240*(240*(f*x + e)/(a^3*c^2) + 5*(18*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^3*c^
2*tan(1/2*f*x + 1/2*e)^3) - 3*(a^12*c^8*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c
^8*tan(1/2*f*x + 1/2*e)^3 + 80*a^12*c^8*tan(1/2*f*x + 1/2*e))/(a^15*c^10))
/f
```

3.37.9 Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{5\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^8+3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^8-30\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6+240\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4 -}{240\,a^3\,c^2\,f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\right.}$$

input
```
int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2),x)
```

output
```
-(5*cos(e/2 + (f*x)/2)^8 + 3*sin(e/2 + (f*x)/2)^8 - 30*cos(e/2 + (f*x)/2)^
2*sin(e/2 + (f*x)/2)^6 + 240*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^4 - 9
0*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^2 - 240*cos(e/2 + (f*x)/2)^5*sin
(e/2 + (f*x)/2)^3*(e + f*x))/(240*a^3*c^2*f*cos(e/2 + (f*x)/2)^5*sin(e/2 +
 (f*x)/2)^3)
```

3.37. $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$

3.38 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$

3.38.1 Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{x}{a^3c^3} + \frac{\cot(e+fx)}{a^3c^3f} - \frac{\cot^3(e+fx)}{3a^3c^3f} + \frac{\cot^5(e+fx)}{5a^3c^3f}$$

output
```
x/a^3/c^3+cot(f*x+e)/a^3/c^3/f-1/3*cot(f*x+e)^3/a^3/c^3/f+1/5*cot(f*x+e)^5
/a^3/c^3/f
```

3.38.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\cot^5(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)}{5a^3c^3f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

output
```
(Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2])/(5*a^3*
c^3*f)
```

3.38.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 4392, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 4392}$$

$$-\frac{\int \cot^6(e+fx)dx}{a^3c^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \tan\left(e+fx+\frac{\pi}{2}\right)^6dx}{a^3c^3}$$

$$\Big\downarrow \text{ 3954}$$

$$-\frac{-\int \cot^4(e+fx)dx-\frac{\cot^5(e+fx)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\int \tan\left(e+fx+\frac{\pi}{2}\right)^4dx-\frac{\cot^5(e+fx)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{ 3954}$$

$$-\frac{\int \cot^2(e+fx)dx-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}}{a^3c^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \tan\left(e+fx+\frac{\pi}{2}\right)^2dx-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}}{a^3c^3}$$

$$\Big\downarrow \text{ 3954}$$

$$-\frac{-\int 1dx-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}-\frac{\cot(e+fx)}{f}}{a^3c^3}$$

$$\Big\downarrow \; 24$$

$$-\frac{-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}-\frac{\cot(e+fx)}{f}-x}{a^3c^3}$$

input
```
Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

output
```
-((-x - Cot[e + f*x]/f + Cot[e + f*x]^3/(3*f) - Cot[e + f*x]^5/(5*f))/(a^3
*c^3))
```

3.38.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.38.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)+\frac{\pi}{2}-\operatorname{arccot}(\cot(fx+e))}{c^3a^3f}$	53
parallelrisch	$\frac{-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5+35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+480fx-330\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+330\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{480f\,c^3a^3}$	91
risch	$\frac{x}{a^3c^3}+\frac{2i\left(45\,e^{8i(fx+e)}-90\,e^{6i(fx+e)}+140\,e^{4i(fx+e)}-70\,e^{2i(fx+e)}+23\right)}{15f\,c^3a^3\left(e^{i(fx+e)}+1\right)^5\left(e^{i(fx+e)}-1\right)^5}$	94
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{ca}+\frac{1}{160acf}-\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{96acf}+\frac{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{16acf}-\frac{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{16acf}+\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{96acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{160acf}}{a^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	160

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/c^3/a^3/f*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)+1/2*Pi-arccot(
cot(f*x+e)))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$$
$$=\frac{23\,\cos(fx+e)^5-35\,\cos(fx+e)^3+15\left(fx\cos(fx+e)^4-2\,fx\cos(fx+e)^2+fx\right)\sin(fx+e)+15\,\cos(fx+e)}{15\left(a^3c^3f\cos(fx+e)^4-2\,a^3c^3f\cos(fx+e)^2+a^3c^3f\right)\sin(fx+e)}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(23*cos(f*x + e)^5 - 35*cos(f*x + e)^3 + 15*(f*x*cos(f*x + e)^4 - 2*f
*x*cos(f*x + e)^2 + f*x)*sin(f*x + e) + 15*cos(f*x + e))/((a^3*c^3*f*cos(f
*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))
```

3.38.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{1}{\sec^6(e+fx)-3\sec^4(e+fx)+3\sec^2(e+fx)-1}\,dx}{a^3c^3}$$

input
```
integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

output
```
-Integral(1/(sec(e + f*x)**6 - 3*sec(e + f*x)**4 + 3*sec(e + f*x)**2 - 1),
 x)/(a**3*c**3)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)}{a^3c^3} + \frac{15\,\tan(fx+e)^4-5\,\tan(fx+e)^2+3}{a^3c^3\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/15*(15*(f*x + e)/(a^3*c^3) + (15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)/
(a^3*c^3*tan(f*x + e)^5))/f
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(63) = 126.

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.93

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\frac{480\,(fx+e)}{a^3c^3} + \frac{330\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-35\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3}{a^3c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5} - \frac{3\,a^{12}c^{12}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a^{12}c^{12}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+330\,a^{12}c^{12}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.}{a^{15}c^{15}}}{480\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/480*(480*(f*x + e)/(a^3*c^3) + (330*tan(1/2*f*x + 1/2*e)^4 - 35*tan(1/2*
f*x + 1/2*e)^2 + 3)/(a^3*c^3*tan(1/2*f*x + 1/2*e)^5) - (3*a^12*c^12*tan(1/
2*f*x + 1/2*e)^5 - 35*a^12*c^12*tan(1/2*f*x + 1/2*e)^3 + 330*a^12*c^12*tan
(1/2*f*x + 1/2*e))/(a^15*c^15))/f
```

3.38.9 Mupad [B] (verification not implemented)

Time = 15.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\frac{5\cos(e+fx)}{24}-\frac{5\cos(3e+3fx)}{48}+\frac{23\cos(5e+5fx)}{240}-\frac{5\sin(3e+3fx)(e+fx)}{16}+\frac{\sin(5e+5fx)(e+fx)}{16}+\frac{5\sin(e+fx)(e+fx)}{8}}{a^3c^3f\sin(e+fx)^5}$$

input
```
int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3),x)
```

output
```
((5*cos(e + f*x))/24 - (5*cos(3*e + 3*f*x))/48 + (23*cos(5*e + 5*f*x))/240
 - (5*sin(3*e + 3*f*x)*(e + f*x))/16 + (sin(5*e + 5*f*x)*(e + f*x))/16 + (
5*sin(e + f*x)*(e + f*x))/8)/(a^3*c^3*f*sin(e + f*x)^5)
```

3.39 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$

3.39.1 Optimal result

Integrand size = 26, antiderivative size = 129

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx &= \frac{x}{a^3c^4}-\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f}+\frac{\cot^5(e+fx)(7+6\sec(e+fx))}{35a^3c^4f}\\ &\quad+\frac{\cot(e+fx)(35+16\sec(e+fx))}{35a^3c^4f}-\frac{\cot^3(e+fx)(35+24\sec(e+fx))}{105a^3c^4f}\end{aligned}$$

output
```
x/a^3/c^4-1/7*cot(f*x+e)^7*(1+sec(f*x+e))/a^3/c^4/f+1/35*cot(f*x+e)^5*(7+6
*sec(f*x+e))/a^3/c^4/f+1/35*cot(f*x+e)*(35+16*sec(f*x+e))/a^3/c^4/f-1/105*
cot(f*x+e)^3*(35+24*sec(f*x+e))/a^3/c^4/f
```

3.39.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = -\frac{\csc^7(e+fx)\left(-106+301\cos(2(e+fx))-70\cos(4(e+fx))+35\cos(6(e+fx))+160\cos^7(e+fx)\right.}{1120a^3c^4f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

output
```
-1/1120*(Csc[e + f*x]^7*(-106 + 301*Cos[2*(e + f*x)] - 70*Cos[4*(e + f*x)]
 + 35*Cos[6*(e + f*x)] + 160*Cos[e + f*x]^7*Hypergeometric2F1[-7/2, 1, -5/
2, -Tan[e + f*x]^2]))/(a^3*c^4*f)
```

3.39.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3042, 4392, 3042, 4370, 25, 3042, 4370, 25, 3042, 4370, 27, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^8(e+fx)(\sec(e+fx)a+a)dx}{a^4c^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}{\cot\left(e+fx+\frac{\pi}{2}\right)^8}dx}{a^4c^4}$$

$$\Big\downarrow \text{4370}$$

$$\frac{\frac{1}{7}\int -\cot^6(e+fx)(6\sec(e+fx)a+7a)dx-\frac{\cot^7(e+fx)(a\sec(e+fx)+a)}{7f}}{a^4c^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{1}{7}\int \cot^6(e+fx)(6\sec(e+fx)a+7a)dx-\frac{\cot^7(e+fx)(a\sec(e+fx)+a)}{7f}}{a^4c^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{1}{7}\int \frac{6\csc\left(e+fx+\frac{\pi}{2}\right)a+7a}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx-\frac{\cot^7(e+fx)(a\sec(e+fx)+a)}{7f}}{a^4c^4}$$

$\Big\downarrow$ 4370

$$\frac{\frac{1}{7}\left(\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}-\frac{1}{5}\int -\cot^4(e+fx)(24\sec(e+fx)a+35a)dx\right)-\frac{\cot^7(e+fx)(a\sec(e+fx)+a)}{7f}}{a^4c^4}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{7}\left(\frac{1}{5}\int \cot^4(e+fx)(24\sec(e+fx)a+35a)dx+\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}\right)-\frac{\cot^7(e+fx)(a\sec(e+fx)+a)}{7f}}{a^4c^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{7}\left(\frac{1}{5}\int \frac{24\csc\left(e+fx+\frac{\pi}{2}\right)a+35a}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx+\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}\right)-\frac{\cot^7(e+fx)(a\sec(e+fx)+a)}{7f}}{a^4c^4}$$

$\Big\downarrow$ 4370

$$\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int -3\cot^2(e+fx)(16\sec(e+fx)a+35a)dx-\frac{\cot^3(e+fx)(24a\sec(e+fx)+35a)}{3f}\right)+\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}\right)-}{a^4c^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{7}\left(\frac{1}{5}\left(-\int \cot^2(e+fx)(16\sec(e+fx)a+35a)dx-\frac{\cot^3(e+fx)(24a\sec(e+fx)+35a)}{3f}\right)+\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}\right)-\text{co}}{a^4c^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{7}\left(\frac{1}{5}\left(-\int \frac{16\csc\left(e+fx+\frac{\pi}{2}\right)a+35a}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx-\frac{\cot^3(e+fx)(24a\sec(e+fx)+35a)}{3f}\right)+\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}\right)-\frac{\cot^7(e+fx)(a\sec(e+f}{7f}}{a^4c^4}$$

$\Big\downarrow$ 4370

$$\frac{\frac{1}{7}\left(\frac{1}{5}\left(-\int -35adx-\frac{\cot^3(e+fx)(24a\sec(e+fx)+35a)}{3f}+\frac{\cot(e+fx)(16a\sec(e+fx)+35a)}{f}\right)+\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}\right)-\cot^7(}{a^4c^4}$$

$\Big\downarrow$ 24

$$\frac{\frac{1}{7}\left(\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}+\frac{1}{5}\left(-\frac{\cot^3(e+fx)(24a\sec(e+fx)+35a)}{3f}+\frac{\cot(e+fx)(16a\sec(e+fx)+35a)}{f}+35ax\right)\right)-\cot^7(e+fx)(}{a^4c^4}$$

3.39. $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$

input
```
Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

output
```
(-1/7*(Cot[e + f*x]^7*(a + a*Sec[e + f*x]))/f + ((Cot[e + f*x]^5*(7*a + 6*
a*Sec[e + f*x]))/(5*f) + (35*a*x + (Cot[e + f*x]*(35*a + 16*a*Sec[e + f*x]
))/f - (Cot[e + f*x]^3*(35*a + 24*a*Sec[e + f*x]))/(3*f))/5)/7)/(a^4*c^4)
```

3.39.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.39.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{-15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}-21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+168\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+280\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-1015\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+6720fx-3045\tan\left(\frac{f}{2}\right.}{6720f\,a^{3}c^{4}}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{8}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{29}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{64}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+128\,a}{64f\,c^{4}a^{3}}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{8}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{29}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{64}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+128\,a}{64f\,c^{4}a^{3}}$
risch	$\frac{x}{a^{3}c^{4}}+\frac{2i\left(105\,\mathrm{e}^{11i(fx+e)}+210\,\mathrm{e}^{10i(fx+e)}-735\,\mathrm{e}^{9i(fx+e)}+1638\,\mathrm{e}^{7i(fx+e)}-196\,\mathrm{e}^{6i(fx+e)}-1882\,\mathrm{e}^{5i(fx+e)}+880\,\mathrm{e}^{4i(fx+}\right.}{105f\,c^{4}a^{3}\left(\mathrm{e}^{i(fx+e)}-1\right)^{7}\left(\mathrm{e}^{i(fx+e)}+1\right)^{5}}$
norman	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{acf}+\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{ca}-\frac{1}{448acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{40acf}-\frac{29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{192acf}-\frac{29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{64acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{24acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{320acf}}{a^{2}c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
1/6720*(-15*cot(1/2*f*x+1/2*e)^7-21*tan(1/2*f*x+1/2*e)^5+168*cot(1/2*f*x+1
/2*e)^5+280*tan(1/2*f*x+1/2*e)^3-1015*cot(1/2*f*x+1/2*e)^3+6720*f*x-3045*t
an(1/2*f*x+1/2*e)+6720*cot(1/2*f*x+1/2*e))/f/a^3/c^4
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.80

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$$
$$=\frac{176\cos(fx+e)^{6}-71\cos(fx+e)^{5}-335\cos(fx+e)^{4}+125\cos(fx+e)^{3}+225\cos(fx+e)^{2}+105}{105\left(a^{3}c^{4}f\cos(fx+e)^{5}-a^{3}c^{4}f\cos(fx+e)^{4}-2\,a\right.}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
1/105*(176*cos(f*x + e)^6 - 71*cos(f*x + e)^5 - 335*cos(f*x + e)^4 + 125*c
os(f*x + e)^3 + 225*cos(f*x + e)^2 + 105*(f*x*cos(f*x + e)^5 - f*x*cos(f*x
 + e)^4 - 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(f*x + e)^2 + f*x*cos(f*x + e) -
 f*x)*sin(f*x + e) - 57*cos(f*x + e) - 48)/((a^3*c^4*f*cos(f*x + e)^5 - a^
3*c^4*f*cos(f*x + e)^4 - 2*a^3*c^4*f*cos(f*x + e)^3 + 2*a^3*c^4*f*cos(f*x
+ e)^2 + a^3*c^4*f*cos(f*x + e) - a^3*c^4*f)*sin(f*x + e))
```

3.39.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\int \frac{1}{\sec^7(e+fx)-\sec^6(e+fx)-3\sec^5(e+fx)+3\sec^4(e+fx)+3\sec^3(e+fx)-3\sec^2(e+fx)-\sec(e+fx)+1}\,dx}{a^3c^4}$$

input
```
integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

output
```
Integral(1/(sec(e + f*x)**7 - sec(e + f*x)**6 - 3*sec(e + f*x)**5 + 3*sec(
e + f*x)**4 + 3*sec(e + f*x)**3 - 3*sec(e + f*x)**2 - sec(e + f*x) + 1), x
)/(a**3*c**4)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.45

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = -\frac{\frac{7\left(\frac{435\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^4}-\frac{13440\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^4}-\frac{\left(\frac{168\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1015\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{6720\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right.}{a^3c^4\sin(fx+e)^7}}{6720f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
-1/6720*(7*(435*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^4) - 13440*
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^4) - (168*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 1015*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6720*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e) + 1)^7/(a^3*c^4*sin(f*x
 + e)^7))/f
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\frac{6720\,(fx+e)}{a^3c^4} + \frac{6720\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6 - 1015\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 168\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 15}{a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7} - \frac{7\left(3\,a^{12}c^{16}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 40\,a^{12}c^{16}\tan\left(\frac{1}{2}\,fx+\right.\right.}{a^{15}c^{20}}}{6720\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/6720*(6720*(f*x + e)/(a^3*c^4) + (6720*tan(1/2*f*x + 1/2*e)^6 - 1015*tan
(1/2*f*x + 1/2*e)^4 + 168*tan(1/2*f*x + 1/2*e)^2 - 15)/(a^3*c^4*tan(1/2*f*
x + 1/2*e)^7) - 7*(3*a^12*c^16*tan(1/2*f*x + 1/2*e)^5 - 40*a^12*c^16*tan(1
/2*f*x + 1/2*e)^3 + 435*a^12*c^16*tan(1/2*f*x + 1/2*e))/(a^15*c^20))/f
```

3.39.9 Mupad [B] (verification not implemented)

Time = 15.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.62

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = -\frac{15\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{12} + 21\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{12} - 280\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{10} + 3045\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\right.}{}$$

input
```
int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4),x)
```

output
```
-(15*cos(e/2 + (f*x)/2)^12 + 21*sin(e/2 + (f*x)/2)^12 - 280*cos(e/2 + (f*x
)/2)^2*sin(e/2 + (f*x)/2)^10 + 3045*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2
)^8 - 6720*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^6 + 1015*cos(e/2 + (f*x
)/2)^8*sin(e/2 + (f*x)/2)^4 - 168*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)/2)
^2 - 6720*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^7*(e + f*x))/(6720*a^3*c
^4*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^7)
```

3.40 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$

3.40.1 Optimal result

Integrand size = 26, antiderivative size = 210

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{x}{a^3c^5}+\frac{\cot(e+fx)}{a^3c^5f}-\frac{\cot^3(e+fx)}{3a^3c^5f}+\frac{\cot^5(e+fx)}{5a^3c^5f}-\frac{\cot^7(e+fx)}{7a^3c^5f}+\frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{2\csc(e+fx)}{a^3c^5f}-\frac{8\csc^3(e+fx)}{3a^3c^5f}+\frac{12\csc^5(e+fx)}{5a^3c^5f}-\frac{8\csc^7(e+fx)}{7a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}$$

output
```
x/a^3/c^5+cot(f*x+e)/a^3/c^5/f-1/3*cot(f*x+e)^3/a^3/c^5/f+1/5*cot(f*x+e)^5
/a^3/c^5/f-1/7*cot(f*x+e)^7/a^3/c^5/f+2/9*cot(f*x+e)^9/a^3/c^5/f+2*csc(f*x
+e)/a^3/c^5/f-8/3*csc(f*x+e)^3/a^3/c^5/f+12/5*csc(f*x+e)^5/a^3/c^5/f-8/7*c
sc(f*x+e)^7/a^3/c^5/f+2/9*csc(f*x+e)^9/a^3/c^5/f
```

3.40.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{\cot^9(e+fx)\left(35+35\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(e+fx)\right)+630\sec(e+fx)-1680\sec^3(e-\right.}{315a^3c^5f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

output
```
(Cot[e + f*x]^9*(35 + 35*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*x]^2]
 + 630*Sec[e + f*x] - 1680*Sec[e + f*x]^3 + 2016*Sec[e + f*x]^5 - 1152*Sec
[e + f*x]^7 + 256*Sec[e + f*x]^9))/(315*a^3*c^5*f)
```

3.40.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^{10}(e+fx)(\sec(e+fx)a+a)^2dx}{a^5c^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\cot\left(e+fx+\frac{\pi}{2}\right)^{10}}dx}{a^5c^5}$$

$$\Big\downarrow \text{4374}$$

$$-\frac{\int \left(a^2\cot^{10}(e+fx)+2a^2\csc(e+fx)\cot^9(e+fx)+a^2\csc^2(e+fx)\cot^8(e+fx)\right)dx}{a^5c^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2a^2\cot^9(e+fx)}{9f}+\frac{a^2\cot^7(e+fx)}{7f}-\frac{a^2\cot^5(e+fx)}{5f}+\frac{a^2\cot^3(e+fx)}{3f}-\frac{a^2\cot(e+fx)}{f}-\frac{2a^2\csc^9(e+fx)}{9f}+\frac{8a^2\csc^7(e+fx)}{7f}-\frac{12a^2\text{cs}\cdots}{}}{a^5c^5}$$

input
```
Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

output
```
-((-(a^2*x) - (a^2*Cot[e + f*x])/f + (a^2*Cot[e + f*x]^3)/(3*f) - (a^2*Cot
[e + f*x]^5)/(5*f) + (a^2*Cot[e + f*x]^7)/(7*f) - (2*a^2*Cot[e + f*x]^9)/(
9*f) - (2*a^2*Csc[e + f*x])/f + (8*a^2*Csc[e + f*x]^3)/(3*f) - (12*a^2*Csc
[e + f*x]^5)/(5*f) + (8*a^2*Csc[e + f*x]^7)/(7*f) - (2*a^2*Csc[e + f*x]^9)
/(9*f))/(a^5*c^5))
```

3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.40.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.56

method	result
parallelrisch	$\frac{35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}-405\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}-63\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+2331\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+945\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-9765\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+40}{40320 f\, a^{3} c^{5}}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+256\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{9}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{}{5\tan(}}{128 f\, c^{5} a^{3}}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+256\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{9}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{}{5\tan(}}{128 f\, c^{5} a^{3}}$
risch	$\frac{x}{a^{3} c^{5}}+\frac{4i\left(315\,\mathrm{e}^{13i(fx+e)}-315\,\mathrm{e}^{12i(fx+e)}-1470\,\mathrm{e}^{11i(fx+e)}+3360\,\mathrm{e}^{10i(fx+e)}+1113\,\mathrm{e}^{9i(fx+e)}-6447\,\mathrm{e}^{8i(fx+e)}+2028\,\mathrm{e}^{7}\right.}{315 f\, c^{5} a^{3}\left(\mathrm{e}^{i(fx+e)}-\right.}$
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{ca}+\frac{1}{1152acf}-\frac{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{896acf}+\frac{37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{640acf}-\frac{31\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{128acf}+\frac{163\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{128acf}-\frac{37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{128acf}+\frac{3\tan}{}}{c^{4} a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}$

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output
```
1/40320*(35*cot(1/2*f*x+1/2*e)^9-405*cot(1/2*f*x+1/2*e)^7-63*tan(1/2*f*x+1
/2*e)^5+2331*cot(1/2*f*x+1/2*e)^5+945*tan(1/2*f*x+1/2*e)^3-9765*cot(1/2*f*
x+1/2*e)^3+40320*f*x-11655*tan(1/2*f*x+1/2*e)+51345*cot(1/2*f*x+1/2*e))/f/
a^3/c^5
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{598\cos(fx+e)^7-566\cos(fx+e)^6-1212\cos(fx+e)^5+1310\cos(fx+e)^4+860\cos(fx+e)^3-1}{315\left(a^3c^5f\cos(fx+e)^6-2\,a^3c^5f\cos(\right.}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
1/315*(598*cos(f*x + e)^7 - 566*cos(f*x + e)^6 - 1212*cos(f*x + e)^5 + 131
0*cos(f*x + e)^4 + 860*cos(f*x + e)^3 - 1014*cos(f*x + e)^2 + 315*(f*x*cos
(f*x + e)^6 - 2*f*x*cos(f*x + e)^5 - f*x*cos(f*x + e)^4 + 4*f*x*cos(f*x +
e)^3 - f*x*cos(f*x + e)^2 - 2*f*x*cos(f*x + e) + f*x)*sin(f*x + e) - 197*c
os(f*x + e) + 256)/((a^3*c^5*f*cos(f*x + e)^6 - 2*a^3*c^5*f*cos(f*x + e)^5
 - a^3*c^5*f*cos(f*x + e)^4 + 4*a^3*c^5*f*cos(f*x + e)^3 - a^3*c^5*f*cos(f
*x + e)^2 - 2*a^3*c^5*f*cos(f*x + e) + a^3*c^5*f)*sin(f*x + e))
```

3.40.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = -\frac{\int \frac{1}{\sec^8(e+fx)-2\sec^7(e+fx)-2\sec^6(e+fx)+6\sec^5(e+fx)-6\sec^3(e+fx)+2\sec^2(e+fx)+2\sec(e+fx)-1}\,dx}{a^3c^5}$$

input
```
integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

output
```
-Integral(1/(sec(e + f*x)**8 - 2*sec(e + f*x)**7 - 2*sec(e + f*x)**6 + 6*s
ec(e + f*x)**5 - 6*sec(e + f*x)**3 + 2*sec(e + f*x)**2 + 2*sec(e + f*x) -
1), x)/(a**3*c**5)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = -\frac{\frac{63\left(\frac{185\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{15\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^5}-\frac{80640\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^5}+\frac{\left(\frac{405\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2331\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{9765\,\sin(fx+e}{(\cos(fx+e)+1}\right.}{a^3c^5\sin(fx}}{40320\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
-1/40320*(63*(185*sin(f*x + e)/(cos(f*x + e) + 1) - 15*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^5) - 80640*
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^5) + (405*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 2331*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9765*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 - 51345*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 - 35)*(cos(f*x + e) + 1)^9/(a^3*c^5*sin(f*x + e)^9))/f
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{\frac{40320\,(fx+e)}{a^3c^5} + \frac{51345\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-9765\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+2331\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-405\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35}{a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9} - \frac{63\left(a^{12}c^{20}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{}}{40320\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
1/40320*(40320*(f*x + e)/(a^3*c^5) + (51345*tan(1/2*f*x + 1/2*e)^8 - 9765*
tan(1/2*f*x + 1/2*e)^6 + 2331*tan(1/2*f*x + 1/2*e)^4 - 405*tan(1/2*f*x + 1
/2*e)^2 + 35)/(a^3*c^5*tan(1/2*f*x + 1/2*e)^9) - 63*(a^12*c^20*tan(1/2*f*x
 + 1/2*e)^5 - 15*a^12*c^20*tan(1/2*f*x + 1/2*e)^3 + 185*a^12*c^20*tan(1/2*
f*x + 1/2*e))/(a^15*c^25))/f
```

3.40.9 Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{35\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{14} - 63\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{14} + 945\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{12} - 11655\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\right.}{}$$

input
```
int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5),x)
```

output
```
(35*cos(e/2 + (f*x)/2)^14 - 63*sin(e/2 + (f*x)/2)^14 + 945*cos(e/2 + (f*x)
/2)^2*sin(e/2 + (f*x)/2)^12 - 11655*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2
)^10 + 51345*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^8 - 9765*cos(e/2 + (f
*x)/2)^8*sin(e/2 + (f*x)/2)^6 + 2331*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)
/2)^4 - 405*cos(e/2 + (f*x)/2)^12*sin(e/2 + (f*x)/2)^2 + 40320*cos(e/2 + (
f*x)/2)^5*sin(e/2 + (f*x)/2)^9*(e + f*x))/(40320*a^3*c^5*f*cos(e/2 + (f*x)
/2)^5*sin(e/2 + (f*x)/2)^9)
```

3.41 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$

3.41.1 Optimal result

Integrand size = 26, antiderivative size = 252

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx &= \frac{x}{a^3c^6}+\frac{\cot(e+fx)}{a^3c^6f}-\frac{\cot^3(e+fx)}{3a^3c^6f}+\frac{\cot^5(e+fx)}{5a^3c^6f}-\frac{\cot^7(e+fx)}{7a^3c^6f}\\ &+\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{3\csc(e+fx)}{a^3c^6f}-\frac{16\csc^3(e+fx)}{3a^3c^6f}\\ &+\frac{34\csc^5(e+fx)}{5a^3c^6f}-\frac{36\csc^7(e+fx)}{7a^3c^6f}+\frac{19\csc^9(e+fx)}{9a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}\end{aligned}$$

output
```
x/a^3/c^6+cot(f*x+e)/a^3/c^6/f-1/3*cot(f*x+e)^3/a^3/c^6/f+1/5*cot(f*x+e)^5
/a^3/c^6/f-1/7*cot(f*x+e)^7/a^3/c^6/f+1/9*cot(f*x+e)^9/a^3/c^6/f-4/11*cot(
f*x+e)^11/a^3/c^6/f+3*csc(f*x+e)/a^3/c^6/f-16/3*csc(f*x+e)^3/a^3/c^6/f+34/
5*csc(f*x+e)^5/a^3/c^6/f-36/7*csc(f*x+e)^7/a^3/c^6/f+19/9*csc(f*x+e)^9/a^3
/c^6/f-4/11*csc(f*x+e)^11/a^3/c^6/f
```

3.41.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 11.12 (sec) , antiderivative size = 787, normalized size of antiderivative = 3.12

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{\cot^9(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{11}{2},1,-\frac{9}{2},-\tan^2(e+fx)\right)}{11a^2c^5f(a+a\sec(e+fx))(c-c\sec(e+fx))} + \frac{16\tan(e+fx)}{55a^3f(c-c\sec(e+fx))^6} - \frac{2a^3\tan(e+fx)}{11f(a+a\sec(e+fx))^6(c-c\sec(e+fx))^6} - \frac{3a^3\sec(e+fx)\tan(e+fx)}{11f(a+a\sec(e+fx))^6(c-c\sec(e+fx))^6} - \frac{a^2\tan(e+fx)}{9f(a+a\sec(e+fx))^5(c-c\sec(e+fx))^6} - \frac{a\tan(e+fx)}{63f(a+a\sec(e+fx))^4(c-c\sec(e+fx))^6} - \frac{\tan(e+fx)}{35f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6} - \frac{8\tan(e+fx)}{105af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^6} - \frac{8\tan(e+fx)}{15a^2f(a+a\sec(e+fx))(c-c\sec(e+fx))^6} + \frac{16\tan(e+fx)}{99a^3cf(c-c\sec(e+fx))^5} - \frac{10a^2\sec(e+fx)\tan(e+fx)}{33cf(a+a\sec(e+fx))^5(c-c\sec(e+fx))^5} + \frac{64\tan(e+fx)}{693a^3c^2f(c-c\sec(e+fx))^4} - \frac{80a\sec(e+fx)\tan(e+fx)}{231c^2f(a+a\sec(e+fx))^4(c-c\sec(e+fx))^4} + \frac{64\tan(e+fx)}{1155a^3c^3f(c-c\sec(e+fx))^3} - \frac{32\sec(e+fx)\tan(e+fx)}{77c^3f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3} + \frac{128\tan(e+fx)}{3465a^3c^4f(c-c\sec(e+fx))^2} - \frac{128\sec(e+fx)\tan(e+fx)}{231ac^4f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2} + \frac{128\tan(e+fx)}{3465a^3c^5f(c-c\sec(e+fx))} - \frac{256\sec(e+fx)\tan(e+fx)}{231a^2c^5f(a+a\sec(e+fx))(c-c\sec(e+fx))}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

output
```
(Cot[e + f*x]^9*Hypergeometric2F1[-11/2, 1, -9/2, -Tan[e + f*x]^2])/(11*a^
2*c^5*f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])) + (16*Tan[e + f*x])/(55
*a^3*f*(c - c*Sec[e + f*x])^6) - (2*a^3*Tan[e + f*x])/(11*f*(a + a*Sec[e +
 f*x])^6*(c - c*Sec[e + f*x])^6) - (3*a^3*Sec[e + f*x]*Tan[e + f*x])/(11*f
*(a + a*Sec[e + f*x])^6*(c - c*Sec[e + f*x])^6) - (a^2*Tan[e + f*x])/(9*f*
(a + a*Sec[e + f*x])^5*(c - c*Sec[e + f*x])^6) - (a*Tan[e + f*x])/(63*f*(a
 + a*Sec[e + f*x])^4*(c - c*Sec[e + f*x])^6) - Tan[e + f*x]/(35*f*(a + a*S
ec[e + f*x])^3*(c - c*Sec[e + f*x])^6) - (8*Tan[e + f*x])/(105*a*f*(a + a*
Sec[e + f*x])^2*(c - c*Sec[e + f*x])^6) - (8*Tan[e + f*x])/(15*a^2*f*(a +
a*Sec[e + f*x])*(c - c*Sec[e + f*x])^6) + (16*Tan[e + f*x])/(99*a^3*c*f*(c
 - c*Sec[e + f*x])^5) - (10*a^2*Sec[e + f*x]*Tan[e + f*x])/(33*c*f*(a + a*
Sec[e + f*x])^5*(c - c*Sec[e + f*x])^5) + (64*Tan[e + f*x])/(693*a^3*c^2*f
*(c - c*Sec[e + f*x])^4) - (80*a*Sec[e + f*x]*Tan[e + f*x])/(231*c^2*f*(a
+ a*Sec[e + f*x])^4*(c - c*Sec[e + f*x])^4) + (64*Tan[e + f*x])/(1155*a^3*
c^3*f*(c - c*Sec[e + f*x])^3) - (32*Sec[e + f*x]*Tan[e + f*x])/(77*c^3*f*(
a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3) + (128*Tan[e + f*x])/(3465*a
^3*c^4*f*(c - c*Sec[e + f*x])^2) - (128*Sec[e + f*x]*Tan[e + f*x])/(231*a*
c^4*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2) + (128*Tan[e + f*x])/
(3465*a^3*c^5*f*(c - c*Sec[e + f*x])) - (256*Sec[e + f*x]*Tan[e + f*x])/(2
31*a^2*c^5*f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]))
```

3.41.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^6}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^6}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^{12}(e+fx)(\sec(e+fx)a+a)^3dx}{a^6c^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3}{\cot\left(e+fx+\frac{\pi}{2}\right)^{12}}dx}{a^6c^6}$$

$$\Big\downarrow \text{4374}$$

$$\frac{\int \left(a^3\cot^{12}(e+fx)+3a^3\csc(e+fx)\cot^{11}(e+fx)+3a^3\csc^2(e+fx)\cot^{10}(e+fx)+a^3\csc^3(e+fx)\cot^9(e+f\right.}{a^6c^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{4a^3\cot^{11}(e+fx)}{11f}+\frac{a^3\cot^9(e+fx)}{9f}-\frac{a^3\cot^7(e+fx)}{7f}+\frac{a^3\cot^5(e+fx)}{5f}-\frac{a^3\cot^3(e+fx)}{3f}+\frac{a^3\cot(e+fx)}{f}-\frac{4a^3\csc^{11}(e+fx)}{11f}+\frac{19a^3\csc}{9}}{a^6c^6}$$

input
```
Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

output
```
(a^3*x + (a^3*Cot[e + f*x])/f - (a^3*Cot[e + f*x]^3)/(3*f) + (a^3*Cot[e +
f*x]^5)/(5*f) - (a^3*Cot[e + f*x]^7)/(7*f) + (a^3*Cot[e + f*x]^9)/(9*f) -
(4*a^3*Cot[e + f*x]^11)/(11*f) + (3*a^3*Csc[e + f*x])/f - (16*a^3*Csc[e +
f*x]^3)/(3*f) + (34*a^3*Csc[e + f*x]^5)/(5*f) - (36*a^3*Csc[e + f*x]^7)/(7
*f) + (19*a^3*Csc[e + f*x]^9)/(9*f) - (4*a^3*Csc[e + f*x]^11)/(11*f))/(a^6
*c^6)
```

3.41.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.41.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.52

method	result
parallelrisch	$\frac{-315\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}+3850\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}-22770\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}-693\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+90090\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+11550\tan\left(\frac{f}{}\right.}{887040 f\,a^{3}c^{6}}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-46\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+512\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{10}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{}{7\tan}}{256 f\,a^{3}c^{6}}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-46\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+512\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{10}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{}{7\tan}}{256 f\,a^{3}c^{6}}$
risch	$\frac{x}{a^{3}c^{6}}+\frac{2i\left(10395\,\mathrm{e}^{15i(fx+e)}-31185\,\mathrm{e}^{14i(fx+e)}+1155\,\mathrm{e}^{13i(fx+e)}+148995\,\mathrm{e}^{12i(fx+e)}-190113\,\mathrm{e}^{11i(fx+e)}-117117\,\mathrm{e}^{10i(\right.}{}$
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{ca}-\frac{1}{2816acf}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{1152acf}-\frac{23\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{896acf}+\frac{13\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{128acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{3acf}+\frac{191\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{128acf}-\frac{23\tan}{}}{c^{5}a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}$

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x,method=_RETURNVERBOSE)
```

output
```
1/887040*(-315*cot(1/2*f*x+1/2*e)^11+3850*cot(1/2*f*x+1/2*e)^9-22770*cot(1
/2*f*x+1/2*e)^7-693*tan(1/2*f*x+1/2*e)^5+90090*cot(1/2*f*x+1/2*e)^5+11550*
tan(1/2*f*x+1/2*e)^3-295680*cot(1/2*f*x+1/2*e)^3+887040*f*x-159390*tan(1/2
*f*x+1/2*e)+1323630*cot(1/2*f*x+1/2*e))/f/a^3/c^6
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.23

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{7453\cos(fx+e)^8 - 11964\cos(fx+e)^7 - 11866\cos(fx+e)^6 + 30542\cos(fx+e)^5 + 90\cos(fx+e)}{3465\left(a^3c^6f\cos(fx+e)^7\right.}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fricas")
```

output
```
1/3465*(7453*cos(f*x + e)^8 - 11964*cos(f*x + e)^7 - 11866*cos(f*x + e)^6
+ 30542*cos(f*x + e)^5 + 90*cos(f*x + e)^4 - 26438*cos(f*x + e)^3 + 8539*c
os(f*x + e)^2 + 3465*(f*x*cos(f*x + e)^7 - 3*f*x*cos(f*x + e)^6 + f*x*cos(
f*x + e)^5 + 5*f*x*cos(f*x + e)^4 - 5*f*x*cos(f*x + e)^3 - f*x*cos(f*x + e
)^2 + 3*f*x*cos(f*x + e) - f*x)*sin(f*x + e) + 7671*cos(f*x + e) - 3712)/(
(a^3*c^6*f*cos(f*x + e)^7 - 3*a^3*c^6*f*cos(f*x + e)^6 + a^3*c^6*f*cos(f*x
 + e)^5 + 5*a^3*c^6*f*cos(f*x + e)^4 - 5*a^3*c^6*f*cos(f*x + e)^3 - a^3*c^
6*f*cos(f*x + e)^2 + 3*a^3*c^6*f*cos(f*x + e) - a^3*c^6*f)*sin(f*x + e))
```

3.41.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{\int \frac{1}{\sec^9(e+fx) - 3\sec^8(e+fx) + 8\sec^6(e+fx) - 6\sec^5(e+fx) - 6\sec^4(e+fx) + 8\sec^3(e+fx) - 3\sec(e+fx) + 1}\,dx}{a^3c^6}$$

input
```
integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

output
```
Integral(1/(sec(e + f*x)**9 - 3*sec(e + f*x)**8 + 8*sec(e + f*x)**6 - 6*se
c(e + f*x)**5 - 6*sec(e + f*x)**4 + 8*sec(e + f*x)**3 - 3*sec(e + f*x) + 1
), x)/(a**3*c**6)
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a + a\sec(e + fx))^3 (c - c\sec(e + fx))^6}\,dx = -\frac{\frac{231\left(\frac{690\,\sin(fx+e)}{\cos(fx+e)+1} - \frac{50\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^6} - \frac{1774080\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^6} - \frac{5\left(\frac{770\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{4554\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{18018\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right.}{}}{887040\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="maxima")
```

output
```
-1/887040*(231*(690*sin(f*x + e)/(cos(f*x + e) + 1) - 50*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^6) - 17
74080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^6) - 5*(770*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 4554*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 18
018*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 59136*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 + 264726*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63)*(cos(f*x +
e) + 1)^11/(a^3*c^6*sin(f*x + e)^11))/f
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a + a\sec(e + fx))^3 (c - c\sec(e + fx))^6}\,dx = \frac{\frac{887040\,(fx+e)}{a^3c^6} + \frac{5\left(264726\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10} - 59136\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8} + 18018\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6} - 4554\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4} + 770\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right.}{a^3c^6\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}}{887040\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="giac")
```

output
```
1/887040*(887040*(f*x + e)/(a^3*c^6) + 5*(264726*tan(1/2*f*x + 1/2*e)^10 -
 59136*tan(1/2*f*x + 1/2*e)^8 + 18018*tan(1/2*f*x + 1/2*e)^6 - 4554*tan(1/
2*f*x + 1/2*e)^4 + 770*tan(1/2*f*x + 1/2*e)^2 - 63)/(a^3*c^6*tan(1/2*f*x +
 1/2*e)^11) - 231*(3*a^12*c^24*tan(1/2*f*x + 1/2*e)^5 - 50*a^12*c^24*tan(1
/2*f*x + 1/2*e)^3 + 690*a^12*c^24*tan(1/2*f*x + 1/2*e))/(a^15*c^30))/f
```

3.41.9 Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = -\frac{315\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{16}+693\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{16}-11550\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}+159390\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\text{s}}{}$$

input
```
int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^6),x)
```

output
```
-(315*cos(e/2 + (f*x)/2)^16 + 693*sin(e/2 + (f*x)/2)^16 - 11550*cos(e/2 +
(f*x)/2)^2*sin(e/2 + (f*x)/2)^14 + 159390*cos(e/2 + (f*x)/2)^4*sin(e/2 + (
f*x)/2)^12 - 1323630*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^10 + 295680*c
os(e/2 + (f*x)/2)^8*sin(e/2 + (f*x)/2)^8 - 90090*cos(e/2 + (f*x)/2)^10*sin
(e/2 + (f*x)/2)^6 + 22770*cos(e/2 + (f*x)/2)^12*sin(e/2 + (f*x)/2)^4 - 385
0*cos(e/2 + (f*x)/2)^14*sin(e/2 + (f*x)/2)^2 - 887040*cos(e/2 + (f*x)/2)^5
*sin(e/2 + (f*x)/2)^11*(e + f*x))/(887040*a^3*c^6*f*cos(e/2 + (f*x)/2)^5*s
in(e/2 + (f*x)/2)^11)
```

3.42 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx$

3.42.1 Optimal result

Integrand size = 28, antiderivative size = 175

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = \frac{2\sqrt{a}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^3c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} + \frac{2a^4c^4\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}$$

output
```
2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f-2*a*c^4*
tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^2*c^4*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)-2/5*a^3*c^4*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)+2/7*a^4*c^4*
tan(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)
```

3.42.2 Mathematica [A] (verified)

Time = 5.73 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.70

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx$$
$$= \frac{2ac^4\left(105\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+\sqrt{c-c\sec(e+fx)}(-176+122\sec(e+fx)-66\sec^2(e+fx)+1\right.}{105f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^4,x]
```

output
```
(2*a*c^4*(105*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c -
 c*Sec[e + f*x]]*(-176 + 122*Sec[e + f*x] - 66*Sec[e + f*x]^2 + 15*Sec[e +
 f*x]^3))*Tan[e + f*x])/(105*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e
 + f*x]])
```

3.42.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4392, 3042, 4375, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$
$$\Big\downarrow \text{4392}$$
$$a^4c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^{7/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$a^4c^4\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^8}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2a^5c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{254}$$

$$-\frac{2a^5c^4\int \left(\frac{\tan^6(e+fx)}{a(\sec(e+fx)a+a)^3}-\frac{\tan^4(e+fx)}{a^2(\sec(e+fx)a+a)^2}+\frac{\tan^2(e+fx)}{a^3(\sec(e+fx)a+a)}+\frac{1}{a^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}-\frac{1}{a^4}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^5c^4\left(-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{9/2}}+\frac{\tan(e+fx)}{a^4\sqrt{a\sec(e+fx)+a}}-\frac{\tan^3(e+fx)}{3a^3(a\sec(e+fx)+a)^{3/2}}+\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)^{5/2}}-\frac{\tan^7(e+fx)}{7a(a\sec(e+fx)+a)^{7}}\right)}{f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^4,x]
```

output
```
(-2*a^5*c^4*(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(
9/2)) + Tan[e + f*x]/(a^4*Sqrt[a + a*Sec[e + f*x]]) - Tan[e + f*x]^3/(3*a^
3*(a + a*Sec[e + f*x])^(3/2)) + Tan[e + f*x]^5/(5*a^2*(a + a*Sec[e + f*x])
^(5/2)) - Tan[e + f*x]^7/(7*a*(a + a*Sec[e + f*x])^(7/2))))/f
```

3.42.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.42.4 Maple [A] (warning: unable to verify)

Time = 7.13 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.33

method	result
default	$c^4\left(105\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{7}{2}}-758(1-\cos(fx+e))^7\csc(fx+e)^7+1078(1\ldots$ / $105f(-\cot(fx+e)+\csc(fx+e)-1)^3\ldots$
parts	$\frac{2c^4\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}+$ $2c^4\left(16\cos(fx+e)^3+8\cos(fx+e)^2+6\cos(fx\ldots\right.$ / $35f(\cos\ldots$

input
```
int((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*c^4/f*(105*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(7/2)-7
58*(1-cos(f*x+e))^7*csc(f*x+e)^7+1078*(1-cos(f*x+e))^5*csc(f*x+e)^5-770*(1
-cos(f*x+e))^3*csc(f*x+e)^3+210*csc(f*x+e)-210*cot(f*x+e))*(-2*a/((1-cos(f
*x+e))^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+csc(f*x+e)-1)^3/(-cot(f*x+e)+
csc(f*x+e)+1)^3
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.13

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx$$
$$= \left[\frac{105\left(c^4\cos(fx+e)^4+c^4\cos(fx+e)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx\cdots}{\cos(fx+e)+1}\right.}{105\left(f\cos(fx+\cdots\right.}\right.$$
$$-\frac{2\left(105\left(c^4\cos(fx+e)^4+c^4\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(176\,c^4\cos(fx+e)^3\cdots\right.}{105\left(f\cos(fx+e)^4+f\cos(fx\cdots\right.}$$

input
```
integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(-a)*log((2*a*co
s(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(176*c^4*c
os(f*x + e)^3 - 122*c^4*cos(f*x + e)^2 + 66*c^4*cos(f*x + e) - 15*c^4)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*c
os(f*x + e)^3), -2/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt
(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*s
in(f*x + e))) + (176*c^4*cos(f*x + e)^3 - 122*c^4*cos(f*x + e)^2 + 66*c^4*
cos(f*x + e) - 15*c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e
))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]
```

3.42.6 Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx$$
$$= c^4\left(\int\left(-4\sqrt{a\sec(e+fx)+a}\sec(e+fx)\right)dx + \int 6\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\,dx + \int\left(-4\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\right)dx + \int\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)\,dx + \int\sqrt{a\sec(e+fx)+a}\,dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**4*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
c**4*(Integral(-4*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(6*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(-4*sqrt(a*sec(e + f
*x) + a)*sec(e + f*x)**3, x) + Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f
*x)**4, x) + Integral(sqrt(a*sec(e + f*x) + a), x))
```

3.42.7 Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = \int \sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^4\,dx$$

input
```
integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/210*(105*((c^4*cos(2*f*x + 2*e)^2 + c^4*sin(2*f*x + 2*e)^2 + 2*c^4*cos(
2*f*x + 2*e) + c^4)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) -
(c^4*cos(2*f*x + 2*e)^2 + c^4*sin(2*f*x + 2*e)^2 + 2*c^4*cos(2*f*x + 2*e)
+ c^4)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^4*f*cos(
2*f*x + 2*e)^2 + c^4*f*sin(2*f*x + 2*e)^2 + 2*c^4*f*cos(2*f*x + 2*e) + c^4
*f)*integrate((((cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*
cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e
)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2
*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x +
 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(9/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(10*f*
x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6
*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin
(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e...
```

3.42.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = \int \sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^4\,dx$$

input
```
integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^4\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^4,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^4, x)
```

3.43 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx$

3.43.1 Optimal result

Integrand size = 28, antiderivative size = 140

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx = \frac{2\sqrt{a}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^3c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

output
```
2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f-2*a*c^3*
tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^2*c^3*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)-2/5*a^3*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

3.43.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx$$
$$= \frac{2ac^3\left(15\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+\sqrt{c-c\sec(e+fx)}(-23+11\sec(e+fx)-3\sec^2(e+fx))\right)\tan(e}{15f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a*c^3*(15*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c -
c*Sec[e + f*x]]*(-23 + 11*Sec[e + f*x] - 3*Sec[e + f*x]^2))*Tan[e + f*x])/
(15*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.43.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4392, 3042, 4375, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4392}$$

$$-a^3c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^6}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^4c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{254}$$

$$\frac{2a^4c^3\int \left(\frac{\tan^4(e+fx)}{a(\sec(e+fx)a+a)^2}-\frac{\tan^2(e+fx)}{a^2(\sec(e+fx)a+a)}-\frac{1}{a^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}+\frac{1}{a^3}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4c^3\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{7/2}}-\frac{\tan(e+fx)}{a^3\sqrt{a\sec(e+fx)+a}}+\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}-\frac{\tan^5(e+fx)}{5a(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a^4*c^3*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(7/2
) - Tan[e + f*x]/(a^3*Sqrt[a + a*Sec[e + f*x]]) + Tan[e + f*x]^3/(3*a^2*(a
 + a*Sec[e + f*x])^(3/2)) - Tan[e + f*x]^5/(5*a*(a + a*Sec[e + f*x])^(5/2)
)))/f
```

3.43.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.43.4 Maple [A] (warning: unable to verify)

Time = 6.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.51

method	result
default	$\frac{c^3\left(15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{5}{2}}-74(1-\cos(fx+e))^5\csc(fx+e)^5+80(1-\cos\right.}{15f(-\cot(fx+e)+\csc(fx+e)-1)^2(-\cot(fx+e)+\csc(f}$
parts	$\frac{2c^3\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}+\frac{6c^3\sqrt{a(\sec(fx+e)+1)}\,(\cot(fx+e)-\csc(fx+e}{f}$

input
```
int((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*c^3/f*(15*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(
1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(5/2)-74*
(1-cos(f*x+e))^5*csc(f*x+e)^5+80*(1-cos(f*x+e))^3*csc(f*x+e)^3-30*csc(f*x+
e)+30*cot(f*x+e))*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f*x
+e)+csc(f*x+e)-1)^2/(-cot(f*x+e)+csc(f*x+e)+1)^2
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.48

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx$$

$$=\left[\frac{15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+}{\cos(fx+e)+1}\right.}{15\left(f\cos(fx+e)^3+f\cos(\right.}\right.$$

$$-\frac{2\left(15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(23\,c^3\cos(fx+e)^2-\right.\right.}{15\left(f\cos(fx+e)^3+f\cos(fx+e)^2\right)}$$

input
```
integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(
f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(23*c^3*cos(
f*x + e)^2 - 11*c^3*cos(f*x + e) + 3*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*(15*(c^
3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (23*c^3*cos(f*x
 + e)^2 - 11*c^3*cos(f*x + e) + 3*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

3.43.6 Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx = -c^3\left(\int 3\sqrt{a\sec(e+fx)+a}\sec(e+fx)\,dx + \int\left(-3\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\right)dx + \int\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\,dx + \int\left(-\sqrt{a\sec(e+fx)+a}\right)dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**3*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
-c**3*(Integral(3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-3*
sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(sqrt(a*sec(e + f*x
) + a)*sec(e + f*x)**3, x) + Integral(-sqrt(a*sec(e + f*x) + a), x))
```

3.43.7 Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx = \int -\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^3\,dx$$

input
```
integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/30*(15*((c^3*cos(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*
f*x + 2*e) + c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c
^3*cos(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*f*x + 2*e) +
c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^3*f*cos(2*
f*x + 2*e)^2 + c^3*f*sin(2*f*x + 2*e)^2 + 2*c^3*f*cos(2*f*x + 2*e) + c^3*f
)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(
2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 +
sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) +
3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e)
+ 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e
) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e
) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)...
```

3.43.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx = \int -\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^3\,dx$$

input
```
integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx=\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^3\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3, x)
```

3.44 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx$

3.44.1 Optimal result

Integrand size = 28, antiderivative size = 105

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \frac{2\sqrt{a}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f-2*a*c^2*
tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^2*c^2*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \frac{2ac^2\left(3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+(-4+\sec(e+fx))\sqrt{c-c\sec(e+fx)}\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2,x]
```

output
```
(2*a*c^2*(3*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + (-4 + Sec[
e + f*x])*Sqrt[c - c*Sec[e + f*x]])*Tan[e + f*x])/(3*f*Sqrt[a*(1 + Sec[e +
 f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.44.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4392, 3042, 4375, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^4}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2a^3c^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{254}$$

$$-\frac{2a^3c^2\int \left(\frac{\tan^2(e+fx)}{a(\sec(e+fx)a+a)}+\frac{1}{a^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}-\frac{1}{a^2}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3c^2\left(-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}}-\frac{\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a^3*c^2*(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(
5/2)) + Tan[e + f*x]/(a^2*Sqrt[a + a*Sec[e + f*x]]) - Tan[e + f*x]^3/(3*a*
(a + a*Sec[e + f*x])^(3/2))))/f
```

3.44.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.44.4 Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.52

method	result
parts	$\frac{2c^2\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}+\frac{2c^2\sqrt{a(\sec(fx+e)+1)}\,(2\sin(fx+e)+\tan(fx-}{3f(\cos(fx+e)+1)}$
default	$-\frac{2c^2\sqrt{a(\sec(fx+e)+1)}\left(-3\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)-3\operatorname{arctanh}\left(\frac{\sin(fx+}{(\cos(fx+e)+1)\sqrt{-}}\right.\right.}{3f(\cos(fx+e)+1)}$

input
```
int((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*c^2/f*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctan
h(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+2/3*c^2/f*
(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(2*sin(f*x+e)+tan(f*x+e))+4*c^2/f*
(a*(sec(f*x+e)+1))^(1/2)*(cot(f*x+e)-csc(f*x+e))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.98

$$\int\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx$$

$$=\left[\frac{3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-}{\cos(fx+e)+1}\right.}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}\right.$$

$$-\frac{2\left(3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(4\,c^2\cos\left(fx+e\right)-c^2\right)\sqrt{\right.}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}$$

input
```
integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x
+ e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(4*c^2*cos(f*x +
 e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*
x + e)^2 + f*cos(f*x + e)), -2/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e)
)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqr
t(a)*sin(f*x + e))) + (4*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]
```

3.44.6 Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = c^2\left(\int \left(-2\sqrt{a\sec(e+fx)+a}\sec(e+fx)\right)dx + \int \sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\,dx + \int \sqrt{a\sec(e+fx)+a}\,dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**2*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
c**2*(Integral(-2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(sqr
t(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(sqrt(a*sec(e + f*x) +
 a), x))
```

3.44.7 Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \int \sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^2\,dx$$

input
```
integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/6*(3*(2*c^2*f*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f
*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f
*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(
5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6
*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2
*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*s
in(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*s
in(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (c
os(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + c
os(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)
*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)))/(((2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*
f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e)
+ cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x
 + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*s
in(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1))^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos...
```

3.44.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \int \sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^2\,dx$$

input
```
integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2, x)
```

3.45 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$

3.45.1 Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx=\frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f-2*a*c*tan(
f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$$
$$=\frac{2c\sqrt{a(1+\sec(e+fx))}\left(\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)-\sqrt{c-c\sec(e+fx)}\right)\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]),x]
```

output
```
(2*c*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/
Sqrt[c]] - Sqrt[c - c*Sec[e + f*x]])*Tan[(e + f*x)/2])/(f*Sqrt[c - c*Sec[e
 + f*x]])
```

3.45.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4392, 3042, 4375, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4392}$$

$$-ac\int \frac{\tan^2(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^2c\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{262}$$

$$\frac{2a^2c\left(-\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{2a^2c\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}}-\frac{\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]),x]
```

output
```
(2*a^2*c*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(3/2)
- Tan[e + f*x]/(a*Sqrt[a + a*Sec[e + f*x]])))/f
```

3.45.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.45.4 Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

method	result
default	$\frac{2c\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}+\cot(fx+e)-\csc(fx+e)\right)}{f}$
parts	$\frac{2c\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}+\frac{2c\sqrt{a(\sec(fx+e)+1)}(\cot(fx+e)-\csc(fx+e))}{f}$

input
```
int((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*c/f*(a*(sec(f*x+e)+1))^(1/2)*(arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*
x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cot(f*x+e)-
csc(f*x+e))
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.55

$$\int\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$$
$$=\left[\frac{(c\cos(fx+e)+c)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-2\,c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cdots}{f\cos(fx+e)+f}\right.$$
$$\left.-\frac{2\left((c\cos(fx+e)+c)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)\right)}{f\cos(fx+e)+f}\right]$$

input
```
integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[((c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x +
 e) - a)/(cos(f*x + e) + 1)) - 2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e))/(f*cos(f*x + e) + f), -2*((c*cos(f*x + e) + c)*sqrt(a)*arct
an(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
 e))) + c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
 + e) + f)]
```

3.45.6 Sympy [F]

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))\,dx = -c\left(\int \sqrt{a\sec(e + fx) + a}\sec(e + fx)\,dx + \int \left(-\sqrt{a\sec(e + fx) + a}\right)dx\right)$$

input
```
integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
-c*(Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-sqrt(a*
sec(e + f*x) + a), x))
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(58) = 116$.

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.23

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))\,dx$$
$$= \frac{\sqrt{a}c\arctan\left(\left(\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(\sin(2fx + 2e\right.\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*c*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
+ sin(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
cos(f*x + e))/f
```

3.45.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx = \int -\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)\,dx$$

input
```
integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)), x)
```

3.46 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$

3.46.1 Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c/f+2*cot(f*x+
e)*(a+a*sec(f*x+e))^(1/2)/c/f
```

3.46.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx =\frac{2\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\sec(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{cf}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x]),x]
```

output
```
(2*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, 1 - Sec[e + f*x]]*Sqrt[a*(
1 + Sec[e + f*x])])/(c*f)
```

3.46.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4392, 3042, 4375, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^2(e+fx)(\sec(e+fx)a+a)^{3/2}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx}{ac}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}$$

$$\Big\downarrow \text{264}$$

$$\frac{2\left(\cot(e+fx)\sqrt{a\sec(e+fx)+a}-a\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{cf}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)}{cf}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x]),x]
```

output
```
(2*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] + Cot[
e + f*x]*Sqrt[a + a*Sec[e + f*x]]))/(c*f)
```

3.46.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.46. $\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$

3.46.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}+\cot(fx+e)\right)}{cf}$	87

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/c/f*(a*(sec(f*x+e)+1))^(1/2)*(arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*
x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cot(f*x+e))
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.86

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$$
$$=\left[\frac{\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\,\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7\,a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin\left(fx+e\right)+4\ldots}{2\,cf\sin\left(fx+e\right)}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x +
e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*co
s(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), (sqrt(a)*arctan(2*sqr
t(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*
a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e))]
```

3.46.6 Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c - c\sec(e + fx)}\, dx = -\frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)-1}\, dx}{c}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x)/c
```

3.46.7 Maxima [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c - c\sec(e + fx)}\, dx = \int -\frac{\sqrt{a\sec(fx + e) + a}}{c\sec(fx + e) - c}\, dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c), x)
```

3.46.8 Giac [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c - c\sec(e + fx)}\, dx = \int -\frac{\sqrt{a\sec(fx + e) + a}}{c\sec(fx + e) - c}\, dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
sage0*x
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c - c\sec(e + fx)}\, dx = \int \frac{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}{c - \frac{c}{\cos(e+f\,x)}}\, dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x)), x)
```

3.47 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx$

3.47.1 Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^2f}$$

output
```
-2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a/c^2/f+2*arctan(a^(1/2)*tan(f*x+
e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c^2/f+2*cot(f*x+e)*(a+a*sec(f*x+e))^(1/
2)/c^2/f
```

3.47.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx = -\frac{2a\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\sec(e+fx)\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}(c-c\sec(e+fx))^2}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a*Hypergeometric2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(3*
f*Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x])^2)
```

3.47.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4392, 3042, 4375, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 4392}$$

$$\frac{\int \cot^4(e+fx)(\sec(e+fx)a+a)^{5/2}dx}{a^2c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{ 4375}$$

$$-\frac{2\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{ac^2f}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{2\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{ac^2f}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{2\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\cot(e+fx)\sqrt{a\sec(e+fx)+a}-a\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx}{\sqrt{\sec(e+fx)}}\right.\right.\right.}{ac^2f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)\right)}{ac^2f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*((Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/3 - a*(Sqrt[a]*ArcTan[(Sq
rt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] + Cot[e + f*x]*Sqrt[a + a*Se
c[e + f*x]])))/(a*c^2*f)
```

3.47.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.47.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.73

method	result
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(3\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos(fx+e)-3\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(f}{\cos(fx}}}\right.\right.}{3c^2f(\cos(fx+e)-1)}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3/c^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)*(3*arctanh(sin(f*x+e)/(c
os(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*cos(f*x+e)-3*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos
(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+4*cos(f*x+e)*cot(f*x
+e)-3*cot(f*x+e))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.26

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx$$
$$= \left[\frac{3\sqrt{-a}(\cos(fx+e)-1)\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\,\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\sin(fx+e)-7\,a\cos(fx+e)+a}{\cos(fx+e)+1}\right.}{6\left(c^2f\cos(fx+e)-c^2f\right)\sin(fx+e}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(-a)*(cos(f*x + e) - 1)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*
x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*
(4*cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(3*sqrt(a)*(cos(f*x +
e) - 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
 e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) +
 2*(4*cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

3.47.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{\sqrt{a\sec{(e+fx)}+a}}{\sec^2{(e+fx)}-2\sec{(e+fx)}+1}\,dx}{c^2}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),
x)/c**2
```

3.47.7 Maxima [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx = \int \frac{\sqrt{a\sec{(fx+e)}+a}}{(c\sec{(fx+e)}-c)^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c)^2, x)
```

3.47.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(c\sec\left(fx+e\right)-c\right)^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^2, x)
```

3.48 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx$

3.48.1 Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}+\frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f}-\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^3f}+\frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^3f}$$

output
```
-2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a/c^3/f+2/5*cot(f*x+e)^5*(a+a*sec
(f*x+e))^(5/2)/a^2/c^3/f+2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2
))*a^(1/2)/c^3/f+2*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^3/f
```

3.48.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx=\frac{2a\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{5c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a*Hypergeometric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(5*c
^3*f*(-1 + Sec[e + f*x])^3*Sqrt[a*(1 + Sec[e + f*x])])
```

3.48.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4392, 3042, 4375, 264, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(\sec(e+fx)a+a)^{7/2}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2c^3f}$$

$$\Big\downarrow \text{264}$$

$$\frac{2\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^2c^3f}$$

$$\Big\downarrow \text{264}$$

$$\frac{2\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\int\frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(\right.\right.\right.}{a^2c^3f}$$

$\Big\downarrow$ 264

$$\frac{2\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\cot(e+fx)\sqrt{a\sec(e+f\right.\right.\right.}{a^2c^3f}$$

$\Big\downarrow$ 216

$$\frac{2\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right.\right.\right.\right.}{a^2c^3f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*((Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/5 - a*((Cot[e + f*x]^3*(a
+ a*Sec[e + f*x])^(3/2))/3 - a*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt
[a + a*Sec[e + f*x]]] + Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]]))))/(a^2*c^3
*f)
```

3.48.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.48. $\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx$

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.48.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(123) = 246.

Time = 2.65 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.90

method	result
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(15\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)^2-30\ \operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos}{\cos}}}\right.\right.}{}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
2/15/c^3/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^2*(15*(-cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+
e)+1))^(1/2))*cos(f*x+e)^2-30*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+
e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+15
*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)+23*cos(f*x+e)^2*cot(f*x+e)-35*cos(f*x+e)*c
ot(f*x+e)+15*cot(f*x+e))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.91

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx$$
$$= \left[\frac{15\left(\cos\left(fx+e\right)^2-2\cos\left(fx+e\right)+1\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\mathrm{si}}{\cos(fx+e)+1}\right.}{30\left(c^3f\cos\left(fx+e\right)^2-2\,c^3\right.}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/30*(15*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(8*a*cos(f*x
 + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) +
1))*sin(f*x + e) + 4*(23*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 15*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c
^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/15*(15*(cos(f*x + e)^2 - 2*cos
(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))
*sin(f*x + e) + 2*(23*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 15*cos(f*x + e)
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*
f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

3.48.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx}{c^3}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**3,x)
```

output
```
-Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 +
3*sec(e + f*x) - 1), x)/c**3
```

3.48.7 Maxima [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = \int -\frac{\sqrt{a\sec(fx+e)+a}}{(c\sec(fx+e)-c)^3}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c)^3, x)
```

3.48.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = \int -\frac{\sqrt{a\sec(fx+e)+a}}{(c\sec(fx+e)-c)^3}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^3, x)
```

3.49 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx$

3.49.1 Optimal result

Integrand size = 28, antiderivative size = 174

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^4f} - \frac{2\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^3c^4f}$$

output
```
-2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a/c^4/f+2/5*cot(f*x+e)^5*(a+a*sec
(f*x+e))^(5/2)/a^2/c^4/f-2/7*cot(f*x+e)^7*(a+a*sec(f*x+e))^(7/2)/a^3/c^4/f
+2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c^4/f+2*cot(f
*x+e)*(a+a*sec(f*x+e))^(1/2)/c^4/f
```

3.49.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx = -\frac{2a\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},1-\sec(e+fx)\right)\tan(e+fx)}{7f\sqrt{a(1+\sec(e+fx))}(c-c\sec(e+fx))^4}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^4,x]
```

output
```
(-2*a*Hypergeometric2F1[-7/2, 1, -5/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(7*
f*Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x])^4)
```

3.49.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3042, 4392, 3042, 4375, 264, 264, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^8(e+fx)(\sec(e+fx)a+a)^{9/2}dx}{a^4c^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{9/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^8}dx}{a^4c^4}$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2\int \frac{\cot^8(e+fx)(\sec(e+fx)a+a)^4}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^3c^4f}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2\left(\frac{1}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^3c^4f}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2\left(\frac{1}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}\right.\right.}{a^3c^4f}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2\left(\frac{1}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e-\right.\right.\right.}{a^3c^4f}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2\left(\frac{1}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e-\right.\right.\right.}{a^3c^4f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\left(\frac{1}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+\right.\right.\right.}{a^3c^4f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^4,x]
```

output
```
(-2*((Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/7 - a*((Cot[e + f*x]^5*(a
 + a*Sec[e + f*x])^(5/2))/5 - a*((Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2
))/3 - a*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]
+ Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])))))/(a^3*c^4*f)
```

3.49. $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx$

3.49.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.49.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(154) = 308$.

Time = 2.80 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.00

method	result
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(105\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)^3-315\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\right.\right.}{}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
2/105/c^4/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^3*(105*(-cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*
x+e)+1))^(1/2))*cos(f*x+e)^3-315*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctan
h(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+e)
^2+315*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2
))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+176*cos(f*x+e)^3*cot(f*x+
e)-105*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2
))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-406*cos(f*x+e)^2*cot(f*x+e)+350*cos(
f*x+e)*cot(f*x+e)-105*cot(f*x+e))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx$$
$$= \left[\frac{105\left(\cos(fx+e)^3-3\cos(fx+e)^2+3\cos(fx+e)-1\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+\cdots\right.}{\mathrm{co}\cdots}\right.}{210\left(c^4f\cos(fx+\cdots\right.}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
[1/210*(105*(cos(f*x + e)^3 - 3*cos(f*x + e)^2 + 3*cos(f*x + e) - 1)*sqrt(
-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e)
+ a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(176*cos(f*x + e)^4 - 406*cos(f*
x + e)^3 + 350*cos(f*x + e)^2 - 105*cos(f*x + e))*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f
*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/105*(105*(cos(f*x + e)^3 - 3*cos(f
*x + e)^2 + 3*cos(f*x + e) - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*c
os(f*x + e) - a))*sin(f*x + e) + 2*(176*cos(f*x + e)^4 - 406*cos(f*x + e)^
3 + 350*cos(f*x + e)^2 - 105*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*
x + e) - c^4*f)*sin(f*x + e))]
```

3.49.6 Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^4} \, dx = \frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1} \, dx}{c^4}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**4,x)
```

output
```
Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6
*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)/c**4
```

3.49.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^4} \, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.49.8 Giac [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^4} \, dx = \int \frac{\sqrt{a\sec(fx + e) + a}}{(c\sec(fx + e) - c)^4} \, dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^4,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^4, x)
```

3.50 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx$

3.50.1 Optimal result

Integrand size = 28, antiderivative size = 177

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = \frac{2a^{3/2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^3c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^4c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{2a^5c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}$$

output
```
2*a^(3/2)*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^2*c^
3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^3*c^3*tan(f*x+e)^3/f/(a+a*sec(
f*x+e))^(3/2)-2/5*a^4*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)-2/7*a^5*c^
3*tan(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)
```

3.50.2 Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = -\frac{2a^2\left(-105c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+c^3\sqrt{c-c\sec(e+fx)}(146-32\sec(e+fx)-24\sec^2(e+fx)+1}{105f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3,x]
```

output
```
(-2*a^2*(-105*c^(7/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + c^3*Sqrt
[c - c*Sec[e + f*x]]*(146 - 32*Sec[e + f*x] - 24*Sec[e + f*x]^2 + 15*Sec[e
 + f*x]^3))*Tan[e + f*x])/(105*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec
[e + f*x]])
```

3.50.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4392, 3042, 4375, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4392}$$

$$-a^3c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^6}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^5c^3\int \frac{\tan^6(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{363}$$

$$\frac{2a^5c^3\left(\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{\tan^7(e+fx)}{7(a\sec(e+fx)+a)^{7/2}}\right)}{f}$$

$$\downarrow \text{254}$$

$$\frac{2a^5c^3\left(\int \left(\frac{\tan^4(e+fx)}{a(\sec(e+fx)a+a)^2}-\frac{\tan^2(e+fx)}{a^2(\sec(e+fx)a+a)}-\frac{1}{a^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}+\frac{1}{a^3}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{\tan^7(e+fx)}{7(a\sec(e+fx)+a)^{7/2}}\right)}{f}$$

$$\downarrow \text{2009}$$

$$\frac{2a^5c^3\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{7/2}}-\frac{\tan(e+fx)}{a^3\sqrt{a\sec(e+fx)+a}}+\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}-\frac{\tan^7(e+fx)}{7(a\sec(e+fx)+a)^{7/2}}-\frac{\tan^5(e+fx)}{5a(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a^5*c^3*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(7/2
) - Tan[e + f*x]/(a^3*Sqrt[a + a*Sec[e + f*x]]) + Tan[e + f*x]^3/(3*a^2*(a
 + a*Sec[e + f*x])^(3/2)) - Tan[e + f*x]^5/(5*a*(a + a*Sec[e + f*x])^(5/2)
) - Tan[e + f*x]^7/(7*(a + a*Sec[e + f*x])^(7/2))))/f
```

3.50.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.50.4 Maple [A] (warning: unable to verify)

Time = 8.28 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.32

method	result
default	$\frac{a\,c^3\left(105\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{7}{2}}-278(1-\cos(fx+e))^7\csc(fx+e)^7+1078}{105f(-\cot(fx+e)+\csc(fx+e)+1)}$
parts	$\frac{2c^3a\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos(fx+e)+\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(f}{\cos(fx+}}}\right.\right.}{f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/105*a*c^3/f*(105*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-
1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(7/2)
-278*(1-cos(f*x+e))^7*csc(f*x+e)^7+1078*(1-cos(f*x+e))^5*csc(f*x+e)^5-770*
(1-cos(f*x+e))^3*csc(f*x+e)^3+210*csc(f*x+e)-210*cot(f*x+e))*(-2*a/((1-cos
(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+csc(f*x+e)+1)^3/(-cot(f*x+e
)+csc(f*x+e)-1)^3
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.18

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = \left[\frac{105\left(ac^3\cos(fx+e)^4+ac^3\cos(fx+e)^3\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos\ldots}{\cos(fx+e\ldots}\right.}{}\right.$$

$$-\frac{2\left(105\left(ac^3\cos(fx+e)^4+ac^3\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(146\,ac^3\cos(fx+e)^3\ldots\right.}{105\left(f\cos(fx+e)^4+f\cos(f\ldots\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f*x + e)^3)*sqrt(-a)*log((2*
a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(146*a
*c^3*cos(f*x + e)^3 - 32*a*c^3*cos(f*x + e)^2 - 24*a*c^3*cos(f*x + e) + 15
*a*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x +
 e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f
*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)/(sqrt(a)*sin(f*x + e))) + (146*a*c^3*cos(f*x + e)^3 - 32*a*c^3*cos(f*
x + e)^2 - 24*a*c^3*cos(f*x + e) + 15*a*c^3)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]
```

3.50.6 Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = -c^3\left(\int\left(-a\sqrt{a\sec(e+fx)+a}\right)dx+\int 2a\sqrt{a\sec(e+fx)+a}\sec(e+fx)\,dx+\int\left(-2a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\right)dx+\int a\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**3,x)
```

output
```
-c**3*(Integral(-a*sqrt(a*sec(e + f*x) + a), x) + Integral(2*a*sqrt(a*sec(
e + f*x) + a)*sec(e + f*x), x) + Integral(-2*a*sqrt(a*sec(e + f*x) + a)*se
c(e + f*x)**3, x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4, x
))
```

3.50.7 Maxima [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx=\int -(a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/210*(105*((a*c^3*cos(2*f*x + 2*e)^2 + a*c^3*sin(2*f*x + 2*e)^2 + 2*a*c^
3*cos(2*f*x + 2*e) + a*c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
 + 1) - (a*c^3*cos(2*f*x + 2*e)^2 + a*c^3*sin(2*f*x + 2*e)^2 + 2*a*c^3*cos
(2*f*x + 2*e) + a*c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1)
 - 2*(a*c^3*f*cos(2*f*x + 2*e)^2 + a*c^3*f*sin(2*f*x + 2*e)^2 + 2*a*c^3*f*
cos(2*f*x + 2*e) + a*c^3*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e)
+ 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e
) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x +
 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x +
 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x
 + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f
*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f
*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(9...
```

3.50.8 Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = \int -(a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^3\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3, x)
```

3.51 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx$

3.51.1 Optimal result

Integrand size = 28, antiderivative size = 142

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx = \frac{2a^{3/2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^3c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{2a^4c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

output
```
2*a^(3/2)*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^2*c^
2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^3*c^2*tan(f*x+e)^3/f/(a+a*sec(
f*x+e))^(3/2)+2/5*a^4*c^2*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx = \frac{2a^2\left(15c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+c^2\sqrt{c-c\sec(e+fx)}(-17-\sec(e+fx)+3\sec}{15f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2,x]
```

output
```
(2*a^2*(15*c^(5/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + c^2*Sqrt[c
- c*Sec[e + f*x]]*(-17 - Sec[e + f*x] + 3*Sec[e + f*x]^2))*Tan[e + f*x])/(
15*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.51.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4392, 3042, 4375, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int \frac{\tan^4(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^4}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2a^4c^2\int \frac{\tan^4(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{363}$$

$$-\frac{2a^4c^2\left(\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{\tan^5(e+fx)}{5(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{254}$$

$$-\frac{2a^4c^2\left(\int \left(\frac{\tan^2(e+fx)}{a(\sec(e+fx)a+a)}+\frac{1}{a^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}-\frac{1}{a^2}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{\tan^5(e+fx)}{5(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^4c^2\left(-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}}-\frac{\tan^5(e+fx)}{5(a\sec(e+fx)+a)^{5/2}}-\frac{\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a^4*c^2*(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(
5/2)) + Tan[e + f*x]/(a^2*Sqrt[a + a*Sec[e + f*x]]) - Tan[e + f*x]^3/(3*a*
(a + a*Sec[e + f*x])^(3/2)) - Tan[e + f*x]^5/(5*(a + a*Sec[e + f*x])^(5/2)
)))/f
```

3.51.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.51.4 Maple [A] (verified)

Time = 6.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.33

method	result
default	$\frac{2a\,c^2\sqrt{a(\sec(fx+e)+1)}\left(15\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos(fx+e)+15\,\text{arctanh}\left(\frac{\sin(fx+}{(\cos(fx+e)+1)\sqrt{-}}\right.\right.}{15f(\cos(fx+e)+1)}$
parts	$\frac{2c^2a\sqrt{a(\sec(fx+e)+1)}\left(\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos(fx+e)+\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx}{\cos(fx+}}}\right.\right.}{f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/15*a*c^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(15*arctanh(sin(f*x+e
)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*cos(f*x+e)+15*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)
/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-17*sin(f*x+e)-t
an(f*x+e)+3*sec(f*x+e)*tan(f*x+e))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.50

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e+fx))^2\, dx = \left[\frac{15\left(ac^2\cos(fx+e)^3 + ac^2\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos\cdots}{\cos(fx+e)\cdots}\right.\cdots}{15\left(f\cos\cdots\right.} \right.$$

$$-\frac{2\left(15\left(ac^2\cos(fx+e)^3 + ac^2\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right) + \left(17\,ac^2\cos(fx+e)^2 + \cdots\right.\right.}{15\left(f\cos(fx+e)^3 + f\cos(fx+e)^2\right)}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a)*log((2*a*
cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(17*a*c^
2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 3*a*c^2)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*
(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (17*
a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 3*a*c^2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

3.51.6 Sympy [F]

$$\int (a + a\sec(e+fx))^{3/2}(c - c\sec(e+fx))^2\, dx = c^2\left(\int a\sqrt{a\sec(e+fx)+a}\, dx + \int\left(-a\sqrt{a\sec(e+fx)+a}\sec(e+fx)\right)dx + \int\left(-a\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\right)dx + \int a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\, dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**2,x)
```

output
```
c**2*(Integral(a*sqrt(a*sec(e + f*x) + a), x) + Integral(-a*sqrt(a*sec(e +
 f*x) + a)*sec(e + f*x), x) + Integral(-a*sqrt(a*sec(e + f*x) + a)*sec(e +
 f*x)**2, x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3, x))
```

3.51.7 Maxima [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/30*(15*((a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*
cos(2*f*x + 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
 1) - (a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*cos(2
*f*x + 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) -
 2*(a*c^2*f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2*a*c^2*f*co
s(2*f*x + 2*e) + a*c^2*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) +
2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e
)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e
)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2
*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6
*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*...
```

3.51.8 Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c - c\sec(e+fx))^2\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2, x)
```

3.52 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx$

3.52.1 Optimal result

Integrand size = 26, antiderivative size = 101

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = \frac{2a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2a^3c\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

output
```
2*a^(3/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^2*c*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*a^3*c*tan(f*x+e)^3/f/(a+a*sec(f*x+e)
)^(3/2)
```

3.52.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = -\frac{2a^2c\left(-3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+(2+\sec(e+fx))\sqrt{c-c\sec(e+fx)}\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x]),x]
```

output
```
(-2*a^2*c*(-3*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + (2 + Sec
[e + f*x])*Sqrt[c - c*Sec[e + f*x]])*Tan[e + f*x])/(3*f*Sqrt[a*(1 + Sec[e
+ f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.52.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 4392, 3042, 4375, 363, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4392}$$

$$-ac\int \sqrt{\sec(e+fx)a+a}\tan^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int \cot\left(e+fx+\frac{\pi}{2}\right)^2\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^3c\int \frac{\tan^2(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{363}$$

$$\frac{2a^3c\left(\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{\tan^3(e+fx)}{3(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{262}$$

$$\frac{2a^3c\left(-\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{\tan^3(e+fx)}{3(a\sec(e+fx)+a)^{3/2}}-\frac{\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{2a^3c\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}}-\frac{\tan^3(e+fx)}{3(a\sec(e+fx)+a)^{3/2}}-\frac{\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x]),x]
```

output
```
(2*a^3*c*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(3/2)
- Tan[e + f*x]/(a*Sqrt[a + a*Sec[e + f*x]]) - Tan[e + f*x]^3/(3*(a + a*Sec
[e + f*x])^(3/2))))/f
```

3.52.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.52.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(89) = 178$.

Time = 1.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.04

method	result
default	$\frac{2ca\sqrt{a(\sec(fx+e)+1)}\,\Big(\operatorname{arctanh}\Big(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\Big)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+\operatorname{arctanh}\Big(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx-}{\cos(fx+e}}}}{f(\cos(fx+e)+1)}$
parts	$\frac{2ca\sqrt{a(\sec(fx+e)+1)}\,\Big(\operatorname{arctanh}\Big(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\Big)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+\operatorname{arctanh}\Big(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx-}{\cos(fx+e}}}}{f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2*c/f*a*(a*(sec(f*x+e)+1))^(1/2)*(arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(
f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e
)+arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+sin(f*x+e))/(cos(f*x+e)+1)-2/3*c/f*a*(a*(
sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(5*sin(f*x+e)+tan(f*x+e))
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.00

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = \left[\frac{3\left(ac\cos(fx+e)^2+ac\cos(fx+e)\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\cos(fx+e)+1}\right.}{3\left(f\cos(fx+e)^2+f\right.}\right.$$

$$-\frac{2\left(3\left(ac\cos(fx+e)^2+ac\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+(2ac\cos(fx+e)+ac)\sqrt{\frac{a}{}}\right.}{3\left(f\cos(fx+e)^2+f\cos(fx+e)\right)}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x
+ e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(2*a*c*cos(f*x +
 e) + a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*
x + e)^2 + f*cos(f*x + e)), -2/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e)
)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqr
t(a)*sin(f*x + e))) + (2*a*c*cos(f*x + e) + a*c)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]
```

3.52.6 Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = -c\left(\int\left(-a\sqrt{a\sec(e+fx)+a}\right)dx+\int a\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e)),x)
```

output
```
-c*(Integral(-a*sqrt(a*sec(e + f*x) + a), x) + Integral(a*sqrt(a*sec(e + f
*x) + a)*sec(e + f*x)**2, x))
```

3.52.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. 2(89) = 178.

Time = 0.44 (sec) , antiderivative size = 998, normalized size of antiderivative = 9.88

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*((a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))) + 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) - 1) - a*arctan2((cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e),...
```

3.52.8 Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = \int -(a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
sage0*x
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx=\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x)), x)
```

3.53 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx$

3.53.1 Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{4a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}$$

output
```
2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f+4*a*cot(f*
x+e)*(a+a*sec(f*x+e))^(1/2)/c/f
```

3.53.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = \frac{2a\cot(e+fx)\sqrt{a(1+\sec(e+fx))}\left(2\sqrt{c}-\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)\sqrt{c-c\cdots}\right)}{c^{3/2}f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x]),x]
```

output
```
(2*a*Cot[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*(2*Sqrt[c] - ArcTanh[Sqrt[c -
 c*Sec[e + f*x]]/Sqrt[c]]*Sqrt[c - c*Sec[e + f*x]]))/(c^(3/2)*f)
```

3.53.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4392, 3042, 4375, 359, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^2(e+fx)(\sec(e+fx)a+a)^{5/2}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx}{ac}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}$$

$$\Big\downarrow \text{359}$$

$$\frac{2a\left(2\cot(e+fx)\sqrt{a\sec(e+fx)+a}-a\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{cf}$$

$$\Big\downarrow \text{216}$$

$$\frac{2a\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+2\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)}{cf}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x]),x]
```

output
```
(2*a*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] + 2*
Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]]))/(c*f)
```

3.53.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.53.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{2a\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}+2\cot(fx+e)\right)}{cf}$	90

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/c/f*a*(a*(sec(f*x+e)+1))^(1/2)*(arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(
f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*cot(f*x
+e))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.84

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = \left[\frac{\sqrt{-a}a\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\,\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7\,a}{\cos(fx+e)+1}\right.}{2\,cf\sin(fx+e)}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a)*a*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x
+ e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*
cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 8*a*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), (a^(3/2)*arctan(2
*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 4*a*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e))]
```

3.53.6 Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{c - c\sec(e + fx)}\,dx = -\frac{\int \frac{a\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)-1}\,dx + \int \frac{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec(e+fx)-1}\,dx}{c}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e)),x)
```

output
```
-(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x) + Integral(a*
sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x) - 1), x))/c
```

3.53.7 Maxima [F]

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{c - c\sec(e + fx)}\,dx = \int -\frac{(a\sec(fx + e) + a)^{\frac{3}{2}}}{c\sec(fx + e) - c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-integrate((a*sec(f*x + e) + a)^(3/2)/(c*sec(f*x + e) - c), x)
```

3.53.8 Giac [F]

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{c - c\sec(e + fx)}\,dx = \int -\frac{(a\sec(fx + e) + a)^{\frac{3}{2}}}{c\sec(fx + e) - c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
sage0*x
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x)), x)
```

3.54 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx$

3.54.1 Optimal result

Integrand size = 28, antiderivative size = 102

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{4\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f}$$

output
```
2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^2/f-4/3*cot(
f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^2/f+2*a*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2
)/c^2/f
```

3.54.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{2a^2\left(-2+3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\sec(e+fx)\right)\left(-1+\sec(e+f\right.\right.}{3c^2f(-1+\sec(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^2,x]
```

output
```
(2*a^2*(-2 + 3*Hypergeometric2F1[-1/2, 1, 1/2, 1 - Sec[e + f*x]]*(-1 + Sec
[e + f*x]))*Tan[e + f*x])/(3*c^2*f*(-1 + Sec[e + f*x])^2*Sqrt[a*(1 + Sec[e
 + f*x])])
```

3.54.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4392, 3042, 4375, 359, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^4(e+fx)(\sec(e+fx)a+a)^{7/2}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{c^2f}$$

$$\Big\downarrow \text{359}$$

$$-\frac{2\left(\frac{2}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{c^2f}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2\left(\frac{2}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\cot(e+fx)\sqrt{a\sec(e+fx)+a}-a\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx}{\sqrt{\sec(e+fx)}}\right.\right.\right.}{c^2f}$$

$$\Big\downarrow 216$$

$$-\frac{2\left(\frac{2}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)\right)}{c^2f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*((2*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/3 - a*(Sqrt[a]*ArcTan[(
Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] + Cot[e + f*x]*Sqrt[a + a*
Sec[e + f*x]])))/(c^2*f)
```

3.54.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.54.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.77

method	result
default	$\frac{2a\sqrt{a(\sec(fx+e)+1)}\left(3\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)-3\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(}{\cos(f}}}\right.\right.}{3c^2f(\cos(fx+e)-1)}$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3/c^2/f*a*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)*(3*arctanh(sin(f*x+e)/
(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*cos(f*x+e)-3*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(c
os(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*cos(f*x+e)*cot(f
*x+e)-3*cot(f*x+e))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.44

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \left[\frac{3\left(a\cos\left(fx+e\right)-a\right)\sqrt{-a}\log\left(-\frac{8a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{a}}{\cos(fx+e)+1}\right.}{6\left(c^2f\right.}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/6*(3*(a*cos(f*x + e) - a)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(
f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) +
4*(5*a*cos(f*x + e)^2 - 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(3*(a*cos(f*x +
e) - a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*
x + e) + 2*(5*a*cos(f*x + e)^2 - 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

3.54.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{a\sqrt{a\sec{(e+fx)}+a}}{\sec^2{(e+fx)}-2\sec{(e+fx)}+1}\,dx + \int \frac{a\sqrt{a\sec{(e+fx)}+a}\sec{(e+fx)}}{\sec^2{(e+fx)}-2\sec{(e+fx)}+1}\,dx}{c^2}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**2,x)
```

output
```
(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1
), x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x)**2
- 2*sec(e + f*x) + 1), x))/c**2
```

3.54.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.54.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx=\int \frac{(a\sec\left(fx+e\right)+a)^{\frac{3}{2}}}{(c\sec\left(fx+e\right)-c)^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^2, x)
```

3.55 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx$

3.55.1 Optimal result

Integrand size = 28, antiderivative size = 137

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3 f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3 f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^3 f} + \frac{4\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^3 f}$$

output
```
2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^3/f-2/3*cot(
f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^3/f+4/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5
/2)/a/c^3/f+2*a*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^3/f
```

3.55.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.96 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = -\frac{2a^2\left(-6+5\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\sec(e+fx)\right)(-1+\sec(e+fx))\right)\tan(e+fx)}{15c^3 f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^3,x]
```

output
```
(-2*a^2*(-6 + 5*Hypergeometric2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]]*(-1 + S
ec[e + f*x]))*Tan[e + f*x])/(15*c^3*f*(-1 + Sec[e + f*x])^3*Sqrt[a*(1 + Se
c[e + f*x])])
```

3.55.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4392, 3042, 4375, 359, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(\sec(e+fx)a+a)^{9/2}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{9/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{ac^3f}$$

$$\Big\downarrow \text{359}$$

$$\frac{2\left(\frac{2}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{ac^3f}$$

$\downarrow$ 264

$$\frac{2\left(\frac{2}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\int\frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(\right.\right.\right.}{ac^3f}$$

$\downarrow$ 264

$$\frac{2\left(\frac{2}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\cot(e+fx)\sqrt{a\sec(e+f}\right.\right.\right.}{ac^3f}$$

$\downarrow$ 216

$$\frac{2\left(\frac{2}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right.\right.\right.\right.}{ac^3f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*((2*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/5 - a*((Cot[e + f*x]^3*(
a + a*Sec[e + f*x])^(3/2))/3 - a*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sq
rt[a + a*Sec[e + f*x]]] + Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]]))))/(a*c^3
*f)
```

3.55.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.55.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(121) = 242$.

Time = 2.74 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.93

method	result
default	$\frac{2a\sqrt{a(\sec(fx+e)+1)}\left(15\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)^2-30\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{c}{co}}}\right.\right.}{}$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
2/15/c^3/f*a*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^2*(15*(-cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*
x+e)+1))^(1/2))*cos(f*x+e)^2-30*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*
x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+
15*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(
-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+26*cos(f*x+e)^2*cot(f*x+e)-35*cos(f*x+e)
*cot(f*x+e)+15*cot(f*x+e))
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.04

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \left[\frac{15\left(a\cos\left(fx+e\right)^2-2\,a\cos\left(fx+e\right)+a\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\,\cos(f\right.}{}\right.}{}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/30*(15*(a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(-a)*log(-(8*a*cos
(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e
) + 1))*sin(f*x + e) + 4*(26*a*cos(f*x + e)^3 - 35*a*cos(f*x + e)^2 + 15*a
*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x +
e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/15*(15*(a*cos(f*x +
e)^2 - 2*a*cos(f*x + e) + a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(
f*x + e) - a))*sin(f*x + e) + 2*(26*a*cos(f*x + e)^3 - 35*a*cos(f*x + e)^2
 + 15*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos
(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

3.55.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{a\sqrt{a\sec(e+fx)+a}}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx}{c^3}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**3,x)
```

output
```
-(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2
 + 3*sec(e + f*x) - 1), x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

3.55.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.55.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \int -\frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(c\sec(fx+e)-c)^3}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^3, x)
```

3.56 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx$

3.56.1 Optimal result

Integrand size = 28, antiderivative size = 172

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^4f} - \frac{4\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^2c^4f}$$

output
```
2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^4/f-2/3*cot(
f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^4/f+2/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5
/2)/a/c^4/f-4/7*cot(f*x+e)^7*(a+a*sec(f*x+e))^(7/2)/a^2/c^4/f+2*a*cot(f*x+
e)*(a+a*sec(f*x+e))^(1/2)/c^4/f
```

3.56.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.60 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.42

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{2a^2\left(-10+7\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\sec(e+fx)\right)(-1+\sec(e+\right.}{35c^4f(-1+\sec(e+fx))^4\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^4,x]
```

output
```
(2*a^2*(-10 + 7*Hypergeometric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]]*(-1 + S
ec[e + f*x]))*Tan[e + f*x])/(35*c^4*f*(-1 + Sec[e + f*x])^4*Sqrt[a*(1 + Se
c[e + f*x])])
```

3.56.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {3042, 4392, 3042, 4375, 359, 264, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^8(e+fx)(\sec(e+fx)a+a)^{11/2}dx}{a^4c^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{11/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^8}dx}{a^4c^4}$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2\int \frac{\cot^8(e+fx)(\sec(e+fx)a+a)^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2c^4f}$$

$$\Big\downarrow \text{359}$$

$$-\frac{2\left(\frac{2}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^2c^4f}$$

$\Big\downarrow$ 264

$$-\frac{2\left(\frac{2}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\int\frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}\right.\right.}{a^2c^4f}$$

$\Big\downarrow$ 264

$$-\frac{2\left(\frac{2}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e-\right.\right.\right.}{a^2c^4f}$$

$\Big\downarrow$ 264

$$-\frac{2\left(\frac{2}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e-\right.\right.\right.}{a^2c^4f}$$

$\Big\downarrow$ 216

$$-\frac{2\left(\frac{2}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+\right.\right.\right.}{a^2c^4f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^4,x]
```

output
```
(-2*((2*Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/7 - a*((Cot[e + f*x]^5*
(a + a*Sec[e + f*x])^(5/2))/5 - a*((Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3
/2))/3 - a*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]
] + Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])))))/(a^2*c^4*f)
```

3.56.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.56.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(152) = 304$.

Time = 2.46 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.03

method	result
default	$\frac{2a\sqrt{a(\sec(fx+e)+1)}\left(105\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)^3-315\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\right.\right.}{}$

3.56. $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
2/105/c^4/f*a*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^3*(105*(-cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(
f*x+e)+1))^(1/2))*cos(f*x+e)^3-315*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arct
anh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+
e)^2+191*cos(f*x+e)^3*cot(f*x+e)+315*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-c
os(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*
x+e)-406*cos(f*x+e)^2*cot(f*x+e)-105*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-c
os(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+350*co
s(f*x+e)*cot(f*x+e)-105*cot(f*x+e))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.88

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^4}\,dx = \left[\frac{105\left(a\cos\left(fx+e\right)^3 - 3\,a\cos\left(fx+e\right)^2 + 3\,a\cos\left(fx+e\right) - a\right)\sqrt{-a}\log\left(-\right.}{}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
[1/210*(105*(a*cos(f*x + e)^3 - 3*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) - a)
*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*s
qrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x
 + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(191*a*cos(f*x + e)^4 - 40
6*a*cos(f*x + e)^3 + 350*a*cos(f*x + e)^2 - 105*a*cos(f*x + e))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x +
e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/105*(105*(a*cos(f*x
+ e)^3 - 3*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) - a)*sqrt(a)*arctan(2*sqrt(
a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*
cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(191*a*cos(f*x + e)
^4 - 406*a*cos(f*x + e)^3 + 350*a*cos(f*x + e)^2 - 105*a*cos(f*x + e))*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos
(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]
```

3.56.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{\int \frac{a\sqrt{a\sec(e+fx)+a}}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \int \frac{a\sqrt{a\sec(e+fx)}}{\sec^4(e+fx)-4\sec^3(e+fx)+6}}{c^4}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**4,x)
```

output
```
(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3
+ 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(a*sqrt(a*sec(e +
f*x) + a)*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*
x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

3.56.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.56.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(c\sec(fx+e)-c)^4}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^4}\, dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c - \frac{c}{\cos(e+f\,x)}\right)^4}\, dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^4,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^4, x)
```

3.57 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx$

3.57.1 Optimal result

Integrand size = 28, antiderivative size = 212

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx=\frac{2a^{5/2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^3c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^4c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{2a^5c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}-\frac{6a^6c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}-\frac{2a^7c^3\tan^9(e+fx)}{9f(a+a\sec(e+fx))^{9/2}}$$

output
```
2*a^(5/2)*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^3*c^
3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^4*c^3*tan(f*x+e)^3/f/(a+a*sec(
f*x+e))^(3/2)-2/5*a^5*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)-6/7*a^6*c^
3*tan(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)-2/9*a^7*c^3*tan(f*x+e)^9/f/(a+a*se
c(f*x+e))^(9/2)
```

3.57.2 Mathematica [A] (verified)

Time = 2.46 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.63

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx=\frac{2a^3c^3\left(315\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+\sqrt{c-c\sec(e+fx)}(-383-34\sec(e+fx)+\cdots}{315f\sqrt{a(1+\sec(e+fx))}\sqrt{c-\cdots}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a^3*c^3*(315*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c
 - c*Sec[e + f*x]]*(-383 - 34*Sec[e + f*x] + 132*Sec[e + f*x]^2 + 5*Sec[e
+ f*x]^3 - 35*Sec[e + f*x]^4))*Tan[e + f*x])/(315*f*Sqrt[a*(1 + Sec[e + f*
x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.57.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4392}$$

$$-a^3c^3\int \frac{\tan^6(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^6}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^6c^3\int \frac{\tan^6(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{2a^6c^3\int\left(\frac{a\tan^8(e+fx)}{(\sec(e+fx)a+a)^4}+\frac{3\tan^6(e+fx)}{(\sec(e+fx)a+a)^3}+\frac{\tan^4(e+fx)}{a(\sec(e+fx)a+a)^2}-\frac{\tan^2(e+fx)}{a^2(\sec(e+fx)a+a)}-\frac{1}{a^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}+\frac{1}{a^3}\right)d\left(-\frac{\mathrm{ta}}{\sqrt{\mathrm{se}}}\right.}{f}$$

$\downarrow$ 2009

$$\frac{2a^6c^3\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{7/2}}-\frac{\tan(e+fx)}{a^3\sqrt{a\sec(e+fx)+a}}+\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}-\frac{a\tan^9(e+fx)}{9(a\sec(e+fx)+a)^{9/2}}-\frac{3\tan^7(e+fx)}{7(a\sec(e+fx)+a)^{7/2}}-\frac{}{5a}\right.}{f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a^6*c^3*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(7/2
) - Tan[e + f*x]/(a^3*Sqrt[a + a*Sec[e + f*x]]) + Tan[e + f*x]^3/(3*a^2*(a
 + a*Sec[e + f*x])^(3/2)) - Tan[e + f*x]^5/(5*a*(a + a*Sec[e + f*x])^(5/2)
) - (3*Tan[e + f*x]^7)/(7*(a + a*Sec[e + f*x])^(7/2)) - (a*Tan[e + f*x]^9)
/(9*(a + a*Sec[e + f*x])^(9/2))))/f
```

3.57.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.57.4 Maple [A] (verified)

Time = 89.61 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.05

method	result
default	$-\frac{2a^2c^3\sqrt{a(\sec(fx+e)+1)}\left(-315\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos(fx+e)-315\,\text{arctanh}\left(\frac{}{(\cos(fx+e)}\right.\right.}{}$
parts	$\frac{2c^3a^2\sqrt{a(\sec(fx+e)+1)}\left(3\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos(fx+e)+3\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-}}\right.\right.}{3f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/315*a^2*c^3/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(-315*arctanh(sin
(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*cos(f*x+e)-315*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos
(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+383*sin(
f*x+e)+34*tan(f*x+e)-132*sec(f*x+e)*tan(f*x+e)-5*sec(f*x+e)^2*tan(f*x+e)+3
5*sec(f*x+e)^3*tan(f*x+e))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.08

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e+fx))^3\,dx = \left[\frac{315\left(a^2c^3\cos(fx+e)^5 + a^2c^3\cos(fx+e)^4\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx \right.}{} \right.$$

$$-\frac{2\left(315\left(a^2c^3\cos(fx+e)^5 + a^2c^3\cos(fx+e)^4\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right) + \left(383\,a^2c^3\cos(fx+\right.}{315\left(f\cos(fx+\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(-a)*log
((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(3
83*a^2*c^3*cos(f*x + e)^4 + 34*a^2*c^3*cos(f*x + e)^3 - 132*a^2*c^3*cos(f*
x + e)^2 - 5*a^2*c^3*cos(f*x + e) + 35*a^2*c^3)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4), -2/315*
(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(a)*arctan(sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) +
 (383*a^2*c^3*cos(f*x + e)^4 + 34*a^2*c^3*cos(f*x + e)^3 - 132*a^2*c^3*cos
(f*x + e)^2 - 5*a^2*c^3*cos(f*x + e) + 35*a^2*c^3)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4)]
```

3.57.6 Sympy [F]

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^3\,dx = -c^3\left(\int \left(-a^2\sqrt{a\sec(e+fx)+a}\right)dx + \int a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)\,dx + \int 2a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\,dx + \int \left(-2a^2\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\right)dx + \int \left(-a^2\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)\right)dx + \int a^2\sqrt{a\sec(e+fx)+a}\sec^5(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**3,x)
```

output
```
-c**3*(Integral(-a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(a**2*sqrt(a*
sec(e + f*x) + a)*sec(e + f*x), x) + Integral(2*a**2*sqrt(a*sec(e + f*x) +
 a)*sec(e + f*x)**2, x) + Integral(-2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e
+ f*x)**3, x) + Integral(-a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4, x
) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**5, x))
```

3.57.7 Maxima [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^3\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.57.8 Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx=\int -(a\sec(fx+e)+a)^{\frac{5}{2}}(c\sec(fx+e)-c)^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^3\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^3, x)
```

3.58 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx$

3.58.1 Optimal result

Integrand size = 28, antiderivative size = 177

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx = \frac{2a^{5/2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^3c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^4c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{6a^5c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} + \frac{2a^6c^2\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}$$

output
```
2*a^(5/2)*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^3*c^
2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^4*c^2*tan(f*x+e)^3/f/(a+a*sec(
f*x+e))^(3/2)+6/5*a^5*c^2*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)+2/7*a^6*c^
2*tan(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)
```

3.58.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx = \frac{2a^3c^2\left(105\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+\sqrt{c-c\sec(e+fx)}(-92-46\sec(e+fx)+1\right.}{105f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2,x]
```

output
```
(2*a^3*c^2*(105*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c
 - c*Sec[e + f*x]]*(-92 - 46*Sec[e + f*x] + 18*Sec[e + f*x]^2 + 15*Sec[e +
 f*x]^3))*Tan[e + f*x])/(105*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e
 + f*x]])
```

3.58.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int \sqrt{\sec(e+fx)a+a}\tan^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cot\left(e+fx+\frac{\pi}{2}\right)^4\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2a^5c^2\int \frac{\tan^4(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{364}$$

$$-\frac{2a^5c^2\int \left(\frac{a\tan^6(e+fx)}{(\sec(e+fx)a+a)^3}+\frac{3\tan^4(e+fx)}{(\sec(e+fx)a+a)^2}+\frac{\tan^2(e+fx)}{a(\sec(e+fx)a+a)}+\frac{1}{a^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}-\frac{1}{a^2}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^5c^2\left(-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}}-\frac{a\tan^7(e+fx)}{7(a\sec(e+fx)+a)^{7/2}}-\frac{3\tan^5(e+fx)}{5(a\sec(e+fx)+a)^{5/2}}-\frac{\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a^5*c^2*(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(
5/2)) + Tan[e + f*x]/(a^2*Sqrt[a + a*Sec[e + f*x]]) - Tan[e + f*x]^3/(3*a*
(a + a*Sec[e + f*x])^(3/2)) - (3*Tan[e + f*x]^5)/(5*(a + a*Sec[e + f*x])^(
5/2)) - (a*Tan[e + f*x]^7)/(7*(a + a*Sec[e + f*x])^(7/2))))/f
```

3.58.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.58.4 Maple [A] (verified)

Time = 22.84 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.17

method	result
default	$-\frac{2a^2c^2\sqrt{a(\sec(fx+e)+1)}\left(-105\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)-105\,\text{arctanh}\left(\frac{\cdots}{(\cos(fx+e)\cdots}\right.\cdots\right.}{105f(\cos(fx\cdots}$
parts	$\frac{2c^2a^2\sqrt{a(\sec(fx+e)+1)}\left(3\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+3\,\text{arctanh}\left(\frac{\sin(fx+e)\cdots}{(\cos(fx+e)+1)\sqrt{-\frac{\cdots}{\cdots}}}\right.\cdots\right.}{3f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/105*a^2*c^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(-105*arctanh(sin
(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*cos(f*x+e)-105*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos
(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+92*sin(f
*x+e)+46*tan(f*x+e)-18*sec(f*x+e)*tan(f*x+e)-15*sec(f*x+e)^2*tan(f*x+e))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.31

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e+fx))^2\, dx = \left[\frac{105\left(a^2c^2\cos(fx+e)^4 + a^2c^2\cos(fx+e)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{\cos(fx}\right.}{}\right.$$

$$-\frac{2\left(105\left(a^2c^2\cos(fx+e)^4 + a^2c^2\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right) + \left(92\,a^2c^2\cos(fx+e\right.}{105\left(f\cos(fx+e)^4 + f\cos(\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(-a)*log
((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(9
2*a^2*c^2*cos(f*x + e)^3 + 46*a^2*c^2*cos(f*x + e)^2 - 18*a^2*c^2*cos(f*x
+ e) - 15*a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(
f*cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a^2*c^2*cos(f*x + e)^4
+ a^2*c^2*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (92*a^2*c^2*cos(f*x + e)^3 +
 46*a^2*c^2*cos(f*x + e)^2 - 18*a^2*c^2*cos(f*x + e) - 15*a^2*c^2)*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f
*x + e)^3)]
```

3.58.6 Sympy [F]

$$\int (a + a\sec(e+fx))^{5/2}(c - c\sec(e+fx))^2\, dx = c^2\left(\int a^2\sqrt{a\sec(e+fx)+a}\, dx + \int\left(-2a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\right)dx + \int a^2\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)\, dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**2,x)
```

output
```
c**2*(Integral(a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(-2*a**2*sqrt(a
*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(a**2*sqrt(a*sec(e + f*x)
 + a)*sec(e + f*x)**4, x))
```

3.58.7 Maxima [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx = \int (a\sec(fx+e)+a)^{\frac{5}{2}}(c\sec(fx+e)-c)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/210*(105*((a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 + 2*
a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) + 1) - (a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 +
 2*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1)) - 1) - 2*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + a^2*c^2*f*sin(2*f*x +
 2*e)^2 + 2*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*integrate((((cos(6*f*x
 + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x
 + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f
*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4
*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2
*f*x + 2*e))*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin
(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*...
```

3.58.8 Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(c\sec(fx+e)-c)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2, x)
```

3.59 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx$

3.59.1 Optimal result

Integrand size = 26, antiderivative size = 132

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx=\frac{2a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^3c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{2a^4c\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}}-\frac{2a^5c\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

output
```
2*a^(5/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^3*c*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2*a^4*c*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^
(3/2)-2/5*a^5*c*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx=-\frac{2a^3c\left(-5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+\sqrt{c-c\sec(e+fx)}(1+3\sec(e+fx)+\sec^2(e+fx))\right)\tan(e+fx)}{5f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x]),x]
```

output
```
(-2*a^3*c*(-5*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c -
 c*Sec[e + f*x]]*(1 + 3*Sec[e + f*x] + Sec[e + f*x]^2))*Tan[e + f*x])/(5*f
*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.59.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4392}$$

$$-ac\int (\sec(e+fx)a+a)^{3/2}\tan^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int \cot\left(e+fx+\frac{\pi}{2}\right)^2\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^4c\int \frac{\tan^2(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{2a^4c\int \left(\frac{a\tan^4(e+fx)}{(\sec(e+fx)a+a)^2}+\frac{3\tan^2(e+fx)}{\sec(e+fx)a+a}+\frac{1}{a}-\frac{1}{a\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4c\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}} - \frac{a\tan^5(e+fx)}{5(a\sec(e+fx)+a)^{5/2}} - \frac{\tan^3(e+fx)}{(a\sec(e+fx)+a)^{3/2}} - \frac{\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x]),x]
```

output
```
(2*a^4*c*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(3/2)
- Tan[e + f*x]/(a*Sqrt[a + a*Sec[e + f*x]]) - Tan[e + f*x]^3/(a + a*Sec[e
+ f*x])^(3/2) - (a*Tan[e + f*x]^5)/(5*(a + a*Sec[e + f*x])^(5/2))))/f
```

3.59.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.59.4 Maple [A] (warning: unable to verify)

Time = 7.99 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.44

method	result
default	$\frac{a^2 c\left(5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{5}{2}}+2(1-\cos(fx+e))^5\csc(fx+e)^5-10\csc(fx+\right.}{5f(-\cot(fx+e)+\csc(fx+e)+1)^2(-\cot(fx+e)+\csc(fx+e)-1)^2}$
parts	$\frac{2c\,a^2\sqrt{a(\sec(fx+e)+1)}\left(3\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+3\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{c}{\cos}}}\right.\right.}{3f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*c/f*(5*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(
1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(5/2)+2*(
1-cos(f*x+e))^5*csc(f*x+e)^5-10*csc(f*x+e)+10*cot(f*x+e))*(-2*a/((1-cos(f*
x+e))^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+csc(f*x+e)+1)^2/(-cot(f*x+e)+c
sc(f*x+e)-1)^2
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.67

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx = \left[\frac{5\left(a^2c\cos(fx+e)^3+a^2c\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(f\right.}{\cos(fx+e)+1}}{5\left(f\cos(f\right.}\right.$$

$$-\frac{2\left(5\left(a^2c\cos(fx+e)^3+a^2c\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(a^2c\cos(fx+e)^2+3\,a^2\right.\right.}{5\left(f\cos(fx+e)^3+f\cos(fx+e)^2\right)}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/5*(5*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a)*log((2*a*co
s(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(a^2*c*cos
(f*x + e)^2 + 3*a^2*c*cos(f*x + e) + a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/5*(5*(a^
2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c*cos(
f*x + e)^2 + 3*a^2*c*cos(f*x + e) + a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

3.59.6 Sympy [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx =$$

$$-c\left(\int\left(-a^2\sqrt{a\sec(e+fx)+a}\right)dx+\int\left(-a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)\right)dx\right.$$
$$+\int a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\,dx$$
$$\left.+\int a^2\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e)),x)
```

output
```
-c*(Integral(-a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(-a**2*sqrt(a*se
c(e + f*x) + a)*sec(e + f*x), x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*
sec(e + f*x)**2, x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*3, x))
```

3.59.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1396 vs. $2(118) = 236$.

Time = 0.42 (sec) , antiderivative size = 1396, normalized size of antiderivative = 10.58

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
1/6*(30*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)
*((12*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x +
 2*e) - 3*a^2*sin(2*f*x + 2*e) - 4*(3*a^2*cos(2*f*x + 2*e) + 4*a^2)*sin(3/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1)) + (12*a^2*sin(2*f*x + 2*e)*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*a^2*cos(2*f*x + 2*e) - a^2 + 4*(3
*a^2*cos(2*f*x + 2*e) + 4*a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(
a) + 3*((a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*f*x
 + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))) + 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*...
```

3.59.8 Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx = \int -(a\sec(fx+e)+a)^{\frac{5}{2}}(c\sec(fx+e)-c)\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
sage0*x
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x)), x)
```

3.60 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx$

3.60.1 Optimal result

Integrand size = 28, antiderivative size = 103

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx=\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{8a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}-\frac{2a^3\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f+8*a^2*cot(
f*x+e)*(a+a*sec(f*x+e))^(1/2)/c/f-2*a^3*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1
/2)
```

3.60.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.64

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx=\frac{2a^2\csc(e+fx)\left(-1+4\cos(e+fx)+\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}\right.\right.}{cf}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x]),x]
```

output
```
(2*a^2*Csc[e + f*x]*(-1 + 4*Cos[e + f*x] + Cos[e + f*x]*Hypergeometric2F1[
-1/2, 1, 1/2, 1 - Sec[e + f*x]])*Sqrt[a*(1 + Sec[e + f*x])])/(c*f)
```

3.60.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^2(e+fx)(\sec(e+fx)a+a)^{7/2}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx}{ac}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^2\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}$$

$$\Big\downarrow \text{364}$$

$$\frac{2a^2\int \left(4(\sec(e+fx)a+a)\cot^2(e+fx)+a-\frac{a}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+4\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{a\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x]),x]
```

output
```
(2*a^2*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] +
4*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]] - (a*Tan[e + f*x])/Sqrt[a + a*Sec[
e + f*x]]))/(c*f)
```

3.60.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.60.4 Maple [A] (verified)

Time = 5.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2a^2\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}+5\cot(fx+e)-\csc(fx+e)\right)}{cf}$	100

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/c/f*a^2*(a*(sec(f*x+e)+1))^(1/2)*(arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-co
s(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*cot(f
*x+e)-csc(f*x+e))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.83

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = \left[\frac{\sqrt{-a}a^2\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7}{\cos(fx+e)+1}\right.}{2\,cf\sin\left(fx\right.}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a)*a^2*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*
x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*
a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(5*a^2*cos(f*x +
e) - a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(c*f*sin(f*x + e)), (a^
(5/2)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2
*(5*a^2*cos(f*x + e) - a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(c*f*
sin(f*x + e))]
```

3.60.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = -\frac{\int \frac{a^2\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)-1}\,dx + \int \frac{2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)}{\sec(e+fx)-1}\,dx}{c}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e)),x)
```

output
```
-(Integral(a**2*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x) + Integral
(2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x) - 1), x) + Int
egral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2/(sec(e + f*x) - 1), x)
)/c
```

3.60.7 Maxima [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = \int -\frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{c\sec(fx+e)-c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-integrate((a*sec(f*x + e) + a)^(5/2)/(c*sec(f*x + e) - c), x)
```

3.60.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = \int -\frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{c\sec(fx+e)-c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
sage0*x
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x)), x)
```

3.61 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx$

3.61.1 Optimal result

Integrand size = 28, antiderivative size = 74

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} - \frac{8a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f}$$

output
```
2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^2/f-8/3*a*co
t(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^2/f
```

3.61.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = -\frac{2a^3\left(\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\sec(e+fx)\right)+3\sec(e+fx)\right)\tan(e+fx)}{3c^2f(-1+\sec(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a^3*(Hypergeometric2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]] + 3*Sec[e + f*
x])*Tan[e + f*x])/(3*c^2*f*(-1 + Sec[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])
])
```

3.61.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^4(e+fx)(\sec(e+fx)a+a)^{9/2}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{9/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2a\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{c^2f}$$

$$\Big\downarrow \text{364}$$

$$-\frac{2a\int \left(4(\sec(e+fx)a+a)^2\cot^4(e+fx)+\frac{a^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{c^2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a\left(\frac{4}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)\right)}{c^2f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a*(-(a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])
+ (4*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/3))/(c^2*f)
```

3.61.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.61.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(64) = 128$.

Time = 13.92 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.36

method	result
default	$\frac{2a^2\sqrt{a(\sec(fx+e)+1)}\left(3\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)-3\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos}{\cos(}}}\right.\right.}{3c^2f(\cos(fx+e)-1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3/c^2/f*a^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)*(3*arctanh(sin(f*x+e
)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*cos(f*x+e)-3*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/
(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+4*cos(f*x+e)*cot
(f*x+e))
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(64) = 128$.

Time = 0.32 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.58

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = \left[\frac{16\,a^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)^2+3\,(a^2\cos(fx+e)-a^2)\sqrt{-a}\log\left(-\frac{8\,a\,c}{}\right.}{6\,(c^2f\cos(fx+}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/6*(16*a^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)^2 + 3*(a
^2*cos(f*x + e) - a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x +
e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(
f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e))/((c^2*f
*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(8*a^2*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*cos(f*x + e)^2 + 3*(a^2*cos(f*x + e) - a^2)*sqrt(a)*arctan(
2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e))/((c^2*f*cos(f*x
 + e) - c^2*f)*sin(f*x + e))]
```

3.61.6 Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^2}\,dx = \frac{\int \frac{a^2\sqrt{a\sec(e+fx)+a}}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{a^2\sqrt{a\sec(e+fx)}\cdots}{\sec^2(e+fx)-2\cdots}}{c^2}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**2,x)
```

output
```
(Integral(a**2*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x)
+ 1), x) + Integral(2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e +
f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)
*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

3.61.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.61.8 Giac [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^2}\,dx = \int \frac{(a\sec(fx + e) + a)^{\frac{5}{2}}}{(c\sec(fx + e) - c)^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^2, x)
```

3.62 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx$

3.62.1 Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx=\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}+\frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f}+\frac{8\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5c^3f}$$

output
```
2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^3/f+8/5*cot(
f*x+e)^5*(a+a*sec(f*x+e))^(5/2)/c^3/f+2*a^2*cot(f*x+e)*(a+a*sec(f*x+e))^(1
/2)/c^3/f
```

3.62.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx=\frac{2a^3\left(4+3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\sec(e+fx)\right)+5\sec(e+fx)\right)}{15c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a^3*(4 + 3*Hypergeometric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]] + 5*Sec[e
 + f*x])*Tan[e + f*x])/(15*c^3*f*(-1 + Sec[e + f*x])^3*Sqrt[a*(1 + Sec[e +
 f*x])])
```

3.62.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(\sec(e+fx)a+a)^{11/2}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{11/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{c^3f}$$

$$\Big\downarrow \text{364}$$

$$\frac{2\int \left(4(\sec(e+fx)a+a)^3\cot^6(e+fx)+a^2(\sec(e+fx)a+a)\cot^2(e+fx)-\frac{a^3}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{c^3f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}+\frac{4}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}\right)}{c^3f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*(a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] + a^2*
Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]] + (4*Cot[e + f*x]^5*(a + a*Sec[e + f
*x])^(5/2))/5))/(c^3*f)
```

3.62.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.62.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(92) = 184.

Time = 70.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.57

method	result
default	$\frac{2a^2\sqrt{a(\sec(fx+e)+1)}\left(5\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)^2-10\ \operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{c}{co}}}\right.\right.}{}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
2/5/c^3/f*a^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^2*(5*(-cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*
x+e)+1))^(1/2))*cos(f*x+e)^2-10*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*
x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+
9*cos(f*x+e)^2*cot(f*x+e)+5*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)
/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-10*cos(f*x+e)*c
ot(f*x+e)+5*cot(f*x+e))
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(92) = 184.

Time = 0.31 (sec) , antiderivative size = 441, normalized size of antiderivative = 4.24

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = \left[\frac{5\left(a^2\cos(fx+e)^2-2\,a^2\cos(fx+e)+a^2\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\cos\right.}{}\right.}{}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/10*(5*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(-a)*log(-(8*
a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*
x + e) + 1))*sin(f*x + e) + 4*(9*a^2*cos(f*x + e)^3 - 10*a^2*cos(f*x + e)^
2 + 5*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*c
os(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/5*(5*(a^2*c
os(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x +
e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(9*a^2*cos(f*x + e)^3 - 10*a^
2*cos(f*x + e)^2 + 5*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))
]
```

3.62.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx =$$
$$-\frac{\int \frac{a^2\sqrt{a\sec(e+fx)+a}}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{a^2\sqrt{a\sec(e+fx)+a}\sec}{\sec^3(e+fx)-3\sec^2(e+fx)+3}}{c^3}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**3,x)
```

output
```
-(Integral(a**2*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)
**2 + 3*sec(e + f*x) - 1), x) + Integral(2*a**2*sqrt(a*sec(e + f*x) + a)*s
ec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x)
 + Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2/(sec(e + f*x)**3
 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

3.62.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.62.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = \int -\frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(c\sec(fx+e)-c)^3}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^3, x)
```

3.63 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx$

3.63.1 Optimal result

Integrand size = 28, antiderivative size = 140

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx=\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f}+\frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f}-\frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f}-\frac{8\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7ac^4f}$$

output
```
2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^4/f-2/3*a*co
t(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^4/f-8/7*cot(f*x+e)^7*(a+a*sec(f*x+e))^
(7/2)/a/c^4/f+2*a^2*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^4/f
```

3.63.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.70 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx=-\frac{2a^3\left(8+5\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},1-\sec(e+fx)\right)+7\sec(e+fx)\right)\tan(e+fx)}{35c^4f(-1+\sec(e+fx))^4\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^4,x]
```

output
```
(-2*a^3*(8 + 5*Hypergeometric2F1[-7/2, 1, -5/2, 1 - Sec[e + f*x]] + 7*Sec[
e + f*x])*Tan[e + f*x])/(35*c^4*f*(-1 + Sec[e + f*x])^4*Sqrt[a*(1 + Sec[e
+ f*x])])
```

3.63.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^8(e+fx)(\sec(e+fx)a+a)^{13/2}dx}{a^4c^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{13/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^8}dx}{a^4c^4}$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2\int \frac{\cot^8(e+fx)(\sec(e+fx)a+a)^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{ac^4f}$$

$$\Big\downarrow \text{364}$$

$$-\frac{2\int \Big(4(\sec(e+fx)a+a)^4\cot^8(e+fx)+a^2(\sec(e+fx)a+a)^2\cot^4(e+fx)-a^3(\sec(e+fx)a+a)\cot^2(e+$$
$$ac^4f$$

$\Big\downarrow$ 2009

$$-\frac{2\left(-a^{7/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)-a^3\cot(e+fx)\sqrt{a\sec(e+fx)+a}+\frac{1}{3}a^2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}\right.}{ac^4f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^4,x]
```

output
```
(-2*(-(a^(7/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]) -
a^3*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]] + (a^2*Cot[e + f*x]^3*(a + a*Sec
[e + f*x])^(3/2))/3 + (4*Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/7))/(a
*c^4*f)
```

3.63.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.63.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(124) = 248$.

Time = 1.64 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.51

$$\frac{2a^2\sqrt{a\left(\sec\left(fx+e\right)+1\right)}\left(21\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos\left(fx+e\right)^3-63\sqrt{-\frac{\cos}{\cos(}}\right.}{}$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x)
```

output
```
2/21/c^4/f*a^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^3*(21*(-cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(
f*x+e)+1))^(1/2))*cos(f*x+e)^3-63*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arcta
nh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+e
)^2+40*cos(f*x+e)^3*cot(f*x+e)+63*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(
f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e
)-77*cos(f*x+e)^2*cot(f*x+e)-21*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*
x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+70*cos(f*x+
e)*cot(f*x+e)-21*cot(f*x+e))
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(124) = 248$.

Time = 0.34 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.76

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \left[\frac{21\left(a^2\cos\left(fx+e\right)^3-3\,a^2\cos\left(fx+e\right)^2+3\,a^2\cos\left(fx+e\right)-a^2\right)\sqrt{-a}\log}{}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
[1/42*(21*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e)
- a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x +
 e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*c
os(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(40*a^2*cos(f*x + e)
^4 - 77*a^2*cos(f*x + e)^3 + 70*a^2*cos(f*x + e)^2 - 21*a^2*cos(f*x + e))*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*
cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/21*(21*(a^
2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) - a^2)*sqrt(a
)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*si
n(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(40
*a^2*cos(f*x + e)^4 - 77*a^2*cos(f*x + e)^3 + 70*a^2*cos(f*x + e)^2 - 21*a
^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x
+ e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x +
e))]
```

3.63.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**4,x)
```

output
```
Timed out
```

3.63.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.63.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(c\sec(fx+e)-c)^4}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^4,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^4, x)
```

3.64 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx$

3.64.1 Optimal result

Integrand size = 28, antiderivative size = 172

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^5f} + \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^5f} - \frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^5f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5c^5f} + \frac{8\cot^9(e+fx)(a+a\sec(e+fx))^{9/2}}{9a^2c^5f}$$

output
```
2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^5/f-2/3*a*co
t(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^5/f+2/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^
(5/2)/c^5/f+8/9*cot(f*x+e)^9*(a+a*sec(f*x+e))^(9/2)/a^2/c^5/f+2*a^2*cot(f*
x+e)*(a+a*sec(f*x+e))^(1/2)/c^5/f
```

3.64.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.42

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \frac{2a^3\left(12+7\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},1-\sec(e+fx)\right)+9\sec(e+fx\right.}{63c^5f(-1+\sec(e+fx))^5\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^5,x]
```

output
```
(2*a^3*(12 + 7*Hypergeometric2F1[-9/2, 1, -7/2, 1 - Sec[e + f*x]] + 9*Sec[
e + f*x])*Tan[e + f*x])/(63*c^5*f*(-1 + Sec[e + f*x])^5*Sqrt[a*(1 + Sec[e
+ f*x])])
```

3.64.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^{10}(e+fx)(\sec(e+fx)a+a)^{15/2}dx}{a^5c^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{15/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^{10}}dx}{a^5c^5}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2\int \frac{\cot^{10}(e+fx)(\sec(e+fx)a+a)^5\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2c^5f}$$

$$\Big\downarrow \text{364}$$

$$\frac{2\int \left(4(\sec(e+fx)a+a)^5\cot^{10}(e+fx)+a^2(\sec(e+fx)a+a)^3\cot^6(e+fx)-a^3(\sec(e+fx)a+a)^2\cot^4(e+\right.}{a^2c^5f}$$

$$\downarrow 2009$$

$$\frac{2\left(a^{9/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+a^4\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{3}a^3\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}+\frac{1}{5}\right.}{a^2c^5f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^5,x]
```

output
```
(2*(a^(9/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] + a^4*
Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]] - (a^3*Cot[e + f*x]^3*(a + a*Sec[e +
 f*x])^(3/2))/3 + (a^2*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/5 + (4*C
ot[e + f*x]^9*(a + a*Sec[e + f*x])^(9/2))/9))/(a^2*c^5*f)
```

3.64.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.64.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(152) = 304$.

Time = 1.35 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.53

$$2a^2\sqrt{a\left(\sec\left(fx+e\right)+1\right)}\left(45\ \operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \cos\left(fx+e\right)^4-180\sqrt{-\frac{c}{}}\right.$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x)
```

output
```
2/45/c^5/f*a^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^4*(45*arctanh(sin(f
*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*cos(f*x+e)^4-180*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arct
anh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+
e)^3+270*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)
+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+e)^2+89*cos(f*x+e)^4*cot(f
*x+e)-180*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(
1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-243*cos(f*x+e)^3*cot(f
*x+e)+45*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1
/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+324*cos(f*x+e)^2*cot(f*x+e)-195*co
s(f*x+e)*cot(f*x+e)+45*cot(f*x+e))
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.49

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \left[\frac{45\left(a^2\cos\left(fx+e\right)^4-4\,a^2\cos\left(fx+e\right)^3+6\,a^2\cos\left(fx+e\right)^2-4\,a^2\cos\left(fx\right.\right.}{}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
[1/90*(45*(a^2*cos(f*x + e)^4 - 4*a^2*cos(f*x + e)^3 + 6*a^2*cos(f*x + e)^
2 - 4*a^2*cos(f*x + e) + a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos
(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) +
 4*(89*a^2*cos(f*x + e)^5 - 243*a^2*cos(f*x + e)^4 + 324*a^2*cos(f*x + e)^
3 - 195*a^2*cos(f*x + e)^2 + 45*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e)))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f
*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e)), 1/45*(45*(a
^2*cos(f*x + e)^4 - 4*a^2*cos(f*x + e)^3 + 6*a^2*cos(f*x + e)^2 - 4*a^2*co
s(f*x + e) + a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) -
a))*sin(f*x + e) + 2*(89*a^2*cos(f*x + e)^5 - 243*a^2*cos(f*x + e)^4 + 324
*a^2*cos(f*x + e)^3 - 195*a^2*cos(f*x + e)^2 + 45*a^2*cos(f*x + e))*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e)))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*
x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x
+ e))]
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^5}\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**5,x)
```

output
```
Timed out
```

3.64.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^5}\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
Timed out
```

3.64.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \int -\frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(c\sec(fx+e)-c)^5}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^5}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^5,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^5, x)
```

3.65 $\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.65.1 Optimal result

Integrand size = 28, antiderivative size = 185

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{16\sqrt{2}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}+\frac{14c^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{2ac^4\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}}+\frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

output
```
2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-16*c^4*a
rctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(
1/2)+14*c^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2*a*c^4*tan(f*x+e)^3/f/(a+
a*sec(f*x+e))^(3/2)+2/5*a^2*c^4*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

3.65.2 Mathematica [A] (verified)

Time = 5.43 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.83

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$=\frac{c^4\cot\left(\frac{1}{2}(e+fx)\right)\left(100-155\cos(e+fx)+96\cos(2(e+fx))-41\cos(3(e+fx))+20\arctan\left(\sqrt{-1+\right.\right.}{1}$$

input
```
Integrate[(c - c*Sec[e + f*x])^4/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(c^4*Cot[(e + f*x)/2]*(100 - 155*Cos[e + f*x] + 96*Cos[2*(e + f*x)] - 41*C
os[3*(e + f*x)] + 20*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^3*Sqrt[-
1 + Sec[e + f*x]] - 160*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Co
s[e + f*x]^3*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^3)/(10*f*Sqrt[a*(1 + Se
c[e + f*x])])
```

3.65.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4392, 3042, 4375, 381, 27, 444, 27, 444, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4392}$$

$$a^4c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^8}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{9/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2a^4c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{381}$$

$$-\frac{2a^4c^4\left(-\frac{\int \frac{5\tan^4(e+fx)\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{5a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$-\frac{2a^4c^4\left(-\frac{\int \frac{\tan^4(e+fx)\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

$$\Big\downarrow \; 444$$

$$-\frac{2a^4c^4\left(-\frac{-\frac{\int \frac{3a\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+6\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{3a^2}-\frac{\tan^3(e+fx)}{a(a\sec(e+fx)+a)^{3/2}}}{a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$-\frac{2a^4c^4\left(-\frac{-\frac{\int \frac{\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+6\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{\tan^3(e+fx)}{a(a\sec(e+fx)+a)^{3/2}}}{a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

$$\Big\downarrow \; 444$$

$$-\frac{2a^4c^4\left(-\frac{-\frac{-\frac{\int \frac{a\left(\frac{15a\tan^2(e+fx)}{\sec(e+fx)a+a}+14\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{\tan^3(e+fx)}{a(a\sec(e+fx)+a)^{3/2}}}{a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)}\right.}{f}$$

3.65. $\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$

$\big\downarrow$ 27

$$-\frac{2a^4c^4\left(-\frac{-\frac{-\frac{\int \frac{\frac{15a\tan^2(e+fx)}{\sec(e+fx)a+a}+14}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{\tan^3(e+fx)}{a(a\sec(e+fx)+a)^{3/2}}}{a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

$\big\downarrow$ 397

$$-\frac{2a^4c^4\left(-\frac{-\frac{-\frac{16\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{\tan^3(e+fx)}{a(a\sec(e+fx)+a)^{3/2}}}{a^2}-\right)}{f}$$

$\big\downarrow$ 216

$$-\frac{2a^4c^4\left(-\frac{-\frac{-\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}-\frac{8\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{\tan^3(e+fx)}{a(a\sec(e+fx)+a)^{3/2}}}{a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

input

```
Int[(c - c*Sec[e + f*x])^4/Sqrt[a + a*Sec[e + f*x]],x]
```

output

```
(-2*a^4*c^4*(-1/5*Tan[e + f*x]^5/(a^2*(a + a*Sec[e + f*x])^(5/2)) - (-(Tan
[e + f*x]^3/(a*(a + a*Sec[e + f*x])^(3/2))) - (-((ArcTan[(Sqrt[a]*Tan[e +
f*x])/Sqrt[a + a*Sec[e + f*x]]]/Sqrt[a] - (8*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e
 + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/Sqrt[a])/a) - (7*Tan[e + f*x
])/(a*Sqrt[a + a*Sec[e + f*x]]))/a)/a^2))/f
```

3.65. $\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.65. $\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.65.4 Maple [A] (warning: unable to verify)

Time = 6.49 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.52

method	result
default	$\frac{c^4\left(5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{5}{2}}-80\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{(1-\cos(f\cdots\right.\right.}{5f\cdots}$
parts	$-\frac{c^4\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\cdots\right.}{fa}$

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*c^4/f/a*(5*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(
1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(5/2)-80*
ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(
f*x+e))^2*csc(f*x+e)^2-1)^(5/2)+98*(1-cos(f*x+e))^5*csc(f*x+e)^5-160*(1-co
s(f*x+e))^3*csc(f*x+e)^3+70*csc(f*x+e)-70*cot(f*x+e))*(-2*a/((1-cos(f*x+e)
)^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+csc(f*x+e)+1)^2/(-cot(f*x+e)+csc(f
*x+e)-1)^2
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.98

$$\int \frac{(c - c\sec(e + fx))^4}{\sqrt{a + a\sec(e + fx)}}\,dx$$

$$= \left[\frac{40\,\sqrt{2}\left(ac^4\cos\left(fx+e\right)^3 + ac^4\cos\left(fx+e\right)^2\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\,\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\,\cos(fx+e)^2+}{\cos(fx+e)^2+2\,\cos(fx+e)+1}\right.}{}\right.$$

$$-\frac{2\left(5\left(c^4\cos\left(fx+e\right)^3 + c^4\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right) - \left(41\,c^4\cos\left(fx+e\right)^2 - 7\right.}{5\left(af\cos\left(fx+\right.\right.}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/5*(40*sqrt(2)*(a*c^4*cos(f*x + e)^3 + a*c^4*cos(f*x + e)^2)*sqrt(-1/a)*
log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x
+ e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2
 + 2*cos(f*x + e) + 1)) - 5*(c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*sqrt
(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)
) + 2*(41*c^4*cos(f*x + e)^2 - 7*c^4*cos(f*x + e) + c^4)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e)^3 + a*f*cos(f*x + e
)^2), -2/5*(5*(c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*sqrt(a)*arctan(sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))
- (41*c^4*cos(f*x + e)^2 - 7*c^4*cos(f*x + e) + c^4)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sin(f*x + e) - 40*sqrt(2)*(a*c^4*cos(f*x + e)^3 + a*c^4
*cos(f*x + e)^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x + e)^3 + a*f*cos(
f*x + e)^2)]
```

3.65.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx = c^4\left(\int\left(-\frac{4\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)dx + \int\frac{6\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx + \int\left(-\frac{4\sec^3(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)dx + \int\frac{\sec^4(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx + \int\frac{1}{\sqrt{a\sec(e+fx)+a}}\,dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
c**4*(Integral(-4*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(6*s
ec(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(-4*sec(e + f*x)**3/
sqrt(a*sec(e + f*x) + a), x) + Integral(sec(e + f*x)**4/sqrt(a*sec(e + f*x
) + a), x) + Integral(1/sqrt(a*sec(e + f*x) + a), x))
```

3.65.7 Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(c\sec(fx+e)-c)^4}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(f*x + e) - c)^4/sqrt(a*sec(f*x + e) + a), x)
```

3.65.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(1/2), x)
```

3.66 $\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.66.1 Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{8\sqrt{2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{6c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-8*c^3*ar
ctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1
/2)+6*c^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*a*c^3*tan(f*x+e)^3/f/(a+
a*sec(f*x+e))^(3/2)
```

3.66.2 Mathematica [A] (verified)

Time = 2.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.09

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{4c^3\cos\left(\frac{e}{2}\right)\cos(e)\cot\left(\frac{1}{2}(e+fx)\right)\left(-6+11\cos(e+fx)-5\cos(2(e+fx))+3\arctan\left(\sqrt{-1+\sec(e+f\right.\right.}{3f\left(\cos\left(\frac{e}{2}\right)+\mathrm{co}\right.}$$

input
```
Integrate[(c - c*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(4*c^3*Cos[e/2]*Cos[e]*Cot[(e + f*x)/2]*(-6 + 11*Cos[e + f*x] - 5*Cos[2*(e
 + f*x)] + 3*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[
e + f*x]] - 12*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e + f*x
]^2*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2)/(3*f*(Cos[e/2] + Cos[(3*e)/2]
)*Sqrt[a*(1 + Sec[e + f*x])])
```

3.66.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4392, 3042, 4375, 381, 27, 444, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{ 4392}$$

$$-a^3c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$-a^3c^3\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^6}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4375}$$

$$\frac{2a^3c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{ 381}$$

$$\frac{2a^3c^3\left(-\frac{\int \frac{3\tan^2(e+fx)\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{3a^2}-\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{2a^3c^3\left(-\frac{\int \frac{\tan^2(e+fx)\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

$$\Big\downarrow \; 444$$

$$\frac{2a^3c^3\left(-\frac{-\frac{\int \frac{a\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+6\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{3\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}-\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{2a^3c^3\left(-\frac{-\frac{\int \frac{\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+6}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{3\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}-\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{2a^3c^3\left(-\frac{-\frac{8\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{3\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}-\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

$$\Big\downarrow \; 216$$

3.66. $\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$

$$\frac{2a^3c^3\left(-\frac{-\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}-\frac{4\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{a}-\frac{3\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}-\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*a^3*c^3*(-1/3*Tan[e + f*x]^3/(a^2*(a + a*Sec[e + f*x])^(3/2)) - (-((Arc
Tan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/Sqrt[a] - (4*Sqrt[2]*
ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/Sqrt[a]
)/a) - (3*Tan[e + f*x])/(a*Sqrt[a + a*Sec[e + f*x]]))/a^2))/f
```

3.66.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.66.4 Maple [A] (warning: unable to verify)

Time = 5.12 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.71

method	result
default	$\frac{c^3\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}-24\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{(1-\cos(f}\right.\right.}{3fa(-\cot(fx+e)+}$
parts	$-\frac{c^3\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\right.}{fa}$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*c^3/f/a*(3*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(
1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)-24*
ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(
f*x+e))^2*csc(f*x+e)^2-1)^(3/2)+22*(1-cos(f*x+e))^3*csc(f*x+e)^3-18*csc(f*
x+e)+18*cot(f*x+e))*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f
*x+e)+csc(f*x+e)+1)/(-cot(f*x+e)+csc(f*x+e)-1)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.41

$$\int \frac{(c - c\sec(e+fx))^3}{\sqrt{a + a\sec(e+fx)}}\,dx$$

$$= \left[\frac{12\sqrt{2}\left(ac^3\cos\left(fx+e\right)^2 + ac^3\cos\left(fx+e\right)\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{} \right.$$

$$- \frac{2\left(3\left(c^3\cos\left(fx+e\right)^2 + c^3\cos\left(fx+e\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right) - \left(10\,c^3\cos\left(fx+e\right) - c^3\right)}{3\left(af\cos\left(fx+e\right)^2 + af\,c\right.}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(12*sqrt(2)*(a*c^3*cos(f*x + e)^2 + a*c^3*cos(f*x + e))*sqrt(-1/a)*lo
g((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x +
e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 +
 2*cos(f*x + e) + 1)) - 3*(c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*sqrt(-a)
*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) +
2*(10*c^3*cos(f*x + e) - c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(
f*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e)), -2/3*(3*(c^3*cos(f*x +
e)^2 + c^3*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (10*c^3*cos(f*x + e) - c^3)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 12*sqrt(2)*(a*c^3*co
s(f*x + e)^2 + a*c^3*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x
 + e)^2 + a*f*cos(f*x + e))]
```

3.66.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = -c^3\left(\int \frac{3\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx + \int\left(-\frac{3\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)dx + \int \frac{\sec^3(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx + \int\left(-\frac{1}{\sqrt{a\sec(e+fx)+a}}\right)dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
-c**3*(Integral(3*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(-3*
sec(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(sec(e + f*x)**3/sq
rt(a*sec(e + f*x) + a), x) + Integral(-1/sqrt(a*sec(e + f*x) + a), x))
```

3.66.7 Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx=\int -\frac{(c\sec(fx+e)-c)^3}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-integrate((c*sec(f*x + e) - c)^3/sqrt(a*sec(f*x + e) + a), x)
```

3.66.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2), x)
```

3.67 $\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.67.1 Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{4\sqrt{2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-4*c^2*ar
ctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1
/2)+2*c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.84

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^2\left(\operatorname{arctanh}\left(\sqrt{1-\sec(e+fx)}\right)-2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)+\sqrt{1-\sec(e+fx)}\right)\tan(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*c^2*(ArcTanh[Sqrt[1 - Sec[e + f*x]]] - 2*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[e
 + f*x]]/Sqrt[2]] + Sqrt[1 - Sec[e + f*x]])*Tan[e + f*x])/(f*Sqrt[1 - Sec[
e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])])
```

3.67.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4392, 3042, 4375, 381, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^4}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2a^2c^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{381}$$

$$-\frac{2a^2c^2\left(-\frac{\int \frac{\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

$$\Big\downarrow \text{397}$$

$$-\frac{2a^2c^2\left(-\frac{4\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2a^2c^2\left(-\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{a^2}-\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(-2*a^2*c^2*(-((ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/Sq
rt[a] - (2*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
 + f*x]])])/Sqrt[a])/a^2) - Tan[e + f*x]/(a^2*Sqrt[a + a*Sec[e + f*x]])))/
f
```

3.67.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.67.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(102) = 204$.

Time = 5.18 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.73

method	result
default	$-\frac{2c^2\sqrt{a(\sec(fx+e)+1)}\left(2\sqrt{2}\,\cos(fx+e)\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\sqrt{-}\right.}{}$
parts	$-\frac{c^2\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\right.}{fa}$

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c^2/f/a*(a*(sec(f*x+e)+1))^(1/2)*(2*2^(1/2)*cos(f*x+e)*ln(csc(f*x+e)-co
t(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))*(-co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e
)^2-1)^(1/2))-arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1
))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-arctanh(sin(f*x+e)
/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)-sin(f*x+e))/(cos(f*x+e)+1)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.68

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$= \left[\frac{2\,c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin\left(fx+e\right)+2\sqrt{2}(ac^2\cos\left(fx+e\right)+ac^2)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(\cdots}{\cos(fx+e)^2+2\cdots}\right.}{} \right.$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[(2*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + 2*sqrt(2)*(
a*c^2*cos(f*x + e) + a*c^2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^
2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - (c^2*cos(
f*x + e) + c^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a
)/(cos(f*x + e) + 1)))/(a*f*cos(f*x + e) + a*f), 2*(c^2*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*sin(f*x + e) - (c^2*cos(f*x + e) + c^2)*sqrt(a)*arct
an(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
 e))) + 2*sqrt(2)*(a*c^2*cos(f*x + e) + a*c^2)*arctan(sqrt(2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/
(a*f*cos(f*x + e) + a*f)]
```

3.67.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = c^2\left(\int \left(-\frac{2\sec{(e+fx)}}{\sqrt{a\sec{(e+fx)}+a}}\right)dx + \int \frac{\sec^2{(e+fx)}}{\sqrt{a\sec{(e+fx)}+a}}\,dx + \int \frac{1}{\sqrt{a\sec{(e+fx)}+a}}\,dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
c**2*(Integral(-2*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(sec
(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(1/sqrt(a*sec(e + f*x)
 + a), x))
```

3.67.7 Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(c\sec{(fx+e)}-c)^2}{\sqrt{a\sec{(fx+e)}+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(f*x + e) - c)^2/sqrt(a*sec(f*x + e) + a), x)
```

3.67.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2), x)
```

3.68 $\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.68.1 Optimal result

Integrand size = 26, antiderivative size = 87

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{2\sqrt{2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-2*c*arctan
(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^{3/2}\left(\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right) - \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\right)\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*c^(3/2)*(ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] - Sqrt[2]*ArcTanh[Sq
rt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])])*Tan[e + f*x])/(f*Sqrt[a*(1 + Se
c[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.68.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4392, 3042, 4375, 383, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4392}$$

$$-ac\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^2}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2ac\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{383}$$

$$\frac{2ac\left(\frac{2\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{2ac\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*a*c*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(3/2) -
(Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]
)/a^(3/2)))/f
```

3.68.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.68.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

method	result
default	$-\frac{2c\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\right.}{fa}$
parts	$-\frac{c\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\right.}{fa}$

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c/f/a*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2^(1
/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x
+e)^2-1)^(1/2))-arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)))
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.43

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$=\left[\frac{\sqrt{2}ac\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)-\sqrt{-a}c\log\left(\frac{2a\cos(fx+e)}{}\right.}{af}\right.$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(2)*a*c*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x
+ e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - sqrt(-a)*c*log((2*a*cos
(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
 e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*f), 2*(sqrt
(2)*sqrt(a)*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f
*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(a)*c*arctan(sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a*f)]
```

3.68.6 Sympy [F]

$$\int \frac{c - c\sec(e + fx)}{\sqrt{a + a\sec(e + fx)}}\, dx = -c\left(\int \frac{\sec{(e + fx)}}{\sqrt{a\sec{(e + fx)} + a}}\, dx + \int \left(-\frac{1}{\sqrt{a\sec{(e + fx)} + a}}\right) dx\right)$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
-c*(Integral(sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(-1/sqrt(
a*sec(e + f*x) + a), x))
```

3.68.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 699, normalized size of antiderivative = 8.03

$$\int \frac{c - c\sec(e + fx)}{\sqrt{a + a\sec(e + fx)}}\, dx = -\frac{\left(\sqrt{2}\sqrt{a}\arctan\left(\frac{\left(\left|2\, e^{(i\, fx + i\, e)} + 2\right|^{4} + 16\, \cos(fx + e)^{4} + 16\, \sin(fx + e)^{4} + 8\left(\cos(fx + e)^{2} - \sin(fx + e)^{2} - 2\, \cos(fx + e) + 1\right)\left|2\, e^{(i\, fx + i\, e)} + 2\right|}{}\right.\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*arctan2(((abs(2*e^(I*f*x + I*e) + 2)^4 + 16*cos(f*x + e)
^4 + 16*sin(f*x + e)^4 + 8*(cos(f*x + e)^2 - sin(f*x + e)^2 - 2*cos(f*x +
e) + 1)*abs(2*e^(I*f*x + I*e) + 2)^2 - 64*cos(f*x + e)^3 + 32*(cos(f*x + e
)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)^2 + 96*cos(f*x + e)^2 - 64*cos(f*x
+ e) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(f*x + e) - 1)*sin(f*x + e)/abs(2*e
^(I*f*x + I*e) + 2)^2, (abs(2*e^(I*f*x + I*e) + 2)^2 + 4*cos(f*x + e)^2 -
4*sin(f*x + e)^2 - 8*cos(f*x + e) + 4)/abs(2*e^(I*f*x + I*e) + 2)^2)) + 2*
sin(f*x + e))/abs(2*e^(I*f*x + I*e) + 2), ((abs(2*e^(I*f*x + I*e) + 2)^4 +
 16*cos(f*x + e)^4 + 16*sin(f*x + e)^4 + 8*(cos(f*x + e)^2 - sin(f*x + e)^
2 - 2*cos(f*x + e) + 1)*abs(2*e^(I*f*x + I*e) + 2)^2 - 64*cos(f*x + e)^3 +
 32*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)^2 + 96*cos(f*x + e)
^2 - 64*cos(f*x + e) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(f*x + e) - 1)*sin(
f*x + e)/abs(2*e^(I*f*x + I*e) + 2)^2, (abs(2*e^(I*f*x + I*e) + 2)^2 + 4*c
os(f*x + e)^2 - 4*sin(f*x + e)^2 - 8*cos(f*x + e) + 4)/abs(2*e^(I*f*x + I*
e) + 2)^2)) + 2*cos(f*x + e) - 2)/abs(2*e^(I*f*x + I*e) + 2)) - sqrt(a)*ar
ctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(f*x +
e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(f*x + e)
))*c/(a*f)
```

3.68.8 Giac [F(-2)]

Exception generated.

$$\int \frac{c - c\sec(e + fx)}{\sqrt{a + a\sec(e + fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c - c\sec(e + fx)}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{c - \frac{c}{\cos(e+f\,x)}}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2), x)
```

3.69 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

3.69.1 Optimal result

Integrand size = 28, antiderivative size = 121

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f/a^(1/2)-1/2*arctan
(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f*2^(1/2)/a^(1/2
)+cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

3.69.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\cot\left(\frac{1}{2}(e+fx)\right)\left(\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)-2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\right.\right.}{cf\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

output
```
-((Cot[(e + f*x)/2]*(Hypergeometric2F1[-1/2, 1, 1/2, (1 - Sec[e + f*x])/2]
 - 2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - Sec[e + f*x]]))/(c*f*Sqrt[a*(1 +
Sec[e + f*x])]))
```

3.69.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3042, 4392, 3042, 4375, 382, 25, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^2(e+fx)\sqrt{\sec(e+fx)a+a}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx}{ac}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{acf}$$

$$\Big\downarrow \text{382}$$

$$\frac{2\left(\frac{1}{2}\int -\frac{a\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)}{acf}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}\int \frac{a\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{acf}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\int \frac{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{acf}$$

$$\Big\downarrow \; 397$$

$$\frac{2\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(2\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)\right)}{acf}$$

$$\Big\downarrow \; 216$$

$$\frac{2\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)\right)}{acf}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

output
```
(2*(-1/2*(a*((-2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/
Sqrt[a] + ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
]/(Sqrt[2]*Sqrt[a]))) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/2))/(a*c*f
)
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.69. $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

3.69.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)-\right.}{2cfa}$

input
```
int(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c/f/a*(a*(sec(f*x+e)+1))^(1/2)*(2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f
*x+e)^2-1)^(1/2))-4*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*
x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-2*cot(f*x+e))
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.60

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$$
$$= \left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)-2\sqrt{-a}\log\right.$$

input
```
integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*
x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) - 2*sqrt(-
a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) +
 a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt(2)*sqrt(a)*arctan(sqr
t(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x
 + e)))*sin(f*x + e) + 2*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x
+ e) - a))*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e))/(a*c*f*sin(f*x + e))]
```

3.69.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

input
```
integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
-Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*sec(e + f*x) +
 a)), x)/c
```

3.69.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int -\frac{1}{\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)}\,dx$$

input
```
integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)), x)
```

3.69.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))), x)
```

3.70 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx$

3.70.1 Optimal result

Integrand size = 28, antiderivative size = 161

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c^2f} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}\sqrt{a}c^2f} + \frac{3\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{2ac^2f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3a^2c^2f}$$

output
```
-1/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^2/c^2/f+2*arctan(a^(1/2)*tan(f*
x+e)/(a+a*sec(f*x+e))^(1/2))/c^2/f/a^(1/2)-1/4*arctan(1/2*a^(1/2)*tan(f*x+
e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/c^2/f*2^(1/2)/a^(1/2)+3/2*cot(f*x+e)*(a
+a*sec(f*x+e))^(1/2)/a/c^2/f
```

3.70.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \frac{\left(\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)-2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\sec(e+fx\right.\right.}{3c^2f(-1+\sec(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2),x]
```

output
```
((Hypergeometric2F1[-3/2, 1, -1/2, (1 - Sec[e + f*x])/2] - 2*Hypergeometri
c2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]])*Tan[e + f*x])/(3*c^2*f*(-1 + Sec[e
+ f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

3.70.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4392, 3042, 4375, 382, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^4(e+fx)(\sec(e+fx)a+a)^{3/2}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2c^2f}$$

$$\Big\downarrow \text{382}$$

$$-\frac{2\left(\frac{1}{6}\int -\frac{3a\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}\right)}{a^2c^2f}$$

$\big\downarrow$ 27

$$-\frac{2\left(\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\int\frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^2c^2f}$$

$\big\downarrow$ 445

$$-\frac{2\left(\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{3}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}\int\frac{a\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+j}{\sec(e+fx)a}\right.}\right.\right.}{a^2c^2f}$$

$\big\downarrow$ 27

$$-\frac{2\left(\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{3}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\int\frac{\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+7}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+}{\sec(e+fx)}\right.}\right.\right.}{a^2c^2f}$$

$\big\downarrow$ 397

$$-\frac{2\left(\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{3}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(4\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{}{\sqrt{s}}\right.\right.\right.\right.}{a^2c^2f}$$

$\big\downarrow$ 216

$$-\frac{2\left(\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{3}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}\right.\right.\right.}{a^2c^2f}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2),x]
```

output
```
(-2*((Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/6 - (a*(-1/2*(a*((-4*ArcT
an[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/Sqrt[a] + ArcTan[(Sqr
t[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(Sqrt[2]*Sqrt[a]))) 
 + (3*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/2))/2))/(a^2*c^2*f)
```

3.70. $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx$

3.70.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.70.4 Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(3\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\right.}{1}$

input
```
int(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/c^2/f/a*(a*(sec(f*x+e)+1))^(1/2)*(3*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+
csc(f*x+e)^2-1)^(1/2))-24*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(
cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+22*cot(f*x+e)^3+4
*csc(f*x+e)*cot(f*x+e)^2-18*csc(f*x+e)^2*cot(f*x+e))
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.23

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx$$
$$= \left[-\frac{3\sqrt{2}\sqrt{-a}(\cos(fx+e)-1)\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin}{}\right.$$

input
```
integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
[-1/24*(3*sqrt(2)*sqrt(-a)*(cos(f*x + e) - 1)*log(-(2*sqrt(2)*sqrt(-a)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f
*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*s
in(f*x + e) + 12*sqrt(-a)*(cos(f*x + e) - 1)*log(-(8*a*cos(f*x + e)^3 + 4*
(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x
+ e) - 4*(11*cos(f*x + e)^2 - 9*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e)), 1/12*(3*sqrt
(2)*sqrt(a)*(cos(f*x + e) - 1)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 12*sqrt(a)
*(cos(f*x + e) - 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*si
n(f*x + e) + 2*(11*cos(f*x + e)^2 - 9*cos(f*x + e))*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e))]
```

3.70.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{1}{\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)-2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+\sqrt{a\sec(e+fx)+a}}\,dx}{c^2}$$

input
```
integrate(1/(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - 2*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x) + sqrt(a*sec(e + f*x) + a)), x)/c**2
```

3.70.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^2}\,dx$$

input
```
integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)^2), x)
```

3.70.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+fx)}}\left(c-\frac{c}{\cos(e+fx)}\right)^2}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2), x)
```

3.71 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx$

3.71.1 Optimal result

Integrand size = 28, antiderivative size = 196

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c^3f}-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}\sqrt{a}c^3f}+\frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4ac^3f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{2a^2c^3f}+\frac{\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^3c^3f}$$

output
```
-1/2*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^2/c^3/f+1/5*cot(f*x+e)^5*(a+a*s
ec(f*x+e))^(5/2)/a^3/c^3/f+2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1
/2))/c^3/f/a^(1/2)-1/8*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+
e))^(1/2))/c^3/f*2^(1/2)/a^(1/2)+7/4*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c
^3/f
```

3.71.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = -\frac{\left(\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)-2\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\sec(e+\right.\right.}{5c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3),x]
```

output
```
-1/5*((Hypergeometric2F1[-5/2, 1, -3/2, (1 - Sec[e + f*x])/2] - 2*Hypergeo
metric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]])*Tan[e + f*x])/(c^3*f*(-1 + Sec
[e + f*x])^3*Sqrt[a*(1 + Sec[e + f*x])])
```

3.71.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4392, 3042, 4375, 382, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(\sec(e+fx)a+a)^{5/2}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^3c^3f}$$

$$\Big\downarrow \text{382}$$

$$\frac{2\left(\frac{1}{10}\int -\frac{5a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}\right)}{a^3c^3f}$$

$\Big\downarrow$ 27

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\int\frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^3c^3f}$$

$\Big\downarrow$ 445

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{6}\int\frac{3a\cot^2(e+fx)(\sec(e+fx)a-}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\right.}\right.\right.}{a^3c^3f}$$

$\Big\downarrow$ 27

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\int\frac{\cot^2(e+fx)(\sec(e+fx)a+}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\right.}\right.\right.}{a^3c^3f}$$

$\Big\downarrow$ 445

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec(}\right.\right.\right.}{a^3c^3f}$$

$\Big\downarrow$ 27

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec(}\right.\right.\right.}{a^3c^3f}$$

$\Big\downarrow$ 397

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec(}\right.\right.\right.}{a^3c^3f}$$

$\Big\downarrow$ 216

3.71. $\displaystyle\int\frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx$

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec(\right.\right.\right.}{a^3c^3f}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3),x]
```

output
```
(2*((Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/10 - (a*((Cot[e + f*x]^3*(
a + a*Sec[e + f*x])^(3/2))/2 - (a*(-1/2*(a*((-8*ArcTan[(Sqrt[a]*Tan[e + f*
x])/Sqrt[a + a*Sec[e + f*x]]])/Sqrt[a] + ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sq
rt[2]*Sqrt[a + a*Sec[e + f*x]])]/(Sqrt[2]*Sqrt[a]))) + (7*Cot[e + f*x]*Sqr
t[a + a*Sec[e + f*x]])/2))/2))/2))/(a^3*c^3*f)
```

3.71.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.71.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(167) = 334$.

Time = 2.85 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.91

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(5\sqrt{2}\cos(fx+e)\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\sqrt{-\frac{\cdots}{\cdots}}\cdots\right.}{\cdots}$

input
```
int(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/40/c^3/f/a*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)*(5*2^(1/2)*cos(f*x+e
)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e
)^2-1)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-80*arctanh(sin(f*x+e)/(co
s(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*cos(f*x+e)-5*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(csc(f*
x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2
))+80*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+98*cos(f*x+e)*cot(f*x+e)^3-62*cot(f*x
+e)^3-90*csc(f*x+e)*cot(f*x+e)^2+70*csc(f*x+e)^2*cot(f*x+e))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.10

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx$$
$$= \left[-\frac{5\sqrt{2}\left(\cos\left(fx+e\right)^2-2\,\cos\left(fx+e\right)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3\,a\cos(fx+e)^2-}{\cos(fx+e)^2+2\,\cos(fx+e)+1}\right.}{}\right.$$

input
```
integrate(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
[-1/80*(5*sqrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(2*s
qrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f
*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*c
os(f*x + e) + 1))*sin(f*x + e) + 40*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*
sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sq
rt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x
+ e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(49*cos(f*x + e)^3 - 80*cos
(f*x + e)^2 + 35*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((
a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e)),
1/40*(5*sqrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt(
2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
 e)))*sin(f*x + e) + 40*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(a)*arct
an(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(49*cos(f
*x + e)^3 - 80*cos(f*x + e)^2 + 35*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*
f)*sin(f*x + e))]
```

3.71.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{1}{\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)-3\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+3\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c^3}$$

input
```
integrate(1/(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
-Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 - 3*sqrt(a*sec(e + f
*x) + a)*sec(e + f*x)**2 + 3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(
a*sec(e + f*x) + a)), x)/c**3
```

3.71.7 Maxima [F]

$$\int \frac{1}{\sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^3} dx = \int -\frac{1}{\sqrt{a\sec(fx + e) + a}(c\sec(fx + e) - c)^3} dx$$

input
```
integrate(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
-integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)^3), x)
```

3.71.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^3} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^3} dx = \int \frac{1}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}\left(c - \frac{c}{\cos(e+f\,x)}\right)^3} dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3), x)
```

3.72 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.72.1 Optimal result

Integrand size = 28, antiderivative size = 203

$$\begin{aligned}\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx &= \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} \\ &+ \frac{12\sqrt{2}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{14c^4\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{8c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{f(a+a\sec(e+fx))^{5/2}}\end{aligned}$$

output
```
2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f+12*c^4*a
rctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(3/
2)/f-14*c^4*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)+8/3*c^4*tan(f*x+e)^3/f/(
a+a*sec(f*x+e))^(3/2)-a*c^4*sec(1/2*f*x+1/2*e)^2*sin(f*x+e)*tan(f*x+e)^4/f
/(a+a*sec(f*x+e))^(5/2)
```

3.72.2 Mathematica [A] (verified)

Time = 4.51 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.97

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c^4\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\left(-22+20\cos(e+fx)-26\cos(2(e+fx\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(c^4*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(-22 + 20*Cos[e + f*x] - 26*Cos[2*(
e + f*x)] + 28*Cos[3*(e + f*x)] + 6*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(Cos[(
e + f*x)/2] + Cos[(3*(e + f*x))/2])^2*Sqrt[-1 + Sec[e + f*x]] + 36*Sqrt[2]
*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*(Cos[(e + f*x)/2] + Cos[(3*(e + f
*x))/2])^2*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2)/(12*a*f*Sqrt[a*(1 + Se
c[e + f*x])])
```

3.72.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4392, 3042, 4375, 372, 27, 444, 27, 444, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^4}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4392}$$

$$a^4c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^8}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{11/2}}dx$$

$\Big\downarrow$ 4375

$$-\frac{2a^3c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$\Big\downarrow$ 372

$$-\frac{2a^3c^4\left(\frac{\int \frac{2\tan^4(e+fx)\left(\frac{4a\tan^2(e+fx)}{\sec(e+fx)a+a}+5\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan^5(e+fx)}{a^2(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\Big\downarrow$ 27

$$-\frac{2a^3c^4\left(\frac{\int \frac{\tan^4(e+fx)\left(\frac{4a\tan^2(e+fx)}{\sec(e+fx)a+a}+5\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}+\frac{\tan^5(e+fx)}{a^2(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\Big\downarrow$ 444

$$-\frac{2a^3c^4\left(\frac{-\frac{\int \frac{3a\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+8\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{3a^2}-\frac{4\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{a^2}+\frac{\tan^5(e+fx)}{a^2(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\Big\downarrow$ 27

$$-\frac{2a^3c^4\left(\frac{-\frac{\int \frac{\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+8\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{4\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{a^2}+\frac{\tan^5(e+fx)}{a^2(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\Big\downarrow$ 444

3.72. $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx$

$$-\frac{2a^3c^4\left(\frac{-\frac{-\frac{\int \frac{a\left(\frac{13a\tan^2(e+fx)}{\sec(e+fx)a+a}+14\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{4\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{a^2}+\frac{\tan^5(e\cdots}{a^2(a\sec(e+fx)+a)^{5/2}}\cdots\right)}{f}$$

$\downarrow$ 27

$$-\frac{2a^3c^4\left(\frac{-\frac{-\frac{\int \frac{\frac{13a\tan^2(e+fx)}{\sec(e+fx)a+a}+14}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{4\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{a^2}+\frac{\tan^5(e\cdots}{a^2(a\sec(e+fx)+a)^{5/2}}\cdots\right)}{f}$$

$\downarrow$ 397

$$-\frac{2a^3c^4\left(\frac{-\frac{-\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+12\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{4\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{a^2}+\cdots\right)}{f}$$

$\downarrow$ 216

$$-\frac{2a^3c^4\left(\frac{-\frac{-\frac{-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}-\frac{6\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{4\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{a^2}+\frac{\tan^5(e+\cdots}{a^2(a\sec(e+fx)+a)^{5/2}(\cdots}\cdots\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(3/2),x]
```

3.72. $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx$

output
```
(-2*a^3*c^4*(Tan[e + f*x]^5/(a^2*(a + a*Sec[e + f*x])^(5/2)*(2 + (a*Tan[e
+ f*x]^2)/(a + a*Sec[e + f*x]))) + ((-4*Tan[e + f*x]^3)/(3*a*(a + a*Sec[e
+ f*x])^(3/2)) - (-((-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*
x]]]/Sqrt[a]) - (6*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[e + f*x]])])/Sqrt[a])/a) - (7*Tan[e + f*x])/(a*Sqrt[a + a*Sec[e + f
*x]]))/a)/a^2))/f
```

3.72.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.72.4 Maple [A] (warning: unable to verify)

Time = 7.02 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.39

method	result
default	$\frac{c^4\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}+12(1-\cos(fx+e))^5\csc(fx+e)^5+36\ln\left(\csc(\right.\right.}{3a^2}$
parts	Expression too large to display

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*c^4/f*(3*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)+1
2*(1-cos(f*x+e))^5*csc(f*x+e)^5+36*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e)
)^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)-58*(1-c
os(f*x+e))^3*csc(f*x+e)^3+42*csc(f*x+e)-42*cot(f*x+e))*(-2*a/((1-cos(f*x+e
))^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+csc(f*x+e)-1)/(-cot(f*x+e)+csc(f*
x+e)+1)
```

3.72.5 Fricas [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 634, normalized size of antiderivative = 3.12

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[\frac{18\sqrt{2}\left(ac^4\cos{(fx+e)}^3+2\,ac^4\cos{(fx+e)}^2+ac^4\cos{(fx+e)}\right)\sqrt{-\frac{1}{a}}\log}{}\right.$$

$$-\frac{2\left(3\left(c^4\cos{(fx+e)}^3+2\,c^4\cos{(fx+e)}^2+c^4\cos{(fx+e)}\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(28\,c^4\cos\right.}{3\left(a^2f\cos{(f}\right.}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[1/3*(18*sqrt(2)*(a*c^4*cos(f*x + e)^3 + 2*a*c^4*cos(f*x + e)^2 + a*c^4*co
s(f*x + e))*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x +
 e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 3*(c^4*cos(f*x + e)^3 +
2*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2
+ 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(28*c^4*cos(f*x + e)^2
+ 15*c^4*cos(f*x + e) - c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f
*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2*f*cos(f*x +
e)), -2/3*(3*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e)
)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqr
t(a)*sin(f*x + e))) + (28*c^4*cos(f*x + e)^2 + 15*c^4*cos(f*x + e) - c^4)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + 18*sqrt(2)*(a*c^4*c
os(f*x + e)^3 + 2*a*c^4*cos(f*x + e)^2 + a*c^4*cos(f*x + e))*arctan(sqrt(2
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
e)))/sqrt(a))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2*f*cos(f
*x + e))]
```

3.72.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = c^4\left(\int\left(-\frac{4\sec{(e+fx)}}{a\sqrt{a\sec{(e+fx)}+a}\sec{(e+fx)}+a\sqrt{a\sec{(e+fx)}+a}}\right)dx + \int\frac{6\sec^2{(e+fx)}}{a\sqrt{a\sec{(e+fx)}+a}\sec{(e+fx)}+a\sqrt{a\sec{(e+fx)}+a}}\,dx + \int\left(-\frac{4\sec^3{(e+fx)}}{a\sqrt{a\sec{(e+fx)}+a}\sec{(e+fx)}+a\sqrt{a\sec{(e+fx)}+a}}\right)dx + \int\frac{\sec^4{(e+fx)}}{a\sqrt{a\sec{(e+fx)}+a}\sec{(e+fx)}+a\sqrt{a\sec{(e+fx)}+a}}\,dx + \int\frac{1}{a\sqrt{a\sec{(e+fx)}+a}\sec{(e+fx)}+a\sqrt{a\sec{(e+fx)}+a}}\,dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
c**4*(Integral(-4*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a*sqrt(a*sec(e + f*x) + a)), x) + Integral(6*sec(e + f*x)**2/(a*sqrt(a*sec
(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(-
4*sec(e + f*x)**3/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(
e + f*x) + a)), x) + Integral(sec(e + f*x)**4/(a*sqrt(a*sec(e + f*x) + a)*
sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a*sqrt(a*sec(
e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

3.72.7 Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(c\sec{(fx+e)}-c)^4}{(a\sec{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(f*x + e) - c)^4/(a*sec(f*x + e) + a)^(3/2), x)
```

3.72.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^4}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(3/2), x)
```

3.73 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.73.1 Optimal result

Integrand size = 28, antiderivative size = 169

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} + \frac{2\sqrt{2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{4c^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f+2*c^3*ar
ctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(3/2
)/f-4*c^3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)+c^3*sec(1/2*f*x+1/2*e)^2*s
in(f*x+e)*tan(f*x+e)^2/f/(a+a*sec(f*x+e))^(3/2)
```

3.73.2 Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.78

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^3\left(-3+\arctan\left(\sqrt{-1+\sec(e+fx)}\right)\cot^2\left(\frac{1}{2}(e+fx)\right)\sqrt{-1+\sec(e+fx)}\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*c^3*(-3 + ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cot[(e + f*x)/2]^2*Sqrt[-1 +
Sec[e + f*x]] + Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cot[(e + f
*x)/2]^2*Sqrt[-1 + Sec[e + f*x]] - Sec[e + f*x])*Tan[(e + f*x)/2])/(a*f*Sq
rt[a*(1 + Sec[e + f*x])])
```

3.73.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4392, 3042, 4375, 372, 27, 444, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^3}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4392}$$

$$-a^3c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^6}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{9/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^2c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{2a^2c^3\left(\frac{\int \frac{2\tan^2(e+fx)\left(\frac{2a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan^3(e+fx)}{a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\downarrow$ 27

$$\frac{2a^2c^3\left(\frac{\int \frac{\tan^2(e+fx)\left(\frac{2a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}+\frac{\tan^3(e+fx)}{a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\downarrow$ 444

$$\frac{2a^2c^3\left(\frac{-\frac{\int \frac{a\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+4\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{2\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}+\frac{\tan^3(e+fx)}{a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\downarrow$ 27

$$\frac{2a^2c^3\left(\frac{-\frac{\int \frac{\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+4}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{2\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}+\frac{\tan^3(e+fx)}{a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\downarrow$ 397

$$\frac{2a^2c^3\left(\frac{-\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+2\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{2\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}+\frac{\tan^3(e+fx)}{a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\downarrow$ 216

3.73. $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$

$$\frac{2a^2c^3\left(\frac{-\frac{-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{a}-\frac{2\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}+\frac{\tan^3(e+fx)}{a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*c^3*((-((-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/
Sqrt[a]) - (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[
e + f*x]])])/Sqrt[a])/a) - (2*Tan[e + f*x])/(a*Sqrt[a + a*Sec[e + f*x]]))/
a^2 + Tan[e + f*x]^3/(a^2*(a + a*Sec[e + f*x])^(3/2)*(2 + (a*Tan[e + f*x]^
2)/(a + a*Sec[e + f*x])))))/f
```

3.73.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.73.4 Maple [A] (warning: unable to verify)

Time = 5.67 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.31

method	result
default	$\frac{c^3 \sqrt{-\frac{2a}{(1-\cos(fx+e))^2 \csc(fx+e)^2-1}} \left(\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2 \csc(fx+e)^2-1}}\right) \sqrt{(1-\cos(fx+e))^2 \csc(fx+e)^2-1}+2(1-\cos(fx+\right.}{}$
parts	$-\frac{c^3 \sqrt{-\frac{2a}{(1-\cos(fx+e))^2 \csc(fx+e)^2-1}} \sqrt{(1-\cos(fx+e))^2 \csc(fx+e)^2-1} \left(-4\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2 \csc(fx+e)^2-1}}\right)-\sqrt{(1-c\right.}{4 f a^2}$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*c^3/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(2^(1/2)*arctan
h(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e))
)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)+2*(1-cos(f*x+e))^3*csc(f*x+e)^3+
2*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-co
s(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)-4*csc(f*x+e)+4*cot(f*x+e))
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.25

$$\int \frac{(c - c\sec(e+fx))^3}{(a + a\sec(e+fx))^{3/2}}\,dx = \left[\frac{\sqrt{2}\left(ac^3\cos(fx+e)^2 + 2\,ac^3\cos(fx+e) + ac^3\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)}{\cos(fx+e}}}{}\right.}{}\right.$$

$$-\frac{2\left(\left(c^3\cos(fx+e)^2 + 2\,c^3\cos(fx+e) + c^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right) + (3\,c^3\cos(fx+e) + c^3)\right.}{a^2 f\cos(fx+e)^2 + 2\,a^2 f\cos}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[(sqrt(2)*(a*c^3*cos(f*x + e)^2 + 2*a*c^3*cos(f*x + e) + a*c^3)*sqrt(-1/a)
*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*
x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)
^2 + 2*cos(f*x + e) + 1)) - (c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3
)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e
) + 1)) - 2*(3*c^3*cos(f*x + e) + c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f),
-2*((c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (3
*c^3*cos(f*x + e) + c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x +
 e) + sqrt(2)*(a*c^3*cos(f*x + e)^2 + 2*a*c^3*cos(f*x + e) + a*c^3)*arctan
(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin
(f*x + e)))/sqrt(a))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)
]
```

3.73.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = -c^3\left(\int \frac{3\sec(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx + \int\left(-\frac{3\sec^2(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx + \int \frac{\sec^3(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx + \int\left(-\frac{1}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
-c**3*(Integral(3*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a*sqrt(a*sec(e + f*x) + a)), x) + Integral(-3*sec(e + f*x)**2/(a*sqrt(a*se
c(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(
sec(e + f*x)**3/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e
+ f*x) + a)), x) + Integral(-1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a*sqrt(a*sec(e + f*x) + a)), x))
```

3.73.7 Maxima [F]

$$\int \frac{(c - c\sec(e+fx))^3}{(a + a\sec(e+fx))^{3/2}}\,dx = \int -\frac{(c\sec(fx+e) - c)^3}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-integrate((c*sec(f*x + e) - c)^3/(a*sec(f*x + e) + a)^(3/2), x)
```

3.73.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e+fx))^3}{(a + a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e+fx))^3}{(a + a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^3}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2), x)
```

3.74 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.74.1 Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{\sqrt{2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f-c^2*arct
an(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/
f-2*c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c^2\left(2\sqrt{1-\sec(e+fx)}-2\text{arctanh}\left(\sqrt{1-\sec(e+fx)}\right)(1+\sec(e+fx))+\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)(1}{f\sqrt{1-\sec(e+fx)}(a(1+\sec(e+fx)))^{3/2}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c^2*(2*Sqrt[1 - Sec[e + f*x]] - 2*ArcTanh[Sqrt[1 - Sec[e + f*x]]]*(1 +
Sec[e + f*x]) + Sqrt[2]*ArcTanh[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]]*(1 + Sec[e
 + f*x]))*Tan[e + f*x])/(f*Sqrt[1 - Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(
3/2)))
```

3.74.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4392, 3042, 4375, 372, 27, 303, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^2}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^4}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2ac^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{372}$$

$$-\frac{2ac^2\left(\frac{\int \frac{2}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2ac^2\left(\frac{\int \frac{1}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$$\Big\downarrow \ 303$$

$$-\frac{2ac^2\left(\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$$\Big\downarrow \ 216$$

$$-\frac{2ac^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{a^2}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*a*c^2*((-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/Sqrt
[a]) + ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(
Sqrt[2]*Sqrt[a]))/a^2 + Tan[e + f*x]/(a^2*Sqrt[a + a*Sec[e + f*x]]*(2 + (a
*Tan[e + f*x]^2)/(a + a*Sec[e + f*x])))))/f
```

3.74.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.74.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.68

method	result
default	$\frac{c^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{(1-\cos(fx}{a^2f}$
parts	$\frac{c^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{(1-\cos(f}{4f\,a^2}$

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*c^2/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))
^2*csc(f*x+e)^2-1)^(1/2)*(2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*
x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))+((1-cos(f*x+e))^2*csc(f*x+e)^2-1
)^(1/2)*(-cot(f*x+e)+csc(f*x+e))-ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2)))
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(102) = 204$.

Time = 0.49 (sec) , antiderivative size = 542, normalized size of antiderivative = 4.55

$$\int \frac{(c - c\sec(e+fx))^2}{(a + a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{4\,c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) - \sqrt{2}\left(ac^2\cos(fx+e)^2 + 2\,ac\right.}{} \right.$$

$$-\frac{2\,c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) + 2\left(c^2\cos(fx+e)^2 + 2\,c^2\cos(fx+e) + c^2\right)\sqrt{a}\arctan\left(\sqrt{\frac{a\cos}{c}}\right.}{a^2 f\cos(fx+e)^2 + 2\,a^2 f\cos(fx}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(4*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) - sqrt(2)*(a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*sqrt(
-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*co
s(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x
+ e)^2 + 2*cos(f*x + e) + 1)) + 2*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e)
 + c^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f
*x + e) + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -(2*
c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 2*
(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(2
)*(a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*arctan(sqrt(2)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/
sqrt(a))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

3.74.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = c^2\left(\int\left(-\frac{2\sec(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx + \int\frac{\sec^2(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx + \int\frac{1}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
c**2*(Integral(-2*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**2/(a*sqrt(a*sec(e
 + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(
a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

3.74.7 Maxima [F]

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{(c\sec(fx + e) - c)^2}{(a\sec(fx + e) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(f*x + e) - c)^2/(a*sec(f*x + e) + a)^(3/2), x)
```

3.74.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^2}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2), x)
```

3.75 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.75.1 Optimal result

Integrand size = 26, antiderivative size = 113

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}-\frac{3c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}a^{3/2}f}-\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f-3/2*c*arct
an(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2
)-c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

3.75.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.28

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\left(4c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)(1+\sec(e+fx))-3\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e}}{\sqrt{2}\sqrt{c}}\right.\right.}{2f(a(1+\sec(e+fx)))^{3/2}\sqrt{c}}$$

input
```
Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
((4*c^(3/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]]*(1 + Sec[e + f*x]) -
 3*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*(1
+ Sec[e + f*x]) - 2*c*Sqrt[c - c*Sec[e + f*x]])*Tan[e + f*x])/(2*f*(a*(1 +
 Sec[e + f*x]))^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

3.75.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 4392, 3042, 4375, 373, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-c\sec(e+fx)}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4392}$$

$$-ac\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^2}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2c\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{2c\left(-\frac{\int \frac{1-\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{2c\left(-\frac{2\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-3\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{2c\left(-\frac{\frac{3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{2a}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*c*(-1/2*((-2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/S
qrt[a] + (3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]
])])/(Sqrt[2]*Sqrt[a]))/a - Tan[e + f*x]/(2*a*Sqrt[a + a*Sec[e + f*x]]*(2
+ (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x])))))/f
```

3.75.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.75.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(96) = 192$.

Time = 2.43 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.78

method	result
default	$-\frac{c\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)-\sqrt{(1-\mathrm{co}}\right.}{2a^2 f}$
parts	$\frac{c\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{(1-\cos(fx}\right.}{4f\,a^2}$

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/a^2/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e)
)^2*csc(f*x+e)^2-1)^(1/2)*(-2*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-((1-cos(f*x+e))^2*csc(f*x+e)
^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e))+3*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(1/2)))
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(96) = 192$.

Time = 0.39 (sec) , antiderivative size = 505, normalized size of antiderivative = 4.47

$$\int \frac{c - c\sec(e+fx)}{(a + a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{4\,c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) + 3\sqrt{2}\left(c\cos(fx+e)^2 + 2\,c\,\mathrm{co}\right.}{} \right.$$

$$-\frac{2\,c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) - 3\sqrt{2}\left(c\cos(fx+e)^2 + 2\,c\cos(fx+e) + c\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{2\left(a^2f\cos(fx+e)^2 + 2\,a^2f\right.}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(4*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e) + 3*sqrt(2)*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(-a)*log(-(2*
sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*
cos(f*x + e) + 1)) + 4*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(-a)*
log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a
^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/2*(2*c*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*sqrt(2)*(c*cos(
f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 4*(c*cos(f*
x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a^2*f*cos(f*x + e)^2
+ 2*a^2*f*cos(f*x + e) + a^2*f)]
```

3.75.6 Sympy [F]

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = -c\left(\int \frac{\sec(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx + \int \left(-\frac{1}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx\right)$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
-c*(Integral(sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqr
t(a*sec(e + f*x) + a)), x) + Integral(-1/(a*sqrt(a*sec(e + f*x) + a)*sec(e
 + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

3.75.7 Maxima [F]

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int -\frac{c\sec(fx+e)-c}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-integrate((c*sec(f*x + e) - c)/(a*sec(f*x + e) + a)^(3/2), x)
```

3.75.8 Giac [F(-2)]

Exception generated.

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{c - \frac{c}{\cos(e+f\,x)}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2), x)
```

3.76 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx$

3.76.1 Optimal result

Integrand size = 28, antiderivative size = 177

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}cf} - \frac{7\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}a^{3/2}cf} + \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4a^2cf} + \frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{4a^2cf}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c/f-7/8*arctan
(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c/f*2^(1/2
)+1/4*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^2/c/f+1/4*cos(f*x+e)*cot(f*x+e)*
sec(1/2*f*x+1/2*e)^2*(a+a*sec(f*x+e))^(1/2)/a^2/c/f
```

3.76.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.58

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = -\frac{\left(-2+7\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))-8\,\text{Hypergeometric2F1}\left(-\right.\right.}{4cf(-1+\sec(e+fx))(a(1+\sec(e+fx)))^{3/2}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])),x]
```

output
```
-1/4*((-2 + 7*Hypergeometric2F1[-1/2, 1, 1/2, (1 - Sec[e + f*x])/2]*(1 + S
ec[e + f*x]) - 8*Hypergeometric2F1[-1/2, 1, 1/2, 1 - Sec[e + f*x]]*(1 + Se
c[e + f*x]))*Tan[e + f*x])/(c*f*(-1 + Sec[e + f*x])*(a*(1 + Sec[e + f*x]))
^(3/2))
```

3.76.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4392, 3042, 4375, 374, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \frac{\cot^2(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{1}{\cot\left(e+fx+\frac{\pi}{2}\right)^2\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{ac}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2cf}$$

$$\Big\downarrow \text{374}$$

$$\frac{2\left(\frac{\int \frac{a\cot^2(e+fx)(\sec(e+fx)a+a)\left(1-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{a^2cf}$$

$\downarrow$ 27

$$\frac{2\left(\frac{1}{4}\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(1-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{a^2cf}$$

$\downarrow$ 445

$$\frac{2\left(\frac{1}{4}\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}\int \frac{a\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+9\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)+\frac{\cot(e+fx)\sqrt{a}}{4\left(\frac{a\tan^2(e}{a\sec(e+f}\right.}\right.}{a^2cf}$$

$\downarrow$ 27

$$\frac{2\left(\frac{1}{4}\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\int \frac{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+9}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)+\frac{\cot(e+fx)\sqrt{a}}{4\left(\frac{a\tan^2(}{a\sec(e+}\right.}\right.}{a^2cf}$$

$\downarrow$ 397

$$\frac{2\left(\frac{1}{4}\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(8\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-7\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(}{\sqrt{\sec(e}}\right.\right.\right.\right.}{a^2cf}$$

$\downarrow$ 216

$$\frac{2\left(\frac{1}{4}\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(\frac{7\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{8\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)\right)+\frac{\cot(e+fx)\sqrt{}}{4\left(\frac{a\tan^2(}{a\sec(e-}\right.}\right.}{a^2cf}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])),x]
```

3.76. $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx$

output
```
(2*((-1/2*(a*((-8*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])
/Sqrt[a] + (7*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*
x]])])/(Sqrt[2]*Sqrt[a]))) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/2)/4
+ (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(4*(2 + (a*Tan[e + f*x]^2)/(a +
a*Sec[e + f*x])))))/(a^2*c*f)
```

3.76.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.76.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(151) = 302$.

Time = 2.18 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.92

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(7\sqrt{2}\cos(fx+e)\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\sqrt{-\frac{c}{co}}\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/8/c/f/a^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(7*2^(1/2)*cos(f*x+e)
*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)
^2-1)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+7*2^(1/2)*(-cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot
(f*x+e)+csc(f*x+e)^2-1)^(1/2))-16*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(
f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e
)-16*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))
*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*cos(f*x+e)*cot(f*x+e)-2*cot(f*x+e))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.90

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = \Bigg[-\frac{7\sqrt{2}\sqrt{-a}(\cos(fx+e)+1)\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{}\right.}{}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[-1/16*(7*sqrt(2)*sqrt(-a)*(cos(f*x + e) + 1)*log(-(2*sqrt(2)*sqrt(-a)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f
*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*s
in(f*x + e) + 8*sqrt(-a)*(cos(f*x + e) + 1)*log(-(8*a*cos(f*x + e)^3 + 4*(
2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x +
 e) - 4*(3*cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e)))/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e)), 1/8*(7*sqrt(2)*s
qrt(a)*(cos(f*x + e) + 1)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 8*sqrt(a)*(cos(
f*x + e) + 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x
+ e) + 2*(3*cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e)))/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e))]
```

3.76.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{a\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)-a\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - a*sqrt(a*sec(e +
 f*x) + a)), x)/c
```

3.76.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = \int -\frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)), x)
```

3.76.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))), x)
```

3.77 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx$

3.77.1 Optimal result

Integrand size = 28, antiderivative size = 214

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx &= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^2f} \\ &- \frac{9\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{8\sqrt{2}a^{3/2}c^2f} + \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{8a^2c^2f} \\ &+ \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{12a^3c^2f} \\ &- \frac{\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{4a^3c^2f}\end{aligned}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c^2/f+1/12*cot
(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^3/c^2/f-1/4*cos(f*x+e)*cot(f*x+e)^3*sec
(1/2*f*x+1/2*e)^2*(a+a*sec(f*x+e))^(3/2)/a^3/c^2/f-9/16*arctan(1/2*a^(1/2)
*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c^2/f*2^(1/2)+7/8*cot(
f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^2/c^2/f
```

3.77.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.47 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \frac{\left(-6+9\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1-\sec(e+f\right.\right.}{}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2),x]
```

output
```
((-6 + 9*Hypergeometric2F1[-3/2, 1, -1/2, (1 - Sec[e + f*x])/2]*(1 + Sec[e
 + f*x]) - 8*Hypergeometric2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]]*(1 + Sec[e
 + f*x]))*Tan[e + f*x])/(12*c^2*f*(-1 + Sec[e + f*x])^2*(a*(1 + Sec[e + f*
x]))^(3/2))
```

3.77.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {3042, 4392, 3042, 4375, 374, 25, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^4(e+fx)\sqrt{\sec(e+fx)a+a}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^3c^2f}$$

$$\Big\downarrow \; 374$$

$$-\frac{2\left(\frac{\int -\frac{a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{a^3c^2f}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\left(\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{\int \frac{a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}\right)}{a^3c^2f}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{1}{4}\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^3c^2f}$$

$$\Big\downarrow \; 445$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{6}\int -\frac{3a\cot^2(e+fx)(\sec(e+fx)a+a)\left(7-\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}\right.\right.}{a^3c^2f}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(7-\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}\right.\right.}{a^3c^2f}$$

$$\Big\downarrow \; 445$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}\int \frac{a\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+23\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)-\frac{1}{6}c\right.\right.}{a^3c^2f}$$

3.77. $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\int\frac{\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+23}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)-\frac{1}{6}c}{a^3c^2f}$$

$\downarrow$ 397

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(16\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-9\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}c}{a^3c^2f}$$

$\downarrow$ 216

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(\frac{9\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{16\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)\right)\right)-\frac{1}{6}}{a^3c^2f}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2),x]
```

output
```
(-2*((-1/6*(Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2)) - (a*(-1/2*(a*((-16
*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/Sqrt[a] + (9*Arc
Tan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*S
qrt[a]))) + (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/2))/2)/4 + (Cot[e +
f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(4*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e
 + f*x])))))/(a^3*c^2*f)
```

3.77.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.77. $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx$

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.77.4 Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.94

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(27\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\right.}{4}$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/48/c^2/f/a^2*(a*(sec(f*x+e)+1))^(1/2)*(27*2^(1/2)*(-cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+
e)+csc(f*x+e)^2-1)^(1/2))-96*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e
)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+62*cot(f*x+e)^
3-4*csc(f*x+e)*cot(f*x+e)^2-42*csc(f*x+e)^2*cot(f*x+e))
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.62

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \left[-\frac{27\sqrt{2}\left(\cos(fx+e)^2-1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx}{\cos(fx}}}{}\right.}{}\right.$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas
")
```

output
```
[-1/96*(27*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*co
s(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)
)*sin(f*x + e) + 48*(cos(f*x + e)^2 - 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3
 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin
(f*x + e) - 4*(31*cos(f*x + e)^3 - 2*cos(f*x + e)^2 - 21*cos(f*x + e))*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2
*f)*sin(f*x + e)), 1/48*(27*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(a)*arctan(sq
rt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*
x + e)))*sin(f*x + e) + 48*(cos(f*x + e)^2 - 1)*sqrt(a)*arctan(2*sqrt(a)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(
f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(31*cos(f*x + e)^3 - 2*
cos(f*x + e)^2 - 21*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))
/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))]
```

3.77.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{1}{a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)-a\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)-a\sqrt{a\sec}}}{c^2}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 - a*sqrt(a*sec(e +
f*x) + a)*sec(e + f*x)**2 - a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sq
rt(a*sec(e + f*x) + a)), x)/c**2
```

3.77.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^2}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima
")
```

output
```
integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)^2), x)
```

3.77.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2), x)
```

3.78 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx$

3.78.1 Optimal result

Integrand size = 28, antiderivative size = 249

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx &= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^3f} \\ &- \frac{11\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{16\sqrt{2}a^{3/2}c^3f} + \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{16a^2c^3f} \\ &- \frac{5\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{24a^3c^3f} - \frac{3\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{20a^4c^3f} \\ &+ \frac{\cos(e+fx)\cot^5(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{5/2}}{4a^4c^3f}\end{aligned}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c^3/f-5/24*cot
(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^3/c^3/f-3/20*cot(f*x+e)^5*(a+a*sec(f*x+
e))^(5/2)/a^4/c^3/f+1/4*cos(f*x+e)*cot(f*x+e)^5*sec(1/2*f*x+1/2*e)^2*(a+a*
sec(f*x+e))^(5/2)/a^4/c^3/f-11/32*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a
+a*sec(f*x+e))^(1/2))/a^(3/2)/c^3/f*2^(1/2)+21/16*cot(f*x+e)*(a+a*sec(f*x+
e))^(1/2)/a^2/c^3/f
```

3.78.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.41

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = -\frac{\left(-10+11\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))-8\,\text{Hypergeometric2F1}\right.}{20c^3f(-1+\sec(e+fx))^3(a(1+\sec(e+fx)))}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3),x]
```

output
```
-1/20*((-10 + 11*Hypergeometric2F1[-5/2, 1, -3/2, (1 - Sec[e + f*x])/2]*(1
 + Sec[e + f*x]) - 8*Hypergeometric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]]*(1
 + Sec[e + f*x]))*Tan[e + f*x])/(c^3*f*(-1 + Sec[e + f*x])^3*(a*(1 + Sec[e
 + f*x]))^(3/2))
```

3.78.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {3042, 4392, 3042, 4375, 374, 25, 27, 445, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(\sec(e+fx)a+a)^{3/2}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \; 4375$$

$$\frac{2\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^4c^3f}$$

$$\Big\downarrow \; 374$$

$$\frac{2\left(\frac{\int -\frac{a\cot^6(e+fx)(\sec(e+fx)a+a)^3\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{a^4c^3f}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{\int \frac{a\cot^6(e+fx)(\sec(e+fx)a+a)^3\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}\right)}{a^4c^3f}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{1}{4}\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^4c^3f}$$

$$\Big\downarrow \; 445$$

$$\frac{2\left(\frac{1}{4}\left(\frac{1}{10}\int -\frac{5a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(5-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{3}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/}\right.\right.}{a^4c^3f}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(5-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{3}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}\right.\right.}{a^4c^3f}$$

$$\Big\downarrow \; 445$$

3.78. $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx$

$$2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{6}\int\frac{3a\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+21\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a}}\right.\right.\right.\right.$$
$$a^4c^3f$$

$\downarrow$ 27

$$2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\int\frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+21\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+}}\right.\right.\right.\right.$$
$$a^4c^3f$$

$\downarrow$ 445

$$2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}\int\frac{a\left(\frac{21a\tan^2(}{\sec(e+fx}\right.}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}\right.\right.\right.\right.$$
$$a^4c$$

$\downarrow$ 27

$$2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\int\frac{\frac{21a\tan^2(}{\sec(e+fx}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}\right.\right.\right.\right.$$
$$a^4c$$

$\downarrow$ 397

$$2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(32\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}}\right.\right.\right.\right.\right.$$

$\downarrow$ 216

$$2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(\frac{11\arctan\left(\frac{\sqrt{a}\,t}{\sqrt{2}\sqrt{a\,s}}\right.}{\sqrt{2}\sqrt{a}}\right.\right.\right.\right.\right.$$
$$a^4$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3),x]
```

3.78. $\int\frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx$

output
```
(2*(((-3*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/10 - (a*((5*Cot[e + f*
x]^3*(a + a*Sec[e + f*x])^(3/2))/6 - (a*(-1/2*(a*((-32*ArcTan[(Sqrt[a]*Tan
[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/Sqrt[a] + (11*ArcTan[(Sqrt[a]*Tan[e
+ f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]))) + (21*Cot
[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/2))/2))/2)/4 + (Cot[e + f*x]^5*(a + a*
Sec[e + f*x])^(5/2))/(4*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x])))))/(
a^4*c^3*f)
```

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.78.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\left(165\sin(fx+e)^5\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)-960\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/480/c^3/f/a^2*(165*sin(f*x+e)^5*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x
+e)^2-1)^(1/2))-960*sin(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctan
h(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))-898*cos(f*
x+e)^5-196*cos(f*x+e)^4+1432*cos(f*x+e)^3+100*cos(f*x+e)^2-630*cos(f*x+e))
*(a*(sec(f*x+e)+1))^(1/2)*csc(f*x+e)^5
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.87

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = \left[-\frac{165\sqrt{2}\left(\cos(fx+e)^3-\cos(fx+e)^2-\cos(fx+e)\right.}{}\right.$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas
")
```

output
```
[-1/960*(165*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*
sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(co
s(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 480*(cos(f*x + e)^3 - c
os(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2
*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x +
e) - 4*(449*cos(f*x + e)^4 - 351*cos(f*x + e)^3 - 365*cos(f*x + e)^2 + 315
*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^3*f*cos(f*
x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*
sin(f*x + e)), 1/480*(165*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f
*x + e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 480*(cos(f*x + e)^3
- cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2
+ a*cos(f*x + e) - a))*sin(f*x + e) + 2*(449*cos(f*x + e)^4 - 351*cos(f*x
+ e)^3 - 365*cos(f*x + e)^2 + 315*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2
*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))]
```

3.78.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{1}{a\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)-2a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)+2a\sqrt{a\sec(e+fx)+a}\sec(e+fx)-a\sqrt{a\sec(e+fx)+a}}\,dx}{c^3}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**3,x)
```

output
```
-Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4 - 2*a*sqrt(a*sec(e
 + f*x) + a)*sec(e + f*x)**3 + 2*a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) -
 a*sqrt(a*sec(e + f*x) + a)), x)/c**3
```

3.78.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = \int -\frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^3}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima
")
```

output
```
-integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)^3), x)
```

3.78.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3), x)
```

3.79 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.79.1 Optimal result

Integrand size = 28, antiderivative size = 260

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c^5\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{23\sqrt{2}c^5\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} + \frac{21c^5\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}} - \frac{19c^5\tan^3(e+fx)}{6af(a+a\sec(e+fx))^{3/2}} + \frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} + \frac{ac^5\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^5(e+fx)}{4f(a+a\sec(e+fx))^{7/2}}$$

output
```
2*c^5*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-23*c^5*a
rctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(5/
2)/f+21*c^5*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)-19/6*c^5*tan(f*x+e)^3/
a/f/(a+a*sec(f*x+e))^(3/2)+3/4*c^5*sec(1/2*f*x+1/2*e)^2*sin(f*x+e)*tan(f*x
+e)^4/f/(a+a*sec(f*x+e))^(5/2)+1/4*a*c^5*sec(1/2*f*x+1/2*e)^4*sin(f*x+e)^2
*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(7/2)
```

3.79.2 Mathematica [A] (verified)

Time = 8.45 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.69

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c^5\cot\left(\frac{1}{2}(e+fx)\right)\left((81-30\cos(e+fx)+52\cos(2(e+fx))-66\cos(3(e+\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(c^5*Cot[(e + f*x)/2]*((81 - 30*Cos[e + f*x] + 52*Cos[2*(e + f*x)] - 66*Co
s[3*(e + f*x)] - 37*Cos[4*(e + f*x)])*Sec[(e + f*x)/2]^4 + 96*ArcTan[Sqrt[
-1 + Sec[e + f*x]]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f*x]] - 1104*Sqrt[2]*
ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f
*x]])*Sec[e + f*x]^2)/(48*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

3.79.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {3042, 4392, 3042, 4375, 372, 27, 440, 25, 27, 444, 27, 444, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^5}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4392}$$

$$-a^5c^5\int \frac{\tan^{10}(e+fx)}{(\sec(e+fx)a+a)^{15/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-a^5c^5\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^{10}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{15/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^3c^5\int \frac{\tan^{10}(e+fx)}{(\sec(e+fx)a+a)^5\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$\downarrow$ 372

$$\frac{2a^3c^5\left(\frac{\int \frac{2\tan^6(e+fx)\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a^2}+\frac{\tan^7(e+fx)}{2a^2(a\sec(e+fx)+a)^{7/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\downarrow$ 27

$$\frac{2a^3c^5\left(\frac{\int \frac{\tan^6(e+fx)\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan^7(e+fx)}{2a^2(a\sec(e+fx)+a)^{7/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\downarrow$ 440

$$\frac{2a^3c^5\left(\frac{\frac{3\tan^5(e+fx)}{2a(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{\int -\frac{a\tan^4(e+fx)\left(\frac{19a\tan^2(e+fx)}{\sec(e+fx)a+a}+15\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}}{2a^2}+\frac{}{2a^2(a\sec(e}\right.}{f}$$

$\downarrow$ 25

$$\frac{2a^3c^5\left(\frac{\frac{\int \frac{a\tan^4(e+fx)\left(\frac{19a\tan^2(e+fx)}{\sec(e+fx)a+a}+15\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{3\tan^5(e+fx)}{2a(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{}{2a^2(a\sec(e+}\right.}{f}$$

$\downarrow$ 27

3.79. $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx$

$$\frac{2a^3c^5\left(\frac{\frac{\int \frac{\tan^4(e+fx)\left(\frac{19a\tan^2(e+fx)}{\sec(e+fx)a+a}+15\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a}+\frac{3\tan^5(e+fx)}{2a(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\ldots}{2a^2(a\sec(e+\ldots}\right.}{f}$$

$$\downarrow 444$$

$$\frac{2a^3c^5\left(\frac{\frac{-\frac{\int \frac{6a\tan^2(e+fx)\left(\frac{21a\tan^2(e+fx)}{\sec(e+fx)a+a}+19\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{3a^2}-\frac{19\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{2a}+\frac{3\tan^5(e+fx)}{2a(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+f\ldots}{a\sec(e+fx)\ldots}\right.}}{2a^2}\ldots\right.}{f}$$

$$\downarrow 27$$

$$\frac{2a^3c^5\left(\frac{\frac{-\frac{2\int \frac{\tan^2(e+fx)\left(\frac{21a\tan^2(e+fx)}{\sec(e+fx)a+a}+19\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{19\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{2a}+\frac{3\tan^5(e+fx)}{2a(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+\ldots}{a\sec(e+fx\ldots}\right.}}{2a^2}\ldots\right.}{f}$$

$$\downarrow 444$$

3.79. $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx$

$$\frac{2a^3c^5\left(\frac{\frac{-\frac{2\left(-\frac{\int \frac{2a\left(\frac{22a\tan^2(e+fx)}{\sec(e+fx)a+a}+21\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{21\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{a}-\frac{19\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{2a}+\frac{3\tan^5(e+\cdots}{2a(a\sec(e+fx)+a)^{5/2}\left(\cdots\right.}}{2a^2}\cdots\right.}{f}$$

$\downarrow$ 27

$$\frac{2a^3c^5\left(\frac{\frac{-\frac{2\left(-\frac{2\int \frac{\frac{22a\tan^2(e+fx)}{\sec(e+fx)a+a}+21}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{21\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{a}-\frac{19\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{2a}+\frac{3\tan^5(e\cdots}{2a(a\sec(e+fx)+a)^{5/2}\left(\cdots\right.}}{2a^2}\cdots\right.}{f}$$

$\downarrow$ 397

3.79. $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx$

$$\frac{2a^3c^5\left(\frac{-\frac{2\left(-\frac{2\left(23\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a}-\frac{21\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{a}}{2a}-\frac{19\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}\cdots}{2a^2}\cdots\right)}{f}$$

$\Big\downarrow$ 216

$$\frac{2a^3c^5\left(\frac{-\frac{2\left(-\frac{2\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}-\frac{23\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}\right)}{a}-\frac{21\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{a}}{2a}-\frac{19\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}+\frac{3\tan^5(e+\cdots}{2a(a\sec(e+fx)+a)^{5/2}\left(\cdots\right.}}{2a^2}\cdots\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(2*a^3*c^5*(Tan[e + f*x]^7/(2*a^2*(a + a*Sec[e + f*x])^(7/2)*(2 + (a*Tan[e
 + f*x]^2)/(a + a*Sec[e + f*x]))^2) + ((3*Tan[e + f*x]^5)/(2*a*(a + a*Sec[
e + f*x])^(5/2)*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x]))) + ((-19*Tan
[e + f*x]^3)/(3*a*(a + a*Sec[e + f*x])^(3/2)) - (2*((-2*(ArcTan[(Sqrt[a]*T
an[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/Sqrt[a] - (23*ArcTan[(Sqrt[a]*Tan[e
 + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a])))/a - (21*
Tan[e + f*x])/(a*Sqrt[a + a*Sec[e + f*x]])))/a)/(2*a))/(2*a^2)))/f
```

3.79. $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.79.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

3.79. $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx$

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.79.4 Maple [A] (warning: unable to verify)

Time = 7.71 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.17

method	result
default	$\frac{c^5\left(-6(1-\cos(fx+e))^7\csc(fx+e)^7+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}-9(1-\cos(\right.}{}$
parts	Expression too large to display

input
```
int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*c^5/a^3/f*(-6*(1-cos(f*x+e))^7*csc(f*x+e)^7+3*2^(1/2)*arctanh(2^(1/2)/
((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(
f*x+e))^2*csc(f*x+e)^2-1)^(3/2)-9*(1-cos(f*x+e))^5*csc(f*x+e)^5-69*ln(csc(
f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*x+e))
^2*csc(f*x+e)^2-1)^(3/2)+82*(1-cos(f*x+e))^3*csc(f*x+e)^3-63*csc(f*x+e)+63
*cot(f*x+e))*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+c
sc(f*x+e)-1)/(-cot(f*x+e)+csc(f*x+e)+1)
```

3.79.5 Fricas [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.85

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[\frac{69\sqrt{2}\left(ac^5\cos(fx+e)^4+3\,ac^5\cos(fx+e)^3+3\,ac^5\cos(fx+e)^2+ac^5\cos\right.}{}\right.$$

$$-\frac{6\left(c^5\cos(fx+e)^4+3\,c^5\cos(fx+e)^3+3\,c^5\cos(fx+e)^2+c^5\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(f}{\sqrt{a}\sin(fx+e)}\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(69*sqrt(2)*(a*c^5*cos(f*x + e)^4 + 3*a*c^5*cos(f*x + e)^3 + 3*a*c^5*
cos(f*x + e)^2 + a*c^5*cos(f*x + e))*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(
f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) -
6*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 + c^5*
cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/
(cos(f*x + e) + 1)) + 4*(37*c^5*cos(f*x + e)^3 + 70*c^5*cos(f*x + e)^2 + 2
0*c^5*cos(f*x + e) - c^5)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e
)^2 + a^3*f*cos(f*x + e)), -1/3*(6*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e
)^3 + 3*c^5*cos(f*x + e)^2 + c^5*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(37*c
^5*cos(f*x + e)^3 + 70*c^5*cos(f*x + e)^2 + 20*c^5*cos(f*x + e) - c^5)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 69*sqrt(2)*(a*c^5*cos(
f*x + e)^4 + 3*a*c^5*cos(f*x + e)^3 + 3*a*c^5*cos(f*x + e)^2 + a*c^5*cos(f
*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*
x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*x + e))]
```

3.79.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx =$$

$$-c^5\left(\int \frac{5\sec(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+\cdots}}\right.$$

$$+\int\left(-\frac{10\sec^2(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+\cdots}}\right.$$

$$+\int \frac{10\sec^3(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,dx$$

$$+\int\left(-\frac{5\sec^4(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+\cdots}}\right.$$

$$+\int \frac{\sec^5(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,dx$$

$$+\int\left(-\frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+\cdots}}\right.$$

input
```
integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
-c**5*(Integral(5*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)
**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f
*x) + a)), x) + Integral(-10*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a
)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sq
rt(a*sec(e + f*x) + a)), x) + Integral(10*sec(e + f*x)**3/(a**2*sqrt(a*sec
(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(-5*sec(e + f*x)**4/(a*
*2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) +
 a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f
*x)**5/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(
e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral
(-1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e +
 f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x))
```

3.79.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.79.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^5}{(a + a\sec(e + fx))^{5/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^5}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^(5/2), x)
```

3.80 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.80.1 Optimal result

Integrand size = 28, antiderivative size = 229

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{11c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}a^{5/2}f} + \frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{c^4\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^3(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}$$

output
```
2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-11/2*c^4
*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f*2
^(1/2)+7/2*c^4*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)-1/4*c^4*sec(1/2*f*x
+1/2*e)^2*sin(f*x+e)*tan(f*x+e)^2/a/f/(a+a*sec(f*x+e))^(3/2)-1/4*c^4*sec(1
/2*f*x+1/2*e)^4*sin(f*x+e)^2*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^(5/2)
```

3.80.2 Mathematica [A] (verified)

Time = 5.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.72

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c^4\cot\left(\frac{1}{2}(e+fx)\right)\left((-4+19\cos(e+fx)-12\cos(2(e+fx))-3\cos(3(e+\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(c^4*Cot[(e + f*x)/2]*((-4 + 19*Cos[e + f*x] - 12*Cos[2*(e + f*x)] - 3*Cos
[3*(e + f*x)])*Sec[(e + f*x)/2]^4 + 32*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos
[e + f*x]*Sqrt[-1 + Sec[e + f*x]] - 88*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*
x]]/Sqrt[2]]*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x])/(16*a^2*f
*Sqrt[a*(1 + Sec[e + f*x])])
```

3.80.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {3042, 4392, 3042, 4375, 372, 27, 440, 25, 27, 444, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^4}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4392}$$

$$a^4c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^{13/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^8}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{13/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2a^2c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$\downarrow$ 372

$$-\frac{2a^2c^4\left(\frac{\int \frac{2\tan^4(e+fx)\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+5\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a^2}+\frac{\tan^5(e+fx)}{2a^2(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\downarrow$ 27

$$-\frac{2a^2c^4\left(\frac{\int \frac{\tan^4(e+fx)\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+5\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan^5(e+fx)}{2a^2(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\downarrow$ 440

$$-\frac{2a^2c^4\left(\frac{\frac{\tan^3(e+fx)}{2a(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{\int -\frac{a\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}}{2a^2}+\frac{\cdots}{2a^2(a\sec\cdots}\right.}{f}$$

$\downarrow$ 25

$$-\frac{2a^2c^4\left(\frac{\frac{\int \frac{a\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan^3(e+fx)}{2a(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\cdots}{2a^2(a\sec(e\cdots}\right.}{f}$$

$\downarrow$ 27

3.80. $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx$

$$-\frac{2a^2c^4\left(\frac{\frac{\int \frac{\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a}+\frac{\tan^3(e+fx)}{2a(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\cdots}{2a^2(a\sec(e\cdots}\right.}{f}$$

$$\Big\downarrow 444$$

$$-\frac{2a^2c^4\left(\frac{\frac{-\frac{\int \frac{2a\left(\frac{9a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{2a}+\frac{\tan^3(e+fx)}{2a(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\cdots}{2a^2(\cdots}\right.}{f}$$

$$\Big\downarrow 27$$

$$-\frac{2a^2c^4\left(\frac{\frac{-\frac{2\int \frac{\frac{9a\tan^2(e+fx)}{\sec(e+fx)a+a}+7}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{2a}+\frac{\tan^3(e+fx)}{2a(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\cdots}{2a^2\cdots}\right.}{f}$$

$$\Big\downarrow 397$$

$$-\frac{2a^2c^4\left(\frac{\frac{-\frac{2\left(11\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-2\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{2a}+\frac{\tan^3(e+\cdots}{2a(a\sec(e+fx)+a)^{3/2}(\cdots}}{2a^2}\cdots\right.}{f}$$

3.80. $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx$

$$\Big\downarrow \text{216}$$

$$-\frac{2a^2c^4\left(\frac{\frac{-\frac{2\left(\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}-\frac{11\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{2a}+\frac{\tan^3(e+fx)}{2a(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{}{2a^2}\right.}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*a^2*c^4*(Tan[e + f*x]^5/(2*a^2*(a + a*Sec[e + f*x])^(5/2)*(2 + (a*Tan[
e + f*x]^2)/(a + a*Sec[e + f*x]))^2) + (((-2*((2*ArcTan[(Sqrt[a]*Tan[e + f
*x])/Sqrt[a + a*Sec[e + f*x]]])/Sqrt[a] - (11*ArcTan[(Sqrt[a]*Tan[e + f*x]
)/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a])))/a - (7*Tan[e +
f*x])/(a*Sqrt[a + a*Sec[e + f*x]]))/(2*a) + Tan[e + f*x]^3/(2*a*(a + a*Sec
[e + f*x])^(3/2)*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x]))))/(2*a^2)))
/f
```

3.80.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.80. $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx$

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.80.4 Maple [A] (warning: unable to verify)

Time = 6.13 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.07

method	result
default	$\frac{c^4\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left(-2(1-\cos(fx+e))^5\csc(fx+e)^5+2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\sqrt{(1-\cos(fx+}\right.}{}$
parts	Expression too large to display

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*c^4/a^3/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-2*(1-cos(f*
x+e))^5*csc(f*x+e)^5+2*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e
)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(
1/2)-(1-cos(f*x+e))^3*csc(f*x+e)^3-11*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x
+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)+7*cs
c(f*x+e)-7*cot(f*x+e))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.86

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[-\frac{11\sqrt{2}\left(c^4\cos(fx+e)^3+3c^4\cos(fx+e)^2+3c^4\cos(fx+e)+c^4\right)\sqrt{-a}}{}\right.$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[-1/4*(11*sqrt(2)*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f
*x + e) + c^4)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*
x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 4*(c^4*cos(f*x + e)^3
 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*sqrt(-a)*log((2*a*cos(
f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(3*c^4*cos(f
*x + e)^2 + 9*c^4*cos(f*x + e) + 2*c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3
*f*cos(f*x + e) + a^3*f), 1/2*(11*sqrt(2)*(c^4*cos(f*x + e)^3 + 3*c^4*cos(
f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 4*(c^4*
cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*sqrt(a)*
arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f
*x + e))) + 2*(3*c^4*cos(f*x + e)^2 + 9*c^4*cos(f*x + e) + 2*c^4)*sqrt((a*
cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^
3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

3.80.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = c^4\left(\int \left(-\frac{4\sec(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec\cdots}\right.\right.$$

$$+\int \frac{6\sec^2(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,dx$$

$$+\int \left(-\frac{4\sec^3(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+\cdots}}\right.$$

$$+\int \frac{\sec^4(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,dx$$

$$+\int \frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
c**4*(Integral(-4*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)
**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f
*x) + a)), x) + Integral(6*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*
sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt
(a*sec(e + f*x) + a)), x) + Integral(-4*sec(e + f*x)**3/(a**2*sqrt(a*sec(e
 + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x
) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**4/(a**2*sq
rt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*s
ec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a**2*sqrt(a
*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e
 + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x))
```

3.80.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.80.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(5/2), x)
```

3.81 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.81.1 Optimal result

Integrand size = 28, antiderivative size = 191

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f}-\frac{7c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}a^{5/2}f}-\frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}}+\frac{c^3\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-7/4*c^3*
arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f*2^
(1/2)-1/4*c^3*sec(1/2*f*x+1/2*e)^2*sin(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)
+1/4*c^3*sec(1/2*f*x+1/2*e)^4*sin(f*x+e)^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))
^(3/2)
```

3.81.2 Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.71

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^3\cot\left(\frac{1}{2}(e+fx)\right)\left((-5+8\cos(e+fx)-3\cos(2(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right)-32\arctan\left(\sqrt{-1+\sec(e}\right.\right.}{16a^2f\sqrt{a(1+\sec(e+fx}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-1/16*(c^3*Cot[(e + f*x)/2]*((-5 + 8*Cos[e + f*x] - 3*Cos[2*(e + f*x)])*Se
c[(e + f*x)/2]^4 - 32*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e + f*
x]] + 28*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e +
 f*x]]))/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

3.81.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4392, 3042, 4375, 372, 27, 440, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^3}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4392}$$

$$-a^3c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^6}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{11/2}}dx$$

$$\Big\downarrow \; 4375$$

$$\frac{2ac^3\int\frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{2ac^3\left(\frac{\int\frac{2\tan^2(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a^2}+\frac{\tan^3(e+fx)}{2a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{2ac^3\left(\frac{\int\frac{\tan^2(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan^3(e+fx)}{2a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$$\Big\downarrow \; 440$$

$$\frac{2ac^3\left(\frac{-\frac{\int\frac{a\left(1-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\tan^3(e+fx)}{2a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan}{a\sec(}\right.}\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{2ac^3\left(\frac{-\frac{\int\frac{1-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\tan^3(e+fx)}{2a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan}{a\sec(}\right.}\right.}{f}$$

3.81. $\displaystyle\int\frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$

$$\Big\downarrow 397$$

$$\frac{2ac^3\left(\frac{-\frac{4\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-7\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{}{2a^2(a\sec}\right.}{f}$$

$$\Big\downarrow 216$$

$$\frac{2ac^3\left(\frac{-\frac{\frac{7\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{2a}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\tan^3(e+fx)}{2a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+}{a\sec(e+f}\right.}\right.}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(2*a*c^3*(Tan[e + f*x]^3/(2*a^2*(a + a*Sec[e + f*x])^(3/2)*(2 + (a*Tan[e +
 f*x]^2)/(a + a*Sec[e + f*x]))^2) + (-1/2*((-4*ArcTan[(Sqrt[a]*Tan[e + f*x
])/Sqrt[a + a*Sec[e + f*x]]])/Sqrt[a] + (7*ArcTan[(Sqrt[a]*Tan[e + f*x])/(
Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]))/a - Tan[e + f*x]/(2
*a*Sqrt[a + a*Sec[e + f*x]]*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x])))
)/(2*a^2)))/f
```

3.81.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.81. $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.81.4 Maple [A] (warning: unable to verify)

Time = 3.89 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.31

method	result
default	$-\frac{c^3 \sqrt{-\frac{2a}{(1-\cos(fx+e))^2 \csc(fx+e)^2-1}}\, \sqrt{(1-\cos(fx+e))^2 \csc(fx+e)^2-1}\, \left(2(1-\cos(fx+e))^3 \sqrt{(1-\cos(fx+e))^2 \csc(fx+e)^2-1}\, \csc(f\right.}{}$
parts	Expression too large to display

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^3*c^3/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^(1/2)*(2*(1-cos(f*x+e))^3*((1-cos(f*x+e))^2*csc(f*x+
e)^2-1)^(1/2)*csc(f*x+e)^3-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f
*x+e)+csc(f*x+e))-4*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2
-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))+7*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x
+e))^2*csc(f*x+e)^2-1)^(1/2)))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 645, normalized size of antiderivative = 3.38

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[-\frac{7\sqrt{2}\left(c^3\cos(fx+e)^3+3\,c^3\cos(fx+e)^2+3\,c^3\cos(fx+e)+c^3\right)\sqrt{-a}\,\mathrm{lo}}{}\right.$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[-1/8*(7*sqrt(2)*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*
x + e) + c^3)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x
 + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c^3*cos(f*x + e)^3
+ 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f
*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(3*c^3*cos(f*
x + e)^2 - c^3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f
*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x
+ e) + a^3*f), 1/4*(7*sqrt(2)*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 +
 3*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 8*(c^3*cos(f*x + e)
^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) -
2*(3*c^3*cos(f*x + e)^2 - c^3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3
*a^3*f*cos(f*x + e) + a^3*f)]
```

3.81.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx =$$

$$-c^3\left(\int \frac{3\sec(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+\right.$$

$$+\int\left(-\frac{3\sec^2(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}\right.$$

$$+\int\frac{\sec^3(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,dx$$

$$+\int\left(-\frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}\right.$$

input
```
integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
-c**3*(Integral(3*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)
**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f
*x) + a)), x) + Integral(-3*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)
*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqr
t(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**3/(a**2*sqrt(a*sec(e +
 f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)
+ a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(-1/(a**2*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a
**2*sqrt(a*sec(e + f*x) + a)), x))
```

3.81.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.81.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2), x)
```

3.82 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.82.1 Optimal result

Integrand size = 28, antiderivative size = 189

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{11c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}a^{5/2}f} - \frac{3c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^2\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-11/8*c^2
*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f*2
^(1/2)-3/8*c^2*sec(1/2*f*x+1/2*e)^2*sin(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2
)-1/4*c^2*cos(f*x+e)*sec(1/2*f*x+1/2*e)^4*sin(f*x+e)/a^2/f/(a+a*sec(f*x+e)
)^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.77

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{c^2\left(22\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)-8\text{arctanh}\left(\sqrt{1-\sec(e+fx)}\right)(1+\sec(e}{4f\sqrt{1-\sec(e+fx)}(a(1+\sec(e+fx)))^{5/2}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-1/4*(c^2*(22*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]]*Cos[(e + f*x
)/2]^4*Sec[e + f*x]^2 - 8*ArcTanh[Sqrt[1 - Sec[e + f*x]]]*(1 + Sec[e + f*x
])^2 + Sqrt[1 - Sec[e + f*x]]*(7 + 3*Sec[e + f*x]))*Tan[e + f*x])/(f*Sqrt[
1 - Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2))
```

3.82.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4392, 3042, 4375, 372, 27, 402, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^2}{(a\sec(e+fx)+a)^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$
$$\Big\downarrow \text{4392}$$
$$a^2c^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^{9/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$a^2c^2\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^4}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{9/2}}dx$$

$$\Big\downarrow \; 4375$$

$$-\frac{2c^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \; 372$$

$$-\frac{2c^2\left(\frac{\int \frac{2\left(1-\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a^2}+\frac{\tan(e+fx)}{2a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$-\frac{2c^2\left(\frac{\int \frac{1-\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan(e+fx)}{2a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$$\Big\downarrow \; 402$$

$$-\frac{2c^2\left(\frac{\frac{\int \frac{a\left(5-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{3\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\tan(e+fx)}{2a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+\cdots}{a\sec(e+fx\cdots}\right.}\right.}{f}$$

$$\Big\downarrow \; 27$$

$$-\frac{2c^2\left(\frac{\frac{1}{4}\int \frac{5-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{3\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\tan(e+fx)}{2a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(\cdots}{a\sec(e+\cdots}\right.}\right.}{f}$$

$$\Big\downarrow \; 397$$

3.82. $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$

$$-\frac{2c^2\left(\frac{\frac{1}{4}\left(8\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-11\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)+\frac{3\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\ldots\right)}{f}$$

$$\downarrow 216$$

$$-\frac{2c^2\left(\frac{\frac{1}{4}\left(\frac{11\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{8\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)+\frac{3\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\tan(e+fx)}{2a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}\ldots\right.}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*c^2*(Tan[e + f*x]/(2*a^2*Sqrt[a + a*Sec[e + f*x]]*(2 + (a*Tan[e + f*x]
^2)/(a + a*Sec[e + f*x]))^2) + (((-8*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a
+ a*Sec[e + f*x]]])/Sqrt[a] + (11*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*S
qrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]))/4 + (3*Tan[e + f*x])/(4*Sqrt
[a + a*Sec[e + f*x]]*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x]))))/(2*a^
2)))/f
```

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.82. $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.82.4 Maple [A] (warning: unable to verify)

Time = 3.55 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.30

method	result
default	$-\frac{c^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}(-\cot(fx+e)+\csc(f$
parts	$-\frac{c^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2(1-\cos(fx+e))^3\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\csc(f$

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^3*c^2/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^(1/2)*(2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(-c
ot(f*x+e)+csc(f*x+e))-3*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+
e)+csc(f*x+e))-8*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
^(1/2)*(-cot(f*x+e)+csc(f*x+e)))+11*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e
))^2*csc(f*x+e)^2-1)^(1/2)))
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 645, normalized size of antiderivative = 3.41

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[-\frac{11\sqrt{2}\left(c^2\cos(fx+e)^3+3c^2\cos(fx+e)^2+3c^2\cos(fx+e)+c^2\right)\sqrt{-a}}{\ }\right.$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[-1/16*(11*sqrt(2)*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(
f*x + e) + c^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f
*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 16*(c^2*cos(f*x + e)
^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*co
s(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(7*c^2*cos
(f*x + e)^2 + 3*c^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos
(f*x + e) + a^3*f), 1/8*(11*sqrt(2)*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x +
e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 16*(c^2*cos(f
*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(a)*arcta
n(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
e))) - 2*(7*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e))*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x +
 e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

3.82.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = c^2\left(\int\left(-\frac{2\sec(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec}\right.\right.$$
$$+\int\frac{\sec^2(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,dx$$
$$+\int\frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
c**2*(Integral(-2*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)
**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f
*x) + a)), x) + Integral(sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*se
c(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a
*sec(e + f*x) + a)), x) + Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e
+ f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec
(e + f*x) + a)), x))
```

3.82.7 Maxima [F]

$$\int \frac{(c - c\sec(e+fx))^2}{(a + a\sec(e+fx))^{5/2}}\,dx = \int \frac{(c\sec(fx+e) - c)^2}{(a\sec(fx+e) + a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(f*x + e) - c)^2/(a*sec(f*x + e) + a)^(5/2), x)
```

3.82.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e+fx))^2}{(a + a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e+fx))^2}{(a + a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^2}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2), x)
```

3.83 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.83.1 Optimal result

Integrand size = 26, antiderivative size = 148

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{23c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{8\sqrt{2}a^{5/2}f} - \frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}} - \frac{7c\tan(e+fx)}{8af(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-23/16*c*ar
ctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f*2^(1
/2)-1/2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)-7/8*c*tan(f*x+e)/a/f/(a+a*se
c(f*x+e))^(3/2)
```

3.83.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.20

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\left(-64c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx) + 46\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\cos^4\left(\right.\right.}{8f(a(1+\sec(e+fx)))^{5/2}\sqrt{c-c\,\text{s}}}$$

input
```
Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-1/8*((-64*c^(3/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]]*Cos[(e + f*x)
/2]^4*Sec[e + f*x]^2 + 46*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]
/(Sqrt[2]*Sqrt[c])]*Cos[(e + f*x)/2]^4*Sec[e + f*x]^2 + c*(11 + 7*Sec[e +
f*x])*Sqrt[c - c*Sec[e + f*x]])*Tan[e + f*x])/(f*(a*(1 + Sec[e + f*x]))^(5
/2)*Sqrt[c - c*Sec[e + f*x]])
```

3.83.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 4392, 3042, 4375, 373, 402, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-c\sec(e+fx)}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4392}$$

$$-ac\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^2}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2c\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{af}$$

$$\Big\downarrow \text{373}$$

$$\frac{2c\left(-\frac{\int \frac{1-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}-\frac{\tan(e+fx)}{4a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{af}$$

$\downarrow$ 402

$$\frac{2c\left(-\frac{\frac{\int \frac{a\left(9-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{7\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{4a}-\frac{\tan(e+fx)}{4a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{af}$$

$\downarrow$ 27

$$\frac{2c\left(-\frac{\frac{1}{4}\int \frac{9-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{7\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{4a}-\frac{\tan(e+fx)}{4a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{af}$$

$\downarrow$ 397

$$\frac{2c\left(-\frac{\frac{1}{4}\left(16\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-23\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)+\frac{7\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{4a}-\frac{\tan(e+fx)}{4a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{af}$$

$\downarrow$ 216

$$\frac{2c\left(-\frac{\frac{1}{4}\left(\frac{23\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{16\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)+\frac{7\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{4a}-\frac{\tan(e+fx)}{4a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{af}$$

input
```
Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

3.83. $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$

output
```
(2*c*(-1/4*Tan[e + f*x]/(a*Sqrt[a + a*Sec[e + f*x]]*(2 + (a*Tan[e + f*x]^2
)/(a + a*Sec[e + f*x]))^2) - (((-16*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a +
 a*Sec[e + f*x]]])/Sqrt[a] + (23*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sq
rt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]))/4 + (7*Tan[e + f*x])/(4*Sqrt[
a + a*Sec[e + f*x]]*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x]))))/(4*a))
)/(a*f)
```

3.83.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.83.4 Maple [A] (warning: unable to verify)

Time = 2.66 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.64

method	result
default	$\frac{c\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-2\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}(-\cot(fx+e)+\csc(fx}{}$
parts	$-\frac{c\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2(1-\cos(fx+e))^3\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\csc(fx}{}$

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/a^3*c/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e)
)^2*csc(f*x+e)^2-1)^(1/2)*(-2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(-co
t(f*x+e)+csc(f*x+e))+16*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+
e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))+7*((1-cos(f*x+e))^2*csc(f*x+e)^2-1
)^(1/2)*(-cot(f*x+e)+csc(f*x+e))-23*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e
))^2*csc(f*x+e)^2-1)^(1/2)))
```

3.83.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(123) = 246$.

Time = 0.56 (sec) , antiderivative size = 605, normalized size of antiderivative = 4.09

$$\int \frac{c - c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[-\frac{23\sqrt{2}\left(c\cos(fx+e)^3 + 3\,c\cos(fx+e)^2 + 3\,c\cos(fx+e) + c\right)\sqrt{-a}\log}{}\right.$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[-1/32*(23*sqrt(2)*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x +
e) + c)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e)
+ a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 32*(c*cos(f*x + e)^3 + 3*c*c
os(f*x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2
*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(11*c*cos(f*x + e)^2 + 7*c
*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*
f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f),
 1/16*(23*sqrt(2)*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e
) + c)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)/(sqrt(a)*sin(f*x + e))) - 32*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)
^2 + 3*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(11*c*cos(f*x + e)^2 + 7*
c*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3
*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
]
```

3.83.6 Sympy [F]

$$\int \frac{c - c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx =$$

$$-c\left(\int \frac{\sec(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx) + 2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx) + a^2\sqrt{a\sec(e+fx)+a}}\right.$$

$$+\int\left(-\frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx) + 2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx) + a^2\sqrt{a\sec(e+fx)+a}}\right.$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
-c*(Integral(sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 +
 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) +
 a)), x) + Integral(-1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*
a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)
), x))
```

3.83.7 Maxima [F]

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\, dx = \int -\frac{c\sec\left(fx + e\right) - c}{\left(a\sec\left(fx + e\right) + a\right)^{\frac{5}{2}}}\, dx$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
-integrate((c*sec(f*x + e) - c)/(a*sec(f*x + e) + a)^(5/2), x)
```

3.83.8 Giac [F(-2)]

Exception generated.

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\,dx = \int \frac{c - \frac{c}{\cos(e+f\,x)}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2), x)
```

3.84 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx$

3.84.1 Optimal result

Integrand size = 28, antiderivative size = 230

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}cf} - \frac{71\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{32\sqrt{2}a^{5/2}cf} - \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{32a^3cf} + \frac{13\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{32a^3cf} + \frac{\cos^2(e+fx)\cot(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{16a^3cf}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c/f-71/64*arct
an(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c/f*2^(1
/2)-7/32*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^3/c/f+13/32*cos(f*x+e)*cot(f*
x+e)*sec(1/2*f*x+1/2*e)^2*(a+a*sec(f*x+e))^(1/2)/a^3/c/f+1/16*cos(f*x+e)^2
*cot(f*x+e)*sec(1/2*f*x+1/2*e)^4*(a+a*sec(f*x+e))^(1/2)/a^3/c/f
```

3.84.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = -\frac{\left(71\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^2-2\left(17+13\sec(e+fx)+32\,\text{H}\right.}{32cf(-1+\sec(e+fx))(a(1+\sec(e}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])),x]
```

output
```
-1/32*((71*Hypergeometric2F1[-1/2, 1, 1/2, (1 - Sec[e + f*x])/2]*(1 + Sec[
e + f*x])^2 - 2*(17 + 13*Sec[e + f*x] + 32*Hypergeometric2F1[-1/2, 1, 1/2,
 1 - Sec[e + f*x]]*(1 + Sec[e + f*x])^2))*Tan[e + f*x])/(c*f*(-1 + Sec[e +
 f*x])*(a*(1 + Sec[e + f*x]))^(5/2))
```

3.84.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4392, 3042, 4375, 374, 27, 441, 25, 27, 445, 25, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \frac{\cot^2(e+fx)}{(\sec(e+fx)a+a)^{3/2}}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{1}{\cot\left(e+fx+\frac{\pi}{2}\right)^2\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{ac}$$

$$\Big\downarrow \; 4375$$

$$\frac{2\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^3cf}$$

$$\Big\downarrow \; 374$$

$$\frac{2\left(\frac{\int \frac{a\cot^2(e+fx)(\sec(e+fx)a+a)\left(3-\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{8a}+\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{8\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{a^3cf}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{1}{8}\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(3-\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{8\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{a^3cf}$$

$$\Big\downarrow \; 441$$

$$\frac{2\left(\frac{1}{8}\left(\frac{\int -\frac{a\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{39a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{13\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)+\frac{\cot(e+fx)\sqrt{a\sec(e+\cdots}}{8\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+\cdots\right.}\cdots\right.}{a^3cf}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{1}{8}\left(\frac{13\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{\int \frac{a\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{39a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}\right)+\frac{\cot(e+fx)\sqrt{a\sec(e+fx\cdots}}{8\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)\cdots}\right.}{a^3cf}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{1}{8}\left(\frac{13\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{1}{4}\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{39a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)+\frac{\cot(e+fx)\sqrt{a\,s\cdots}}{8\left(\frac{a\tan^2(e+\cdots}{a\sec(e+f\cdots}\right.}\cdots\right.}{a^3cf}$$

$$\Big\downarrow \; 445$$

3.84. $\displaystyle \int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx$

$$\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}\int -\frac{a\left(57-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)+\frac{13\cot(e}{4(}\right.\right.}{a^3cf}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{a\left(57-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)+\frac{13\cot(e}{4(}\right.\right.}{a^3cf}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\int \frac{57-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)+\frac{13\cot(}{4(}\right.\right.}{a^3cf}$$

$$\Big\downarrow \color{blue}{397}$$

$$\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(64\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-71\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)-\frac{7}{2}\cot(e+fx)\right.\right.\right.}{a^3cf}$$

$$\Big\downarrow \color{blue}{216}$$

$$\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{71\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{64\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)-\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)+\frac{13c}{}\right.\right.}{a^3cf}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])),x]
```

output
```
(2*((Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(8*(2 + (a*Tan[e + f*x]^2)/(a
+ a*Sec[e + f*x]))^2) + ((-1/2*(a*((-64*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt
[a + a*Sec[e + f*x]]])/Sqrt[a] + (71*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2
]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]))) - (7*Cot[e + f*x]*Sqrt[a
 + a*Sec[e + f*x]])/2)/4 + (13*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(4*(
2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x]))))/8))/(a^3*c*f)
```

3.84. $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx$

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.84.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(199) = 398$.

Time = 2.48 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.17

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(24\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{9}{2}}\sin(fx+e)-24(1-\cos\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/192/c/f/a^3*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^(1/2)/(1-cos(f*x+e))*(24*((1-cos(f*x+e))^2*csc(f*x+e
)^2-1)^(9/2)*sin(f*x+e)-24*(1-cos(f*x+e))^2*((1-cos(f*x+e))^2*csc(f*x+e)^2
-1)^(7/2)*csc(f*x+e)+28*(1-cos(f*x+e))^2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
^(5/2)*csc(f*x+e)-4*(1-cos(f*x+e))^6*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/
2)*csc(f*x+e)^5-35*(1-cos(f*x+e))^2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2
)*csc(f*x+e)+25*(1-cos(f*x+e))^4*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*c
sc(f*x+e)^3-192*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^
(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(1-cos(f*x+e))-42*(1-cos(f*x+e))^2*((1-cos
(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*csc(f*x+e)+213*ln(csc(f*x+e)-cot(f*x+e)+(
(1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*(1-cos(f*x+e)))
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 608, normalized size of antiderivative = 2.64

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = \left[-\frac{71\sqrt{2}\left(\cos(fx+e)^2+2\cos(fx+e)+1\right)\sqrt{-a}\log\left(-\right.}{}\right.$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[-1/128*(71*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(2
*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2
*cos(f*x + e) + 1))*sin(f*x + e) + 64*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1
)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*
sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*
x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(27*cos(f*x + e)^3 + 12*c
os(f*x + e)^2 - 7*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(
(a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e)),
 1/64*(71*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(a)*arctan(sqr
t(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x
 + e)))*sin(f*x + e) + 64*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(a)*ar
ctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*
x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(27*cos
(f*x + e)^3 + 12*cos(f*x + e)^2 - 7*cos(f*x + e))*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c
*f)*sin(f*x + e))]
```

3.84.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)+a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)-a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)-a^2\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 + a**2*sqrt(a*s
ec(e + f*x) + a)*sec(e + f*x)**2 - a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x) - a**2*sqrt(a*sec(e + f*x) + a)), x)/c
```

3.84.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = \int -\frac{1}{(a\sec(fx+e)+a)^{\frac{5}{2}}(c\sec(fx+e)-c)}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-integrate(1/((a*sec(f*x + e) + a)^(5/2)*(c*sec(f*x + e) - c)), x)
```

3.84.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))), x)
```

3.85 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx$

3.85.1 Optimal result

Integrand size = 28, antiderivative size = 269

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx &= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}c^2f} \\ &- \frac{107\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{64\sqrt{2}a^{5/2}c^2f} + \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{64a^3c^2f} \\ &+ \frac{43\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{96a^4c^2f} \\ &- \frac{15\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{32a^4c^2f} \\ &- \frac{\cos^2(e+fx)\cot^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{16a^4c^2f}\end{aligned}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c^2/f+43/96*co
t(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^4/c^2/f-15/32*cos(f*x+e)*cot(f*x+e)^3*
sec(1/2*f*x+1/2*e)^2*(a+a*sec(f*x+e))^(3/2)/a^4/c^2/f-1/16*cos(f*x+e)^2*co
t(f*x+e)^3*sec(1/2*f*x+1/2*e)^4*(a+a*sec(f*x+e))^(3/2)/a^4/c^2/f-107/128*a
rctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c^2/f
*2^(1/2)+21/64*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^3/c^2/f
```

3.85.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.42

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \frac{\cot^3(e+fx)\left(107\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1-\right.\right.}{}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2),x]
```

output
```
(Cot[e + f*x]^3*(107*Hypergeometric2F1[-3/2, 1, -1/2, (1 - Sec[e + f*x])/2
]*(1 + Sec[e + f*x])^2 - 2*(57 + 45*Sec[e + f*x] + 32*Hypergeometric2F1[-3
/2, 1, -1/2, 1 - Sec[e + f*x]]*(1 + Sec[e + f*x])^2)))/(96*a^2*c^2*f*Sqrt[
a*(1 + Sec[e + f*x])])
```

3.85.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.99, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {3042, 4392, 3042, 4375, 374, 27, 441, 25, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4392$$

$$\frac{\int \frac{\cot^4(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{a^2c^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{1}{\cot\left(e+fx+\frac{\pi}{2}\right)^4\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a^2c^2}$$

$$\Big\downarrow \; 4375$$

$$-\frac{2\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^4c^2f}$$

$$\Big\downarrow \; 374$$

$$-\frac{2\left(\frac{\int \frac{a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(1-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{8a}+\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{8\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{a^4c^2f}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{1}{8}\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(1-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{8\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{a^4c^2f}$$

$$\Big\downarrow \; 441$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{\int -\frac{a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{75a\tan^2(e+fx)}{\sec(e+fx)a+a}+43\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{15\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)+\frac{\cot^3(e+fx)(a\cdots}{8\left(\frac{a\tan^2\cdots}{a\sec(e\cdots}\right.}\cdots}{a^4c^2f}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{15\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{\int \frac{a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{75a\tan^2(e+fx)}{\sec(e+fx)a+a}+43\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}\right)+\frac{\cot^3(e+fx)(a\,s\cdots}{8\left(\frac{a\tan^2(\cdots}{a\sec(e+\cdots}\right.}\cdots}{a^4c^2f}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{15\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{1}{4}\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{75a\tan^2(e+fx)}{\sec(e+fx)a+a}+43\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)+\frac{\cot^3(e\cdots}{8\cdots}\cdots}{a^4c^2f}$$

$$\Big\downarrow \; 445$$

3.85. $\displaystyle\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{6}\int -\frac{3a\cot^2(e+fx)(\sec(e+fx)a+a)\left(21-\frac{43a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{43}{6}\cot^3(e+fx)(a\sec(e+fx)+\right.\right.\right.}{a^4c^2f}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(21-\frac{43a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{43}{6}\cot^3(e+fx)(a\sec(e+fx)+\right.\right.\right.}{a^4c^2f}$$

$\downarrow$ 445

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}\int \frac{a\left(\frac{21a\tan^2(e+fx)}{\sec(e+fx)a+a}+149\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)-\right.\right.\right.}{a^4c^2f}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\int \frac{\frac{21a\tan^2(e+fx)}{\sec(e+fx)a+a}+149}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)-\right.\right.\right.}{a^4c^2f}$$

$\downarrow$ 397

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(128\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-107\int \frac{}{\frac{a\tan^2(e}{\sec(e+f}}\right.\right.\right.\right.\right.}{}$$

$\downarrow$ 216

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(\frac{107\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{128\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)\right.\right.\right.\right.}{a^4c^2}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2),x]
```

3.85. $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx$

output
```
(-2*((Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(8*(2 + (a*Tan[e + f*x]^2
)/(a + a*Sec[e + f*x]))^2) + (((-43*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3
/2))/6 - (a*(-1/2*(a*((-128*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e
 + f*x]]])/Sqrt[a] + (107*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a +
a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]))) + (21*Cot[e + f*x]*Sqrt[a + a*Sec[e
 + f*x]])/2))/2)/4 + (15*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(4*(2
+ (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x]))))/8))/(a^4*c^2*f)
```

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.85. $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx$

3.85.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(321\sqrt{2}\cos(fx+e)\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\sqrt{-}\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/384/c^2/f/a^3*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(321*2^(1/2)*cos(
f*x+e)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(
f*x+e)^2-1)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-768*arctanh(sin(f*x+
e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*cos(f*x+e)+321*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*l
n(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2
-1)^(1/2))-768*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+
1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+410*cos(f*x+e)*cot(f*x+e)^3+
142*cot(f*x+e)^3-298*csc(f*x+e)*cot(f*x+e)^2-126*csc(f*x+e)^2*cot(f*x+e))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 706, normalized size of antiderivative = 2.62

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \left[-\frac{321\sqrt{2}\left(\cos(fx+e)^3+\cos(fx+e)^2-\cos(fx+e)\right.}{}\right.$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas
")
```

output
```
[-1/768*(321*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*
sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(co
s(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 384*(cos(f*x + e)^3 + c
os(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2
*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x +
e) - 4*(205*cos(f*x + e)^4 + 71*cos(f*x + e)^3 - 149*cos(f*x + e)^2 - 63*c
os(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x
+ e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*si
n(f*x + e)), 1/384*(321*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x
 + e) - 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 384*(cos(f*x + e)^3 +
cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 +
a*cos(f*x + e) - a))*sin(f*x + e) + 2*(205*cos(f*x + e)^4 + 71*cos(f*x + e
)^3 - 149*cos(f*x + e)^2 - 63*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2
*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))]
```

3.85.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)-2a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+a^2\sqrt{}}}{c^2}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4 - 2*a**2*sqrt(a*
sec(e + f*x) + a)*sec(e + f*x)**2 + a**2*sqrt(a*sec(e + f*x) + a)), x)/c**
2
```

3.85.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{5}{2}}(c\sec(fx+e)-c)^2}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima
")
```

output
```
integrate(1/((a*sec(f*x + e) + a)^(5/2)*(c*sec(f*x + e) - c)^2), x)
```

3.85.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+fx)}\right)^{5/2}\left(c-\frac{c}{\cos(e+fx)}\right)^2}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2), x)
```

3.86 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx$

3.86.1 Optimal result

Integrand size = 30, antiderivative size = 185

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx = \frac{ac^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{ac^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{ac^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} - \frac{ac(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

output
```
-1/2*a*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-1/3*
a*c*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+a*c^4*ln(co
s(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-a*c^3
*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.45

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx=\frac{ac^4(6\log(\cos(e+fx))+18\sec(e+fx)-9\sec^2(e+fx)+2\sec^3(e+fx))\tan(e+fx)}{6f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
(a*c^4*(6*Log[Cos[e + f*x]] + 18*Sec[e + f*x] - 9*Sec[e + f*x]^2 + 2*Sec[e
 + f*x]^3)*Tan[e + f*x])/(6*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e
+ f*x]])
```

3.86.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {3042, 4394, 3042, 4394, 3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{4394}$$

$$c\int \sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{5/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 4394

$$c\left(c\int \sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{3/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 3042

$$c\left(c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 4394

$$c\left(c\left(c\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c}{2f\sqrt{a\sec(e+}}\right.$$
$$\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 3042

$$c\left(c\left(c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e}{2}\right.$$
$$\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 4393

$$c\left(c\left(\frac{ac^2\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e}{2f\sqrt{a\sec(e+fx)-}}\right.$$
$$\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 25

$$c\left(c\left(-\frac{ac^2\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(}{2f\sqrt{a\sec(e+fx}}\right.$$
$$\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

3.86. $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx$

$$\Big\downarrow \text{3042}$$

$$c\left(c\left(-\frac{ac^2\tan(e+fx)\int\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(}{2f\sqrt{a\sec(e+fx}}\right.$$
$$\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3956}$$

$$c\left(c\left(\frac{ac^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(}{2f\sqrt{a\sec(e+fx)}}\right.$$
$$\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
-1/3*(a*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f
*x]]) + c*(-1/2*(a*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[a +
a*Sec[e + f*x]]) + c*((a*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a
*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (a*c*Sqrt[c - c*Sec[e + f*x]]*T
an[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])))
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

3.86.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.90

method	result
default	$\frac{c^3\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^3\left(6\cos(fx+e)^3\ln(-\cot(fx+e)+\csc(fx+e)+1)-6\cos(fx+e)^3\ln\left(\frac{2}{\cos(fx+\cdots}\right.\right.}{6f(\cos(fx+e)-1)\cdots}$
risch	$\frac{c^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,\left(-18ie^{5i(fx+e)}-3\,e^{6i(fx+e)}fx-18ie^{i(fx+e)}-6\,e^{6i(fx+e)}e-9\,e^{4i(fx+e)}fx-9ie^{4i(fx+e)}\ln(1\cdots\right.}{\cdots}$

input
```
int((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/f*c^3*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1
)^3*(6*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)+1)-6*cos(f*x+e)^3*ln(2/(cos(
f*x+e)+1))+6*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)-1)+29*cos(f*x+e)^3+18*
cos(f*x+e)^2-9*cos(f*x+e)+2)/(cos(f*x+e)-1)^3*cot(f*x+e)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.48

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx = \left[-\frac{\left(11\,c^3\cos\left(fx+e\right)^2-7\,c^3\cos\left(fx+e\right)+2\,c^3\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(fx+e\right)-3 \cdots}{6\,(\cdots}\right.$$

$$-\frac{\left(11\,c^3\cos\left(fx+e\right)^2-7\,c^3\cos\left(fx+e\right)+2\,c^3\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(fx+e\right)-6\left(c^3\cos\left(fx+e \cdots\right.\right.}{6\left(f\cos\left(fx+e\right)^3+f\cos\left(fx+ \cdots\right.\right.}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[-1/6*((11*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) + 2*c^3)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x
+ e) - 3*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*(a*c
*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x
 + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -1/6*
((11*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) + 2*c^3)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) -
 6*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a*c)*arctan(sqrt(a*c)*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e)
^3 + f*cos(f*x + e)^2)]
```

3.86.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(7/2)*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.86.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1289 vs. 2(165) = 330.

Time = 0.46 (sec) , antiderivative size = 1289, normalized size of antiderivative = 6.97

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/3*(3*(f*x + e)*c^3*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*c^3*cos(4*f*x + 4*
e)^2 + 27*(f*x + e)*c^3*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*c^3*sin(6*f*x + 6
*e)^2 + 27*(f*x + e)*c^3*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*c^3*sin(2*f*x +
 2*e)^2 + 18*(f*x + e)*c^3*cos(2*f*x + 2*e) + 3*(f*x + e)*c^3 + 18*c^3*sin
(2*f*x + 2*e) - 3*(c^3*cos(6*f*x + 6*e)^2 + 9*c^3*cos(4*f*x + 4*e)^2 + 9*c
^3*cos(2*f*x + 2*e)^2 + c^3*sin(6*f*x + 6*e)^2 + 9*c^3*sin(4*f*x + 4*e)^2
+ 18*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*c^3*sin(2*f*x + 2*e)^2 + 6*
c^3*cos(2*f*x + 2*e) + c^3 + 2*(3*c^3*cos(4*f*x + 4*e) + 3*c^3*cos(2*f*x +
 2*e) + c^3)*cos(6*f*x + 6*e) + 6*(3*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*x
 + 4*e) + 6*(c^3*sin(4*f*x + 4*e) + c^3*sin(2*f*x + 2*e))*sin(6*f*x + 6*e)
)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 6*(3*(f*x + e)*c^3*cos
(4*f*x + 4*e) + 3*(f*x + e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3 + 3*c^3*s
in(4*f*x + 4*e) + 3*c^3*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x +
e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3)*cos(4*f*x + 4*e) + 18*(c^3*sin(6*
f*x + 6*e) + 3*c^3*sin(4*f*x + 4*e) + 3*c^3*sin(2*f*x + 2*e))*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 44*(c^3*sin(6*f*x + 6*e) + 3*c^
3*sin(4*f*x + 4*e) + 3*c^3*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 18*(c^3*sin(6*f*x + 6*e) + 3*c^3*sin(4*f*x + 4*e
) + 3*c^3*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 18*((f*x + e)*c^3*sin(4*f*x + 4*e) + (f*x + e)*c^3*sin(2*f*x +...
```

3.86.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx=\int \sqrt{a\sec\left(fx+e\right)+a}\left(-c\sec\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx=\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{7/2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(7/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(7/2), x)
```

3.87 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx$

3.87.1 Optimal result

Integrand size = 30, antiderivative size = 139

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \frac{ac^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{ac^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{ac(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

output
```
-1/2*a*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+a*c^3*
ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-
a*c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.52

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{ac^3(-2\log(\cos(e+fx))-4\sec(e+fx)+\sec^2(e+fx))\tan(e+fx)}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-1/2*(a*c^3*(-2*Log[Cos[e + f*x]] - 4*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e
 + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.87.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4394, 3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4394}$$

$$c\int \sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{3/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4394}$$

$$c\left(c\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$c\left(c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4393}$$

$$c\left(\frac{ac^2\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$c\left(-\frac{ac^2\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$c\left(-\frac{ac^2\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3956}$$

$$c\left(\frac{ac^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-1/2*(a*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f
*x]]) + c*((a*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*
x]]*Sqrt[c - c*Sec[e + f*x]]) - (a*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x]
)/(f*Sqrt[a + a*Sec[e + f*x]]))
```

3.87. $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx$

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

3.87.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.13

method	result
default	$-\frac{c^2(\sec(fx+e)-1)^2\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{a(\sec(fx+e)+1)}\left(2\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e)-1)-2\cos(fx+e)^2\ln\left(\frac{}{\cos(}\right.\right.}{2f(\cos(fx+e)-1)^2}$
risch	$-\frac{c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(ie^{4i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+e^{4i(fx+e)}fx+2ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+2e^{4i(fx+e)}e+2\right.}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\left(1+e^{2i(fx+e}\right.}$

input
```
int((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*c^2*(sec(f*x+e)-1)^2*(-c*(sec(f*x+e)-1))^(1/2)*(a*(sec(f*x+e)+1))^(
1/2)*(2*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)-1)-2*cos(f*x+e)^2*ln(2/(cos
(f*x+e)+1))+2*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+5*cos(f*x+e)^2+4*c
os(f*x+e)-1)/(cos(f*x+e)-1)^2*cot(f*x+e)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.06

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \left[-\frac{(3\,c^2\cos(fx+e)-c^2)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)-\left(c^2\cos(fx+e)^2+c^2\cos\right.}{2\left(f\cos(fx+\right.}\right.$$

$$-\frac{(3\,c^2\cos(fx+e)-c^2)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)-2\left(c^2\cos(fx+e)^2+c^2\cos(fx+e)\right)\sqrt{a}}{2\left(f\cos(fx+e)^2+f\cos(fx+e)\right)}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[-1/2*((3*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - (c^2*cos(f*x + e)^2
 + c^2*cos(f*x + e))*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e
)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqr
t((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/
(f*cos(f*x + e)^2 + f*cos(f*x + e)), -1/2*((3*c^2*cos(f*x + e) - c^2)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)
)*sin(f*x + e) - 2*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(a*c)*arcta
n(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) -
 c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(
f*cos(f*x + e)^2 + f*cos(f*x + e))]
```

3.87.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(5/2)*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.87.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(125) = 250$.

Time = 0.39 (sec) , antiderivative size = 710, normalized size of antiderivative = 5.11

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx =$$
$$-\frac{\left((fx+e)c^2\cos\left(4\,fx+4\,e\right)^2+4\,(fx+e)c^2\cos\left(2\,fx+2\,e\right)^2+(fx+e)c^2\sin\left(4\,fx+4\,e\right)^2+4\,(fx+e)c^2\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*c^2*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*c^2*cos(2*f*x + 2*e)^2 +
(f*x + e)*c^2*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*c^2*sin(2*f*x + 2*e)^2 + 4*
(f*x + e)*c^2*cos(2*f*x + 2*e) + (f*x + e)*c^2 + 2*c^2*sin(2*f*x + 2*e) -
(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + c^2*sin(4*f*x + 4*e)^
2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4
*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x +
 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*c^
2*cos(2*f*x + 2*e) + (f*x + e)*c^2 + c^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e
) + 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f
*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*
x + e)*c^2*sin(2*f*x + 2*e) - c^2*cos(2*f*x + 2*e))*sin(4*f*x + 4*e) - 4*(
c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*
x + 2*e) + c^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt
(a)*sqrt(c)/((2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*
e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(
2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

3.87.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c - c\sec(e+fx))^{5/2}\,dx = \int \sqrt{a\sec\left(fx+e\right)+a}(-c\sec\left(fx+e\right)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx=\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{5/2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2), x)
```

3.88 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx$

3.88.1 Optimal result

Integrand size = 30, antiderivative size = 93

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\frac{ac^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{ac\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
a*c^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)-a*c*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\frac{ac^2(\log(\cos(e+fx))+\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a*c^2*(Log[Cos[e + f*x]] + Sec[e + f*x])*Tan[e + f*x])/(f*Sqrt[a*(1 + Sec
[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.88.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4394}$$

$$c\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4393}$$

$$\frac{ac^2\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{ac^2\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ac^2\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{ac^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
- c*Sec[e + f*x]]) - (a*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a
 + a*Sec[e + f*x]])
```

3.88.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

3.88.4 Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.42

method	result
default	$\frac{c\sqrt{a(\sec(fx+e)+1)}\,(\sec(fx+e)-1)\sqrt{-c(\sec(fx+e)-1)}\,\left(\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)-1)+\cos(fx+e)\ln(-\cot(fx+e)+c\right.}{f(\cos(fx+e)-1)}$
risch	$\frac{c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(1+e^{2i(fx+e)})\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,x}{(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)}-\frac{2c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(1+e^{2i(fx+e)})\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(fx+e)}{(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)f}-\frac{2ic\sqrt{}}{}$

input
```
int((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*c*(a*(sec(f*x+e)+1))^(1/2)*(sec(f*x+e)-1)*(-c*(sec(f*x+e)-1))^(1/2)*(c
os(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)-1)+cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e
)+1)-cos(f*x+e)*ln(2/(cos(f*x+e)+1))+cos(f*x+e)+1)/(cos(f*x+e)-1)*cot(f*x+
e)
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.76

$$\int\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\left[-\frac{2\,c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)-\sqrt{-ac}(c\cos(fx+e)+c)\log\left(\frac{ac\cos(fx+e)^4-(}{}\right.}{2\,(f\cos(fx+e)+f)}\right.$$

$$-\frac{c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)-\sqrt{ac}(c\cos(fx+e)+c)\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\,\mathrm{co}}{ac\cos(fx+e)^2+ac}\right.}{f\cos(fx+e)+f}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[-1/2*(2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e))*sin(f*x + e) - sqrt(-a*c)*(c*cos(f*x + e) + c)*log(1/2*(a
*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f
*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e) + f), -(c*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x +
e) - sqrt(a*c)*(c*cos(f*x + e) + c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*si
n(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e) + f)]
```

3.88.6 Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\int\sqrt{a\left(\sec\left(e+fx\right)+1\right)}(-c(\sec\left(e+fx\right)-1))^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2), x)
```

3.88.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(85) = 170$.

Time = 0.40 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.61

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=-\frac{\left((fx+e)c\cos\left(2\,fx+2\,e\right)^2+(fx+e)c\sin\left(2\,fx+2\,e\right)^2+2\left(fx+e\right)c\cos\left(2\,fx+2\,e\right)+2\,c\cos\left(\frac{1}{2}\arctan\right.\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*c*cos(2*f*x + 2*e)^2 + (f*x + e)*c*sin(2*f*x + 2*e)^2 + 2*(f*x
 + e)*c*cos(2*f*x + 2*e) + 2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))*sin(2*f*x + 2*e) + (f*x + e)*c - (c*cos(2*f*x + 2*e)^2 + c*sin(2
*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1) - 2*(c*cos(2*f*x + 2*e) + c)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

3.88.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\int \sqrt{a\sec(fx+e)+a}(-c\sec(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\int \sqrt{a+\frac{a}{\cos(e+fx)}}\left(c-\frac{c}{\cos(e+fx)}\right)^{3/2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2), x)
```

3.89 $\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$

3.89.1 Optimal result

Integrand size = 30, antiderivative size = 48

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx=\frac{ac\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
a*c*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx =\frac{c\log(\cos(e+fx))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(c*Log[Cos[e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[
c - c*Sec[e + f*x]])
```

3.89.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4393}$$

$$\frac{ac\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{ac\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ac\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{ac\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(a*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
c*Sec[e + f*x]])
```

3.89.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

3.89.4 Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.81

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-c(\sec(fx+e)-1)}\,\left(\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\ln(-\cot(fx+e)+\csc(fx+e)-1)\right) c$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}\,\left(1+e^{2i(fx+e)}\right)x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)}-\frac{2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}\,\left(1+e^{2i(fx+e)}\right)(fx+e)}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}-\frac{i\sqrt{\frac{a\left(e^{i(\right.}}{1+e}}$

input
```
int((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2)*(ln(-cot(f*x+e)+cs
c(f*x+e)+1)-ln(2/(cos(f*x+e)+1))+ln(-cot(f*x+e)+csc(f*x+e)-1))*cot(f*x+e)
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(44) = 88.

Time = 0.33 (sec) , antiderivative size = 200, normalized size of antiderivative = 4.17

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{-ac}\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)+ac}{2\cos(fx+e)^2}\right)}{2f}, \frac{\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\ldots}}{\ldots}\right)}{\ldots}\right.$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[1/2*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x +
e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2)/f, sqrt(a*c)*arct
an(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c))/f
]
```

3.89.6 Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$$
$$= \int \sqrt{a(\sec(e+fx)+1)}\sqrt{-c(\sec(e+fx)-1)}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1)), x)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{(fx+e-\arctan(\sin(2\,fx+2\,e),\cos(2\,fx+2\,e)+1))\sqrt{a}\sqrt{c}}{f}$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
-(f*x + e - arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sqrt(a)*sqrt(
c)/f
```

3.89.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = \int \sqrt{a\sec(fx+e)+a}\sqrt{-c\sec(fx+e)+c}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2), x)
```

3.90 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$

3.90.1 Optimal result

Integrand size = 30, antiderivative size = 51

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a\log(1-\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
a*ln(1-cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{(\log(\cos(e+fx))+\log(1-\sec(e+fx)))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
((Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]])*Sqrt[a*(1 + Sec[e + f*x])]*Ta
n[(e + f*x)/2])/(f*Sqrt[c - c*Sec[e + f*x]])
```

3.90.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4399}$$

$$\frac{ac\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
- c*Sec[e + f*x]])
```

3.90.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.90.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

method	result
default	$\frac{\sqrt{a(\sec(fx+e)+1)}\left(\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\ln(-\cot(fx+e)+\csc(fx+e))\right)(\cot(fx+e)-\csc(fx+e))}{f\sqrt{-c(\sec(fx+e)-1)}}$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)x}{(e^{i(fx+e)}+1)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}}-\frac{2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)(fx+e)}{(e^{i(fx+e)}+1)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}-\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)\ln(e^{i(fx+e)}-}{(e^{i(fx+e)}+1)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(sec(f*x+e)+1))^(1/2)*(ln(2/(cos(f*x+e)+1))-2*ln(-cot(f*x+e)+csc(f*
x+e)))/(-c*(sec(f*x+e)-1))^(1/2)*(cot(f*x+e)-csc(f*x+e))
```

3.90.5 Fricas [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x +
e) - c), x)
```

3.90.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/sqrt(-c*(sec(e + f*x) - 1)), x)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\frac{2\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{c}} - \frac{\sqrt{-a}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{c}}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
(2*sqrt(-a)*log(sin(f*x + e)/(cos(f*x + e) + 1))/sqrt(c) - sqrt(-a)*log(si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c))/f
```

3.90.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(1/2), x)
```

3.91 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx$

3.91.1 Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx=-\frac{a\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{a\log(1-\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a*ln(1-cos(f
*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx=\frac{a\left(\log(\cos(e+fx))+\log(1-\sec(e+fx))+\frac{1}{-1+\sec(e+fx)}\right)\tan(e+fx)}{cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a*(Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] + (-1 + Sec[e + f*x])^(-1))*
Tan[e + f*x])/(c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.91.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4395}$$

$$\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2))
) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

3.91.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.91.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\left(4\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e))-2\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-4\ln(-\cot(fx+e)+\csc(fx+e))+2\ln\left(\frac{2}{\cos(fx+e)+1}\right)-\right.}{2f\sqrt{-c(\sec(fx+e)-1)}\,c(\sec(fx+e)-1)(\cos(fx+e)+1)}$
risch	$-\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(2ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+e^{2i(fx+e)}fx-4ie^{i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+2\,e^{2i(fx+e)}e-2\,e^{i(fx+e)}fx-2ie^{i(fx+e}\right.}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*(4*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e))-2*cos(f*x+e)*ln(2/(cos(f*x
+e)+1))-4*ln(-cot(f*x+e)+csc(f*x+e))+2*ln(2/(cos(f*x+e)+1))-cos(f*x+e)-1)*
(a*(sec(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/c/(sec(f*x+e)-1)/(cos(f
*x+e)+1)*tan(f*x+e)
```

3.91.5 Fricas [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^{3/2}}\, dx = \int \frac{\sqrt{a\sec\left(fx + e\right) + a}}{\left(-c\sec\left(fx + e\right) + c\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x +
 e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

3.91.6 Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^{3/2}}\, dx = \int \frac{\sqrt{a\left(\sec\left(e + fx\right) + 1\right)}}{\left(-c\left(\sec\left(e + fx\right) - 1\right)\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/(-c*(sec(e + f*x) - 1))**(3/2), x)
```

3.91.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(88) = 176$.

Time = 0.37 (sec) , antiderivative size = 399, normalized size of antiderivative = 4.16

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^{3/2}}\, dx =$$
$$-\frac{\left((fx + e)\cos\left(2\,fx + 2\,e\right)^2 + 4\left(fx + e\right)\cos\left(fx + e\right)^2 + \left(fx + e\right)\sin\left(2\,fx + 2\,e\right)^2 + 4\left(fx + e\right)\sin\left(fx + e\right.\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*cos(2*f*x + 2*e)^2 + 4*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*si
n(2*f*x + 2*e)^2 + 4*(f*x + e)*sin(f*x + e)^2 + f*x + 2*(2*(2*cos(f*x + e)
 - 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(f*x + e)^2 - sin(2*f*x
 + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e)^2 + 4*cos(f*x
 + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*(f*x - 2*(f*x + e)*
cos(f*x + e) + e + sin(f*x + e))*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f*x +
e) - 2*(2*(f*x + e)*sin(f*x + e) + cos(f*x + e))*sin(2*f*x + 2*e) + e + 2*
sin(f*x + e))*sqrt(a)*sqrt(c)/((c^2*cos(2*f*x + 2*e)^2 + 4*c^2*cos(f*x + e
)^2 + c^2*sin(2*f*x + 2*e)^2 - 4*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*c^2
*sin(f*x + e)^2 - 4*c^2*cos(f*x + e) + c^2 - 2*(2*c^2*cos(f*x + e) - c^2)*
cos(2*f*x + 2*e))*f)
```

3.91.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(3/2), x)
```

3.92 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx$

3.92.1 Optimal result

Integrand size = 30, antiderivative size = 142

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-1/2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a*tan(f*
x+e)/c/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a*ln(1-cos(f*x+e))*
tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\left(-2\log(\cos(e+fx))-2\log(1-\sec(e+fx))+\frac{3-2\sec(e+fx)}{(-1+\sec(e+fx))^2}\right)\tan(e+fx)}{2c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-1/2*(a*(-2*Log[Cos[e + f*x]] - 2*Log[1 - Sec[e + f*x]] + (3 - 2*Sec[e + f
*x])/(-1 + Sec[e + f*x])^2)*Tan[e + f*x])/(c^2*f*Sqrt[a*(1 + Sec[e + f*x])
]*Sqrt[c - c*Sec[e + f*x]])
```

3.92.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4395, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a \sec(e+fx)+a}}{(c-c \sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a \csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c \csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4395}$$

$$\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c \sec(e+fx))^{3/2}}dx}{c}-\frac{a \tan(e+fx)}{2 f \sqrt{a \sec(e+fx)+a}(c-c \sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c \csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{a \tan(e+fx)}{2 f \sqrt{a \sec(e+fx)+a}(c-c \sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4395}$$

$$\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c \sec(e+fx)}}dx}{c}-\frac{a \tan(e+fx)}{f \sqrt{a \sec(e+fx)+a}(c-c \sec(e+fx))^{3/2}}}{c}-\frac{a \tan(e+fx)}{2 f \sqrt{a \sec(e+fx)+a}(c-c \sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/
2)) + (-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])
^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]]))/c
```

3.92.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.92.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.67

method	result
default	$-\frac{\sqrt{2}\,\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(8\ln\left((1-\cos(fx+e))^2\csc(fx+e)^2+1\right)(1-\cos(fx+e))^4\csc(fx+e)^4-16\ln(-\right.}{16f\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^2\left(\frac{c(1-\cos(f}{(1-\cos(fx-}\right.}$
risch	$-\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(-4ie^{i(fx+e)}+e^{4i(fx+e)}fx-8ie^{3i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+2\,e^{4i(fx+e)}e-4\,e^{3i(fx+e)}fx+12ie^{2i(fx+e)}\ln\left(e^{i(fx+e}\right.\right.}{}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/f*2^(1/2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f*x
+e))^2*csc(f*x+e)^2-1)^2/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^
2-1)*csc(f*x+e)^2)^(5/2)*(1-cos(f*x+e))*(8*ln((1-cos(f*x+e))^2*csc(f*x+e)^
2+1)*(1-cos(f*x+e))^4*csc(f*x+e)^4-16*ln(-cot(f*x+e)+csc(f*x+e))*(1-cos(f*
x+e))^4*csc(f*x+e)^4-6*(1-cos(f*x+e))^2*csc(f*x+e)^2+1)*csc(f*x+e)
```

3.92.5 Fricas [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{a\sec(fx+e)+a}}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x
+ e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

3.92.6 Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^{5/2}}\, dx = \int \frac{\sqrt{a\left(\sec\left(e + fx\right) + 1\right)}}{\left(-c\left(\sec\left(e + fx\right) - 1\right)\right)^{\frac{5}{2}}}\, dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/(-c*(sec(e + f*x) - 1))**(5/2), x)
```

3.92.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(128) = 256$.

Time = 0.48 (sec) , antiderivative size = 1173, normalized size of antiderivative = 8.26

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(3*f*x + 3*e)^2 + 36*(f*x
 + e)*cos(2*f*x + 2*e)^2 + 16*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin(4*f
*x + 4*e)^2 + 16*(f*x + e)*sin(3*f*x + 3*e)^2 + 36*(f*x + e)*sin(2*f*x + 2
*e)^2 + 16*(f*x + e)*sin(f*x + e)^2 + f*x + 2*(2*(4*cos(3*f*x + 3*e) - 6*c
os(2*f*x + 2*e) + 4*cos(f*x + e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^
2 + 8*(6*cos(2*f*x + 2*e) - 4*cos(f*x + e) + 1)*cos(3*f*x + 3*e) - 16*cos(
3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - 36*cos(2*f*x +
 2*e)^2 - 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) +
 2*sin(f*x + e))*sin(4*f*x + 4*e) - sin(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x +
 2*e) - 2*sin(f*x + e))*sin(3*f*x + 3*e) - 16*sin(3*f*x + 3*e)^2 - 36*sin(
2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e) - 16*sin(f*x + e)^2 + 8*
cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*(f*x - 4*(f*
x + e)*cos(3*f*x + 3*e) + 6*(f*x + e)*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f
*x + e) + e + 2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*co
s(4*f*x + 4*e) - 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f
*x + e) + e)*cos(3*f*x + 3*e) + 12*(f*x - 4*(f*x + e)*cos(f*x + e) + e)*co
s(2*f*x + 2*e) - 8*(f*x + e)*cos(f*x + e) - 2*(4*(f*x + e)*sin(3*f*x + 3*e
) - 6*(f*x + e)*sin(2*f*x + 2*e) + 4*(f*x + e)*sin(f*x + e) + 2*cos(3*f*x
+ 3*e) - 3*cos(2*f*x + 2*e) + 2*cos(f*x + e))*sin(4*f*x + 4*e) - 4*(12*(f*
x + e)*sin(2*f*x + 2*e) - 8*(f*x + e)*sin(f*x + e) - 1)*sin(3*f*x + 3*e...
```

3.92.8 Giac [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^{5/2}} \, dx = \int \frac{\sqrt{a\sec(fx + e) + a}}{(-c\sec(fx + e) + c)^{\frac{5}{2}}} \, dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(5/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(5/2), x)
```

3.93 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx$

3.93.1 Optimal result

Integrand size = 30, antiderivative size = 188

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx =& -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}\\ &-\frac{a\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\ &-\frac{a\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\\ &+\frac{a\log(1-\cos(e+fx))\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output
```
-1/3*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a*ta
n(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a*tan(f*x+e)/c^
2/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a*ln(1-cos(f*x+e))*tan(f
*x+e)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a\left(-6\log(\cos(e+fx))-6\log(1-\sec(e+fx))+\frac{-11+15\sec(e+fx)-6\sec^2(e+fx)}{(-1+\sec(e+fx))^3}\right)\tan(e+fx)}{6c^3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
-1/6*(a*(-6*Log[Cos[e + f*x]] - 6*Log[1 - Sec[e + f*x]] + (-11 + 15*Sec[e
+ f*x] - 6*Sec[e + f*x]^2)/(-1 + Sec[e + f*x])^3)*Tan[e + f*x])/(c^3*f*Sqr
t[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.93.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4395, 3042, 4395, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c-c\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4395}$$

$$\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{5/2}}dx}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc(e+fx+\frac{\pi}{2})a+a}}{(c-c\csc(e+fx+\frac{\pi}{2}))^{5/2}}dx}{c} - \frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4395}$$

$$\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{3/2}}dx}{c} - \frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c} - \frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c} - \frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c} - \frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4395}$$

$$\frac{\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c} - \frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c} - \frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c} - \frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c} - \frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c} - \frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c} - \frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{\frac{\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c} - \frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c} - \frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{16}$$

3.93. $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx$

$$\frac{\frac{\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
-1/3*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/
2)) + (-1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(5/2)) + (-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e
+ f*x])^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*S
ec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))/c)/c
```

3.93.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.93.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.38

method	result
default	$\frac{\sqrt{2}\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(48\ln(-\cot(fx+e)+\csc(fx+e))(1-\cos(fx+e))^6\csc(fx+e)^6-24\ln\left((1-\cos(f\right.\right.}{48f\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right.}$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)x}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}}-\frac{2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)(fx+e)}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}+\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(9\,e^{5i(fx+e)}-27\,e^{4i(fx+e)}\right.}{3c^3\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right.}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f*2^(1/2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^3/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2
-1)*csc(f*x+e)^2)^(7/2)*(1-cos(f*x+e))*(48*ln(-cot(f*x+e)+csc(f*x+e))*(1-c
os(f*x+e))^6*csc(f*x+e)^6-24*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos(f*
x+e))^6*csc(f*x+e)^6+21*(1-cos(f*x+e))^4*csc(f*x+e)^4-6*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+1)*csc(f*x+e)
```

3.93.5 Fricas [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx=\int \frac{\sqrt{a\sec(fx+e)+a}}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x +
 e)^4 - 4*c^4*sec(f*x + e)^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) +
 c^4), x)
```

3.93.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.93.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2444 vs. $2(168) = 336$.

Time = 1.91 (sec) , antiderivative size = 2444, normalized size of antiderivative = 13.00

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxi
ma")
```

output
```
-1/3*(3*(f*x + e)*cos(6*f*x + 6*e)^2 + 108*(f*x + e)*cos(5*f*x + 5*e)^2 +
675*(f*x + e)*cos(4*f*x + 4*e)^2 + 1200*(f*x + e)*cos(3*f*x + 3*e)^2 + 675
*(f*x + e)*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*cos(f*x + e)^2 + 3*(f*x + e)
*sin(6*f*x + 6*e)^2 + 108*(f*x + e)*sin(5*f*x + 5*e)^2 + 675*(f*x + e)*sin
(4*f*x + 4*e)^2 + 1200*(f*x + e)*sin(3*f*x + 3*e)^2 + 675*(f*x + e)*sin(2*
f*x + 2*e)^2 + 108*(f*x + e)*sin(f*x + e)^2 + 3*f*x + 6*(2*(6*cos(5*f*x +
5*e) - 15*cos(4*f*x + 4*e) + 20*cos(3*f*x + 3*e) - 15*cos(2*f*x + 2*e) + 6
*cos(f*x + e) - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 + 12*(15*cos(4*f*
x + 4*e) - 20*cos(3*f*x + 3*e) + 15*cos(2*f*x + 2*e) - 6*cos(f*x + e) + 1)
*cos(5*f*x + 5*e) - 36*cos(5*f*x + 5*e)^2 + 30*(20*cos(3*f*x + 3*e) - 15*c
os(2*f*x + 2*e) + 6*cos(f*x + e) - 1)*cos(4*f*x + 4*e) - 225*cos(4*f*x + 4
*e)^2 + 40*(15*cos(2*f*x + 2*e) - 6*cos(f*x + e) + 1)*cos(3*f*x + 3*e) - 4
00*cos(3*f*x + 3*e)^2 + 30*(6*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - 225*cos
(2*f*x + 2*e)^2 - 36*cos(f*x + e)^2 + 2*(6*sin(5*f*x + 5*e) - 15*sin(4*f*x
 + 4*e) + 20*sin(3*f*x + 3*e) - 15*sin(2*f*x + 2*e) + 6*sin(f*x + e))*sin(
6*f*x + 6*e) - sin(6*f*x + 6*e)^2 + 12*(15*sin(4*f*x + 4*e) - 20*sin(3*f*x
 + 3*e) + 15*sin(2*f*x + 2*e) - 6*sin(f*x + e))*sin(5*f*x + 5*e) - 36*sin(
5*f*x + 5*e)^2 + 30*(20*sin(3*f*x + 3*e) - 15*sin(2*f*x + 2*e) + 6*sin(f*x
 + e))*sin(4*f*x + 4*e) - 225*sin(4*f*x + 4*e)^2 + 120*(5*sin(2*f*x + 2*e)
 - 2*sin(f*x + e))*sin(3*f*x + 3*e) - 400*sin(3*f*x + 3*e)^2 - 225*sin(...
```

3.93.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx=\int \frac{\sqrt{a\sec(fx+e)+a}}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^{7/2}}\,dx = \int \frac{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}{\left(c - \frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(7/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(7/2), x)
```

3.94 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$

3.94.1 Optimal result

Integrand size = 30, antiderivative size = 190

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{a^2c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{a^2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{a^2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} + \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

output
```
-1/2*a^2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+1/3*
a^2*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+a^2*c^3*ln(
cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-a^2
*c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.46

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx=\frac{a^2c^3(2+6\log(\cos(e+fx))+6\sec(e+fx)+3\sec^2(e+fx)-2\sec^3(e+fx))\tan(e+fx)}{6f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
(a^2*c^3*(2 + 6*Log[Cos[e + f*x]] + 6*Sec[e + f*x] + 3*Sec[e + f*x]^2 - 2*
Sec[e + f*x]^3)*Tan[e + f*x])/(6*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*S
ec[e + f*x]])
```

3.94.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {3042, 4397, 3042, 4394, 3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4397}$$

$$a\int \sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{5/2}dx+\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4394}$$

$$a\left(c\int \sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{3/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}\right)+\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}\right)+\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4394}$$

$$a\left(c\left(c\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c}{2f\sqrt{a\sec(e+}}\right.$$
$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(c\left(c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(}{2}\right.$$
$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4393}$$

$$a\left(c\left(\frac{ac^2\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e}{2f\sqrt{a\sec(e+fx)}}\right.$$
$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$a\left(c\left(-\frac{ac^2\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec}{2f\sqrt{a\sec(e+fx}}\right.$$
$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

3.94. $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$

$$\Big\downarrow \text{3042}$$

$$a\left(c\left(-\frac{ac^2\tan(e+fx)\int\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}\right)+\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}+a\left(c\left(\frac{ac^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}\right)$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
(a^2*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]
]) + a*(-1/2*(a*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[a + a*S
ec[e + f*x]]) + c*((a*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Se
c[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (a*c*Sqrt[c - c*Sec[e + f*x]]*Tan[
e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])))
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

rule 4397
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-2*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a   Int[Sqrt[a +
b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

3.94.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.87

method	result
default	$-\frac{a\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^2 c^2 \sqrt{a(\sec(fx+e)+1)}\,\left(6\cos(fx+e)^3\ln(-\cot(fx+e)+\csc(fx+e)+1)+6\cos(fx+e)^3\ln(-\text{co}\right.}{6f(\cos(fx+e)-}$
risch	$-\frac{a\,c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(6ie^{5i(fx+e)}+3\,e^{6i(fx+e)}fx+6ie^{i(fx+e)}+6\,e^{6i(fx+e)}e+9\,e^{4i(fx+e)}fx+6ie^{4i(fx+e)}+18\,\text{e}\right.}{}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/f*a*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^2*c^2*(a*(sec(f*x+e)+1))
^(1/2)*(6*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)+1)+6*cos(f*x+e)^3*ln(-cot
(f*x+e)+csc(f*x+e)-1)-6*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+cos(f*x+e)^3+6*c
os(f*x+e)^2+3*cos(f*x+e)-2)/(cos(f*x+e)-1)^2*csc(f*x+e)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.46

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \left[-\frac{\left(7\,ac^2\cos\left(fx+e\right)^2+ac^2\cos\left(fx+e\right)-2\,ac^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(f\right.}{}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
[-1/6*((7*a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 2*a*c^2)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f
*x + e) - 3*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a*c)*log(1
/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*
sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)
, -1/6*((7*a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 2*a*c^2)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(
f*x + e) - 6*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(a*c)*arcta
n(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) -
 c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(
f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

3.94.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.94.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1356 vs. $2(170) = 340$.

Time = 0.48 (sec) , antiderivative size = 1356, normalized size of antiderivative = 7.14

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
-1/3*(3*(f*x + e)*a*c^2*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*a*c^2*cos(4*f*x
+ 4*e)^2 + 27*(f*x + e)*a*c^2*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*a*c^2*sin(6
*f*x + 6*e)^2 + 27*(f*x + e)*a*c^2*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*a*c^2
*sin(2*f*x + 2*e)^2 + 18*(f*x + e)*a*c^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a*
c^2 - 6*a*c^2*sin(2*f*x + 2*e) - 3*(a*c^2*cos(6*f*x + 6*e)^2 + 9*a*c^2*cos
(4*f*x + 4*e)^2 + 9*a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(6*f*x + 6*e)^2 +
9*a*c^2*sin(4*f*x + 4*e)^2 + 18*a*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
9*a*c^2*sin(2*f*x + 2*e)^2 + 6*a*c^2*cos(2*f*x + 2*e) + a*c^2 + 2*(3*a*c^2
*cos(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*cos(6*f*x + 6*e) + 6
*(3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*cos(4*f*x + 4*e) + 6*(a*c^2*sin(4*f*x
+ 4*e) + a*c^2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1) + 6*(3*(f*x + e)*a*c^2*cos(4*f*x + 4*e) + 3*(f*x
+ e)*a*c^2*cos(2*f*x + 2*e) + (f*x + e)*a*c^2 - a*c^2*sin(4*f*x + 4*e) - a
*c^2*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x + e)*a*c^2*cos(2*f*x
+ 2*e) + (f*x + e)*a*c^2)*cos(4*f*x + 4*e) + 6*(a*c^2*sin(6*f*x + 6*e) + 3
*a*c^2*sin(4*f*x + 4*e) + 3*a*c^2*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4
*f*x + 4*e) + 3*a*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 6*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e)
+ 3*a*c^2*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x...
```

3.94.8 Giac [F]

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^{5/2}\,dx = \int (a\sec(fx + e) + a)^{\frac{3}{2}}(-c\sec(fx + e) + c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^{5/2}\,dx = \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^{3/2}\left(c - \frac{c}{\cos(e + f\,x)}\right)^{5/2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2), x)
```

3.95 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$

3.95.1 Optimal result

Integrand size = 30, antiderivative size = 103

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{a^2c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{a^2c^2\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
a^2*c^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)+1/2*a^2*c^2*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.64

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{a^2c^2(2\log(\cos(e+fx))+\sec^2(e+fx))\tan(e+fx)}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a^2*c^2*(2*Log[Cos[e + f*x]] + Sec[e + f*x]^2)*Tan[e + f*x])/(2*f*Sqrt[a*
(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.95.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.67, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 4393, 25, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4393}$$

$$-\frac{a^2c^2\tan(e+fx)\int -\tan^3(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{a^2c^2\tan(e+fx)\int \tan^3(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2c^2\tan(e+fx)\int \tan(e+fx)^3dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{a^2c^2\tan(e+fx)\left(\frac{\tan^2(e+fx)}{2f}-\int \tan(e+fx)dx\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2c^2\tan(e+fx)\left(\frac{\tan^2(e+fx)}{2f}-\int \tan(e+fx)dx\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{a^2c^2\tan(e+fx)\left(\frac{\tan^2(e+fx)}{2f}+\frac{\log(\cos(e+fx))}{f}\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a^2*c^2*Tan[e + f*x]*(Log[Cos[e + f*x]]/f + Tan[e + f*x]^2/(2*f)))/(Sqrt[
a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

3.95.4 Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.39

method	result
default	$\frac{a\left(2\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e)-1)+2\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e)+1)-2\cos(fx+e)^2\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\mathrm{si}\right.}{2f(\cos(fx+e)-1)}$
risch	$-\frac{ac\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\left(i\mathrm{e}^{4i(fx+e)}\ln\left(1+\mathrm{e}^{2i(fx+e)}\right)+\mathrm{e}^{4i(fx+e)}fx+2\,\mathrm{e}^{4i(fx+e)}e+2i\mathrm{e}^{2i(fx+e)}\ln\left(1+\mathrm{e}^{2i(fx+e)}\right)+2\right.}{\left(1+\mathrm{e}^{2i(fx+e)}\right)\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)f}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*a*(2*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)-1)+2*cos(f*x+e)^2*ln(-co
t(f*x+e)+csc(f*x+e)+1)-2*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+sin(f*x+e)^2)*(
-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)*c*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*
x+e)-1)*csc(f*x+e)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 346, normalized size of antiderivative = 3.36

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \left[\frac{\sqrt{-ac}ac\cos(fx+e)\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+}{\cos(fx}}}{2\cos(fx+e)^2}\right.}{2f\cos(fx+e)}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
[1/2*(sqrt(-a*c)*a*c*cos(f*x + e)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x +
 e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2)
 - a*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)), 1/2*(2*sqrt(a*c)*a*c*arctan(s
qrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c))*cos(f
*x + e) - a*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e))]
```

3.95.6 Sympy [F]

$$\int (a + a\sec(e+fx))^{3/2}(c - c\sec(e+fx))^{3/2}\,dx = \int \left(a(\sec{(e+fx)}+1)\right)^{\frac{3}{2}}\left(-c(\sec{(e+fx)}-1)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*(-c*(sec(e + f*x) - 1))**(3/2), x)
```

3.95.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(93) = 186$.

Time = 0.40 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.63

$$\int (a + a\sec(e+fx))^{3/2}(c - c\sec(e+fx))^{3/2}\,dx = -\frac{\left((fx+e)ac\cos\left(4\,fx+4\,e\right)^2 + 4\,(fx+e)ac\cos\left(2\,fx+2\,e\right)^2 + (fx+e)ac\sin\left(4\,fx+4\,e\right)^2 + 4\,(fx+e)a\cdots\right.}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a*c*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*a*c*cos(2*f*x + 2*e)^2 +
(f*x + e)*a*c*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*a*c*sin(2*f*x + 2*e)^2 + 4*
(f*x + e)*a*c*cos(2*f*x + 2*e) + (f*x + e)*a*c - 2*a*c*sin(2*f*x + 2*e) -
(a*c*cos(4*f*x + 4*e)^2 + 4*a*c*cos(2*f*x + 2*e)^2 + a*c*sin(4*f*x + 4*e)^
2 + 4*a*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a*c*sin(2*f*x + 2*e)^2 + 4
*a*c*cos(2*f*x + 2*e) + a*c + 2*(2*a*c*cos(2*f*x + 2*e) + a*c)*cos(4*f*x +
 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*a*
c*cos(2*f*x + 2*e) + (f*x + e)*a*c - a*c*sin(2*f*x + 2*e))*cos(4*f*x + 4*e
) + 2*(2*(f*x + e)*a*c*sin(2*f*x + 2*e) + a*c*cos(2*f*x + 2*e))*sin(4*f*x
+ 4*e))*sqrt(a)*sqrt(c)/((2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + co
s(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*
f)
```

3.95.8 Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2), x)
```

3.96 $\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx$

3.96.1 Optimal result

Integrand size = 30, antiderivative size = 93

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = \frac{a^2c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{ac\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

output
```
a^2*c*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)-a*c*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{ac\sqrt{a(1+\sec(e+fx))}(-\log(\cos(e+fx))+\sec(e+fx))\tan(e+fx)}{f(1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-((a*c*Sqrt[a*(1 + Sec[e + f*x])]*(-Log[Cos[e + f*x]] + Sec[e + f*x])*Tan[
e + f*x])/(f*(1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]))
```

3.96.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4394}$$

$$a\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4393}$$

$$\frac{a^2c\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2c\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2c\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{a^2 c \tan(e+fx) \log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(a^2*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
- c*Sec[e + f*x]]) - (a*c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c
 - c*Sec[e + f*x]])
```

3.96.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

3.96.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.27

method	result
default	$-\frac{a\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{a(\sec(fx+e)+1)}\,\left(\ln(-\cot(fx+e)+\csc(fx+e)-1)\cot(fx+e)+\ln(-\cot(fx+e)+\csc(fx+e)+1)\cot(fx+e\right.}{f}$
risch	$\frac{a(1+e^{2i(fx+e)})\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}\,x}{(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)} - \frac{2a(1+e^{2i(fx+e)})\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}\,(fx+e)}{(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)f} + \frac{2ia\sqrt{}}{}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*a*(-c*(sec(f*x+e)-1))^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*(ln(-cot(f*x+e)+
csc(f*x+e)-1)*cot(f*x+e)+ln(-cot(f*x+e)+csc(f*x+e)+1)*cot(f*x+e)-cot(f*x+e
)*ln(2/(cos(f*x+e)+1))-cot(f*x+e)-csc(f*x+e))
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.73

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = \left[\frac{2\,a\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)+\sqrt{-ac}(a\cos(fx+e)+a)}{2\,(f\cos}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[1/2*(2*a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c
)/cos(f*x + e))*sin(f*x + e) + sqrt(-a*c)*(a*cos(f*x + e) + a)*log(1/2*(a*
c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*
x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e) + f), (a*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e)
 + sqrt(a*c)*(a*cos(f*x + e) + a)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e) + f)]
```

3.96.6 Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\int (a(\sec(e+fx)+1))^{\frac{3}{2}}\sqrt{-c(\sec(e+fx)-1)}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1)), x)
```

3.96.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(85) = 170.

Time = 0.40 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.61

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{((fx+e)a\cos(2fx+2e)^2+(fx+e)a\sin(2fx+2e)^2+2(fx+e)a\cos(2fx+2e)-2a\cos\left(\frac{1}{2}\operatorname{arcta}\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a*cos(2*f*x + 2*e)^2 + (f*x + e)*a*sin(2*f*x + 2*e)^2 + 2*(f*x
 + e)*a*cos(2*f*x + 2*e) - 2*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))*sin(2*f*x + 2*e) + (f*x + e)*a - (a*cos(2*f*x + 2*e)^2 + a*sin(2
*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1) + 2*(a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

3.96.8 Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}\sqrt{-c\sec(fx+e)+c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2), x)
```

3.97 $\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

3.97.1 Optimal result

Integrand size = 30, antiderivative size = 104

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2a^2\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
a^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+2*a^2*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.62

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a(\log(\cos(e+fx))+2\log(1-\sec(e+fx)))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+\right.}{f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(a*(Log[Cos[e + f*x]] + 2*Log[1 - Sec[e + f*x]])*Sqrt[a*(1 + Sec[e + f*x])
]*Tan[(e + f*x)/2])/(f*Sqrt[c - c*Sec[e + f*x]])
```

3.97.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{a\cos(e+fx)(\sec(e+fx)+1)}{c(1-\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)}{1-\sec(e+fx)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{a^2\tan(e+fx)\int \left(\cos(e+fx)-\frac{2}{\sec(e+fx)-1}\right)d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\tan(e+fx)(\log(\sec(e+fx))-2\log(1-\sec(e+fx)))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-((a^2*(-2*Log[1 - Sec[e + f*x]] + Log[Sec[e + f*x]])*Tan[e + f*x])/(f*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

3.97.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.97.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08

method	result
default	$\frac{a\sqrt{a(\sec(fx+e)+1)}\left(\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\ln(-\cot(fx+e)+\csc(fx+e)+1)-4\ln(-\cot(fx+e)+\csc(fx+e))+\ln(-\cot(fx+e)+\csc(fx-}{f\sqrt{-c(\sec(fx+e)-1)}}$
risch	$\frac{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)x}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}}-\frac{2a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)(fx+e)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}-\frac{4ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)\ln(e^{i(fx+}}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*a*(a*(sec(f*x+e)+1))^(1/2)*(ln(2/(cos(f*x+e)+1))+ln(-cot(f*x+e)+csc(f*
x+e)+1)-4*ln(-cot(f*x+e)+csc(f*x+e))+ln(-cot(f*x+e)+csc(f*x+e)-1))/(-c*(se
c(f*x+e)-1))^(1/2)*(cot(f*x+e)-csc(f*x+e))
```

3.97.5 Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral(-(a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x
+ e) - c), x)
```

3.97.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)/sqrt(-c*(sec(e + f*x) - 1)), x)
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{((fx+e)a + a\arctan(\sin(2fx+2e),\cos(2fx+2e)+1) - 4a\arctan(\sin(fx+e),\cos(fx+e)-1))}{\sqrt{c}f}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a + a*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*a*ar
ctan2(sin(f*x + e), cos(f*x + e) - 1))*sqrt(a)/(sqrt(c)*f)
```

3.97.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(1/2), x)
```

3.98 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

3.98.1 Optimal result

Integrand size = 30, antiderivative size = 100

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{2a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-2*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^2*ln(1
-cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.98.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{a^2\left(\log(\cos(e+fx))+\log(1-\sec(e+fx))+\frac{2}{-1+\sec(e+fx)}\right)\tan(e+fx)}{cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a^2*(Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] + 2/(-1 + Sec[e + f*x]))*T
an[e + f*x])/(c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.98.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4396, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4396}$$

$$\frac{a\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{2a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{2a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4399}$$

$$\frac{a^2\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*a^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/
2)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

3.98.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4396
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a/c   Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.98.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.45

method	result
default	$\frac{a\left(\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e))-\ln\left(\frac{2}{\cos(fx+e)+1}\right)+2\ln(-\cot(fx+e)+\csc(fx+e))+\cos(\right.}{f\sqrt{-c(\sec(fx+e)-1)}\,c(\sec(fx+e)-1)(\cos(fx+e)+1)}$
risch	$-\frac{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(2ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+e^{2i(fx+e)}fx-4ie^{i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+2e^{2i(fx+e)}e-2e^{i(fx+e)}fx-4ie^{i(fx+\right.}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*a*(cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x
+e))-ln(2/(cos(f*x+e)+1))+2*ln(-cot(f*x+e)+csc(f*x+e))+cos(f*x+e)+1)*(a*(s
ec(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/c/(sec(f*x+e)-1)/(cos(f*x+e)
+1)*tan(f*x+e)
```

3.98.5 Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
integral((a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x
 + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

3.98.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)/(-c*(sec(e + f*x) - 1))**(3/2), x)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\frac{2\sqrt{-a}a\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{3}{2}}} - \frac{\sqrt{-a}a\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}} + \frac{\sqrt{-a}a(\cos(fx+e)+1)^2}{c^{\frac{3}{2}}\sin(fx+e)^2}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
(2*sqrt(-a)*a*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(3/2) - sqrt(-a)*a*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + sqrt(-a)*a*(cos(f*x +
 e) + 1)^2/(c^(3/2)*sin(f*x + e)^2))/f
```

3.98.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sec\left(fx+e\right)+a)^{\frac{3}{2}}}{(-c\sec\left(fx+e\right)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(3/2), x)
```

3.99 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

3.99.1 Optimal result

Integrand size = 30, antiderivative size = 146

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a^2*tan(f*
x+e)/c/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^2*ln(1-cos(f*x+e)
)*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.99.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.57

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a^2\left(\log(\cos(e+fx))+\log(1-\sec(e+fx))+\frac{-2+\sec(e+fx)}{(-1+\sec(e+fx))^2}\right)\tan(e+fx)}{c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
(a^2*(Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] + (-2 + Sec[e + f*x])/(-1
+ Sec[e + f*x])^2)*Tan[e + f*x])/(c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
- c*Sec[e + f*x]])
```

3.99.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4396, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4396}$$

$$\frac{a\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{3/2}}dx}{c}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4395}$$

$$\frac{a\left(\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{a\left(\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\left(\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-((a^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2
))) + (a*(-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])))/c
```

3.99.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4396
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a/c   Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.99.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.63

method	result
default	$\sqrt{2}\, a \sqrt{-\frac{2a}{(1-\cos(fx+e))^2 \csc(fx+e)^2-1}}\, (1-\cos(fx+e)) \left(8 \ln(-\cot(fx+e)+\csc(fx+e)) (1-\cos(fx+e))^4 \csc(fx+e)^4 - 4 \ln\left((1-\cos(fx-\right.\right.$ $8f\left((1-\cos(fx+e))^2 \csc(fx+e)^2-1\right)^2 \left(\frac{c(1-\cos(fx+}{(1-\cos(fx+e))}\right.$
risch	$a\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\, \left(6i\mathrm{e}^{i(fx+e)}-\mathrm{e}^{4i(fx+e)} fx-8i\mathrm{e}^{2i(fx+e)}-2\,\mathrm{e}^{4i(fx+e)} e+4\,\mathrm{e}^{3i(fx+e)} fx-2i\mathrm{e}^{4i(fx+e)} \ln\left(\mathrm{e}^{i(fx+e)}-1\right)+8\,\mathrm{e}^{3i(fx+e)} e-\right.$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f*2^(1/2)*a*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f*x
+e))^2*csc(f*x+e)^2-1)^2/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^
2-1)*csc(f*x+e)^2)^(5/2)*(1-cos(f*x+e))*(8*ln(-cot(f*x+e)+csc(f*x+e))*(1-c
os(f*x+e))^4*csc(f*x+e)^4-4*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos(f*x
+e))^4*csc(f*x+e)^4+4*(1-cos(f*x+e))^2*csc(f*x+e)^2-1)*csc(f*x+e)
```

3.99.5 Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral(-(a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*
x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

3.99.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{(-c(\sec(e+fx)-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)/(-c*(sec(e + f*x) - 1))**(5/2), x)
```

3.99.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1786 vs. 2(134) = 268.

Time = 0.53 (sec) , antiderivative size = 1786, normalized size of antiderivative = 12.23

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a*cos(4*f*x + 4*e)^2 + 36*(f*x + e)*a*cos(2*f*x + 2*e)^2 + 16*
(f*x + e)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f
*x + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + (f*x +
e)*a*sin(4*f*x + 4*e)^2 + 36*(f*x + e)*a*sin(2*f*x + 2*e)^2 + 16*(f*x + e)
*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*a
*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*(f*x + e)*a*c
os(2*f*x + 2*e) + (f*x + e)*a - 2*(a*cos(4*f*x + 4*e)^2 + 36*a*cos(2*f*x +
 2*e)^2 + 16*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16
*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a*sin(4*f*x +
4*e)^2 + 12*a*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*a*sin(2*f*x + 2*e)^2
+ 16*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(6*a*cos(2*f*x + 2*e
) + a)*cos(4*f*x + 4*e) + 12*a*cos(2*f*x + 2*e) - 8*(a*cos(4*f*x + 4*e) +
6*a*cos(2*f*x + 2*e) - 4*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + a)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(a*cos
(4*f*x + 4*e) + 6*a*cos(2*f*x + 2*e) + a)*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 8*(a*sin(4*f*x + 4*e) + 6*a*sin(2*f*x + 2*e) - 4*a*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(a*sin(4*f*x + 4*e) + 6*a*sin(2*f*x +
 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*arctan...
```

3.99.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^{5/2}}\,dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c - \frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(5/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(5/2), x)
```

3.100 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

3.100.1 Optimal result

Integrand size = 30, antiderivative size = 196

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a^2\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output

```
-2/3*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a^
2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a^2*tan(f*x
+e)/c^2/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^2*ln(1-cos(f*x+e
))*tan(f*x+e)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.100.2 Mathematica [A] (verified)

Time = 2.51 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a^2\left(-6\log(\cos(e+fx))-6\log(1-\sec(e+fx))+\frac{-13+15\sec(e+fx)-6\sec^2(e+fx)}{(-1+\sec(e+fx))^3}\right)\tan(e+fx)}{6c^3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
-1/6*(a^2*(-6*Log[Cos[e + f*x]] - 6*Log[1 - Sec[e + f*x]] + (-13 + 15*Sec[
e + f*x] - 6*Sec[e + f*x]^2)/(-1 + Sec[e + f*x])^3)*Tan[e + f*x])/(c^3*f*S
qrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.100.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4396, 3042, 4395, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4396}$$

$$\frac{a\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{5/2}}dx}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 4395$$

$$\frac{a\left(\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\left(\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 4395$$

$$\frac{a\left(\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\left(\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 4399$$

$$\frac{a\left(\frac{\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

3.100. $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

$\downarrow$ 16

$$\frac{a\left(\frac{\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
(-2*a^2*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(
7/2)) + (a*(-1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e
 + f*x])^(5/2)) + (-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*S
ec[e + f*x])^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a
+ a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))/c))/c
```

3.100.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4396
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a/c   Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.100.4 Maple [A] (warning: unable to verify)

Time = 2.11 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.33

method	result
default	$\frac{\sqrt{2}\,a\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\Big(48\ln(-\cot(fx+e)+\csc(fx+e))(1-\cos(fx+e))^6\csc(fx+e)^6-24\ln\Big((1-\cos(f}{48f\Big((1-\cos(fx+e))^2\csc(fx+e)^2-}$
risch	$\frac{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)x}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}}-\frac{2a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)(fx+e)}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}+\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(12\,e^{5i(fx+e)}-33\,e^{4i(f}{3c^3\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e}}$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f*2^(1/2)*a*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^3/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)
^2-1)*csc(f*x+e)^2)^(7/2)*(1-cos(f*x+e))*(48*ln(-cot(f*x+e)+csc(f*x+e))*(1
-cos(f*x+e))^6*csc(f*x+e)^6-24*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos(
f*x+e))^6*csc(f*x+e)^6+24*(1-cos(f*x+e))^4*csc(f*x+e)^4-9*(1-cos(f*x+e))^2
*csc(f*x+e)^2+2)*csc(f*x+e)
```

3.100.5 Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="fric
as")
```

output
```
integral((a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x
 + e)^4 - 4*c^4*sec(f*x + e)^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e)
 + c^4), x)
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.100.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3480 vs. 2(176) = 352.

Time = 1.85 (sec) , antiderivative size = 3480, normalized size of antiderivative = 17.76

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxi
ma")
```

output
```
-1/3*(3*(f*x + e)*a*cos(6*f*x + 6*e)^2 + 675*(f*x + e)*a*cos(4*f*x + 4*e)^
2 + 675*(f*x + e)*a*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*a*cos(5/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a*cos(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a*cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3*(f*x + e)*a*sin(6*f*x + 6*e)^2 +
675*(f*x + e)*a*sin(4*f*x + 4*e)^2 + 675*(f*x + e)*a*sin(2*f*x + 2*e)^2 +
108*(f*x + e)*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1
200*(f*x + e)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1
08*(f*x + e)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90
*(f*x + e)*a*cos(2*f*x + 2*e) + 3*(f*x + e)*a - 6*(a*cos(6*f*x + 6*e)^2 +
225*a*cos(4*f*x + 4*e)^2 + 225*a*cos(2*f*x + 2*e)^2 + 36*a*cos(5/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a*cos(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 36*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + a*sin(6*f*x + 6*e)^2 + 225*a*sin(4*f*x + 4*e)^2 + 450*a
*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 225*a*sin(2*f*x + 2*e)^2 + 36*a*sin(5
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*a*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))^2 + 2*(15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2
*e) + a)*cos(6*f*x + 6*e) + 30*(15*a*cos(2*f*x + 2*e) + a)*cos(4*f*x + 4*e
) + 30*a*cos(2*f*x + 2*e) - 12*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4...
```

3.100.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(7/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(7/2), x)
```

3.101 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$

3.101.1 Optimal result

Integrand size = 30, antiderivative size = 153

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{a^3c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{a^3c^3\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{a^3c^3\tan^5(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
a^3*c^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)+1/2*a^3*c^3*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)-1/4*a^3*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.50

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{a^3c^3(-4\log(\cos(e+fx))-4\sec^2(e+fx)+\sec^4(e+fx))\tan(e+fx)}{4f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-1/4*(a^3*c^3*(-4*Log[Cos[e + f*x]] - 4*Sec[e + f*x]^2 + Sec[e + f*x]^4)*T
an[e + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.101.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.56, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4393, 25, 3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4393}$$

$$\frac{a^3c^3\tan(e+fx)\int -\tan^5(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^3c^3\tan(e+fx)\int \tan^5(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^3c^3\tan(e+fx)\int \tan(e+fx)^5dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3954$$

$$-\frac{a^3c^3\tan(e+fx)\left(\frac{\tan^4(e+fx)}{4f}-\int\tan^3(e+fx)dx\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a^3c^3\tan(e+fx)\left(\frac{\tan^4(e+fx)}{4f}-\int\tan(e+fx)^3dx\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3954$$

$$-\frac{a^3c^3\tan(e+fx)\left(\int\tan(e+fx)dx+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a^3c^3\tan(e+fx)\left(\int\tan(e+fx)dx+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3956$$

$$-\frac{a^3c^3\tan(e+fx)\left(\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}-\frac{\log(\cos(e+fx))}{f}\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-((a^3*c^3*Tan[e + f*x]*(-(Log[Cos[e + f*x]]/f) - Tan[e + f*x]^2/(2*f) + T
an[e + f*x]^4/(4*f)))/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

3.101.4 Maple [A] (verified)

Time = 30.57 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.10

method	result
default	$-\frac{a^2(\sec(fx+e)-1)^2\left(4\cos(fx+e)^4\ln(-\cot(fx+e)+\csc(fx+e)-1)-4\cos(fx+e)^4\ln\left(\frac{2}{\cos(fx+e)+1}\right)+4\cos(fx+e)^4\ln(-\cot(fx+e\right.}{4f(\cos(fx+e)-}$
risch	$-\frac{a^2c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(4ie^{4i(fx+e)}+e^{8i(fx+e)}fx+4ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+2e^{8i(fx+e)}e+4e^{6i(fx+e)}fx+\right.}{}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*a^2*(sec(f*x+e)-1)^2*(4*cos(f*x+e)^4*ln(-cot(f*x+e)+csc(f*x+e)-1)-4
*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))+4*cos(f*x+e)^4*ln(-cot(f*x+e)+csc(f*x+e
)+1)-3*cos(f*x+e)^4+4*cos(f*x+e)^2-1)*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*
x+e)-1))^(1/2)*c^2/(cos(f*x+e)-1)^2*sec(f*x+e)*csc(f*x+e)
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.65

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \left[\frac{2\sqrt{-ac}a^2c^2\cos(fx+e)^3\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\,c}{}}}{2\cos(fx+e)^2}\right.}{4f}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
[1/4*(2*sqrt(-a*c)*a^2*c^2*cos(f*x + e)^3*log(1/2*(a*c*cos(f*x + e)^4 - (c
os(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x
 + e)^2) - (3*a^2*c^2*cos(f*x + e)^2 - a^2*c^2)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e))/(f*cos
(f*x + e)^3), 1/4*(4*sqrt(a*c)*a^2*c^2*arctan(sqrt(a*c)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c))*cos(f*x + e)^3 - (3*a^2*c^2*cos(
f*x + e)^2 - a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(
f*x + e) - c)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)]
```

3.101.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.101.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1619 vs. $2(137) = 274$.

Time = 0.46 (sec) , antiderivative size = 1619, normalized size of antiderivative = 10.58

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a^2*c^2*cos(8*f*x + 8*e)^2 + 16*(f*x + e)*a^2*c^2*cos(6*f*x +
6*e)^2 + 36*(f*x + e)*a^2*c^2*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*a^2*c^2*co
s(2*f*x + 2*e)^2 + (f*x + e)*a^2*c^2*sin(8*f*x + 8*e)^2 + 16*(f*x + e)*a^2
*c^2*sin(6*f*x + 6*e)^2 + 36*(f*x + e)*a^2*c^2*sin(4*f*x + 4*e)^2 + 16*(f*
x + e)*a^2*c^2*sin(2*f*x + 2*e)^2 + 8*(f*x + e)*a^2*c^2*cos(2*f*x + 2*e) +
 (f*x + e)*a^2*c^2 - 4*a^2*c^2*sin(2*f*x + 2*e) - (a^2*c^2*cos(8*f*x + 8*e
)^2 + 16*a^2*c^2*cos(6*f*x + 6*e)^2 + 36*a^2*c^2*cos(4*f*x + 4*e)^2 + 16*a
^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(8*f*x + 8*e)^2 + 16*a^2*c^2*sin(6*
f*x + 6*e)^2 + 36*a^2*c^2*sin(4*f*x + 4*e)^2 + 48*a^2*c^2*sin(4*f*x + 4*e)
*sin(2*f*x + 2*e) + 16*a^2*c^2*sin(2*f*x + 2*e)^2 + 8*a^2*c^2*cos(2*f*x +
2*e) + a^2*c^2 + 2*(4*a^2*c^2*cos(6*f*x + 6*e) + 6*a^2*c^2*cos(4*f*x + 4*e
) + 4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(8*f*x + 8*e) + 8*(6*a^2*c^2*
cos(4*f*x + 4*e) + 4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(6*f*x + 6*e)
+ 12*(4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(4*f*x + 4*e) + 4*(2*a^2*c^
2*sin(6*f*x + 6*e) + 3*a^2*c^2*sin(4*f*x + 4*e) + 2*a^2*c^2*sin(2*f*x + 2*
e))*sin(8*f*x + 8*e) + 16*(3*a^2*c^2*sin(4*f*x + 4*e) + 2*a^2*c^2*sin(2*f*
x + 2*e))*sin(6*f*x + 6*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
) + 2*(4*(f*x + e)*a^2*c^2*cos(6*f*x + 6*e) + 6*(f*x + e)*a^2*c^2*cos(4*f*
x + 4*e) + 4*(f*x + e)*a^2*c^2*cos(2*f*x + 2*e) + (f*x + e)*a^2*c^2 - 2*a^
2*c^2*sin(6*f*x + 6*e) - 2*a^2*c^2*sin(4*f*x + 4*e) - 2*a^2*c^2*sin(2*f...
```

3.101.8 Giac [F]

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^{5/2}\,dx = \int (a\sec(fx + e) + a)^{\frac{5}{2}}(-c\sec(fx + e) + c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2), x)
```

3.102 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$

3.102.1 Optimal result

Integrand size = 30, antiderivative size = 190

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = \frac{a^3c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{a^2c^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} + \frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

output
```
-1/2*a*c^2*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+1/3*
c^2*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+a^3*c^2*ln(
cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-a^2
*c^2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.46

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{a^3c^2(2+6\log(\cos(e+fx))-6\sec(e+fx)+3\sec^2(e+fx)+2\sec^3(e+fx))\tan(e+fx)}{6f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a^3*c^2*(2 + 6*Log[Cos[e + f*x]] - 6*Sec[e + f*x] + 3*Sec[e + f*x]^2 + 2*
Sec[e + f*x]^3)*Tan[e + f*x])/(6*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*S
ec[e + f*x]])
```

3.102.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {3042, 4397, 3042, 4394, 3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4397}$$

$$c\int (\sec(e+fx)a+a)^{5/2}\sqrt{c-c\sec(e+fx)}dx+\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$c\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$\downarrow$ 4394

$$c\left(a\int (\sec(e+fx)a+a)^{3/2}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}\right)+\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$\downarrow$ 3042

$$c\left(a\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}\right)+\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$\downarrow$ 4394

$$c\left(a\left(a\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec}{2f\sqrt{c-c\sec}}\right.$$
$$\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$\downarrow$ 3042

$$c\left(a\left(a\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-ac\tan(\right.$$
$$\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$\downarrow$ 4393

$$c\left(a\left(\frac{a^2c\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+}{2f\sqrt{c-c\sec(e+}}\right.$$
$$\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$\downarrow$ 25

$$c\left(a\left(-\frac{a^2c\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+}{2f\sqrt{c-c\sec(e+}}\right.$$
$$\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

3.102. $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$

$$\downarrow \text{3042}$$

$$c\left(a\left(-\frac{a^2c\tan(e+fx)\int\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+}{2f\sqrt{c-c\sec(e}\right.$$
$$\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\downarrow \text{3956}$$

$$c\left(a\left(\frac{a^2c\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+}{2f\sqrt{c-c\sec(e+}\right.$$
$$\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(c^2*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]
]) + c*(-1/2*(a*c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[c - c*S
ec[e + f*x]]) + a*((a^2*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Se
c[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (a*c*Sqrt[a + a*Sec[e + f*x]]*Tan[
e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]])))
```

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

rule 4397
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-2*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a   Int[Sqrt[a +
b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

3.102.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a^2\left(6\cos(fx+e)^3\ln\left(\frac{2}{\cos(fx+e)+1}\right)-6\cos(fx+e)^3\ln(-\cot(fx+e)+\csc(fx+e)-1)-6\cos(fx+e)^3\ln(-\cot(fx+e)+\csc(fx+e)+1)\cdots\right.}{6f(\cos(fx\cdots}$
risch	$-\frac{a^2c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(-6ie^{5i(fx+e)}+3e^{6i(fx+e)}fx+6e^{6i(fx+e)}e-6ie^{i(fx+e)}+9e^{4i(fx+e)}fx+18e^{4i(fx+e)}e+\cdots\right.}{\cdots}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/f*a^2*(6*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-6*cos(f*x+e)^3*ln(-cot(f*x
+e)+csc(f*x+e)-1)-6*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)+1)+7*cos(f*x+e)
^3+6*cos(f*x+e)^2-3*cos(f*x+e)-2)*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)
*c*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)*sec(f*x+e)*csc(f*x+e)
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.46

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e+fx))^{3/2}\,dx = \left[\frac{\left(a^2c\cos(fx+e)^2 - 5\,a^2c\cos(fx+e) - 2\,a^2c\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
[1/6*((a^2*c*cos(f*x + e)^2 - 5*a^2*c*cos(f*x + e) - 2*a^2*c)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*
x + e) + 3*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a*c)*log(1/
2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*s
in(f*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2),
 1/6*((a^2*c*cos(f*x + e)^2 - 5*a^2*c*cos(f*x + e) - 2*a^2*c)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*
x + e) + 6*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(a*c)*arctan(
sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*
cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

3.102.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.102.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1356 vs. $2(170) = 340$.

Time = 0.43 (sec) , antiderivative size = 1356, normalized size of antiderivative = 7.14

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^{3/2}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
-1/3*(3*(f*x + e)*a^2*c*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*a^2*c*cos(4*f*x
+ 4*e)^2 + 27*(f*x + e)*a^2*c*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*a^2*c*sin(6
*f*x + 6*e)^2 + 27*(f*x + e)*a^2*c*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*a^2*c
*sin(2*f*x + 2*e)^2 + 18*(f*x + e)*a^2*c*cos(2*f*x + 2*e) + 3*(f*x + e)*a^
2*c - 6*a^2*c*sin(2*f*x + 2*e) - 3*(a^2*c*cos(6*f*x + 6*e)^2 + 9*a^2*c*cos
(4*f*x + 4*e)^2 + 9*a^2*c*cos(2*f*x + 2*e)^2 + a^2*c*sin(6*f*x + 6*e)^2 +
9*a^2*c*sin(4*f*x + 4*e)^2 + 18*a^2*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
9*a^2*c*sin(2*f*x + 2*e)^2 + 6*a^2*c*cos(2*f*x + 2*e) + a^2*c + 2*(3*a^2*c
*cos(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*cos(6*f*x + 6*e) + 6
*(3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*cos(4*f*x + 4*e) + 6*(a^2*c*sin(4*f*x
+ 4*e) + a^2*c*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1) + 6*(3*(f*x + e)*a^2*c*cos(4*f*x + 4*e) + 3*(f*x
+ e)*a^2*c*cos(2*f*x + 2*e) + (f*x + e)*a^2*c - a^2*c*sin(4*f*x + 4*e) - a
^2*c*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x + e)*a^2*c*cos(2*f*x
+ 2*e) + (f*x + e)*a^2*c)*cos(4*f*x + 4*e) - 6*(a^2*c*sin(6*f*x + 6*e) + 3
*a^2*c*sin(4*f*x + 4*e) + 3*a^2*c*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4
*f*x + 4*e) + 3*a^2*c*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) - 6*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e)
+ 3*a^2*c*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x...
```

3.102.8 Giac [F]

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^{3/2}\, dx = \int (a\sec(fx + e) + a)^{\frac{5}{2}}(-c\sec(fx + e) + c)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{5/2}\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2), x)
```

3.103 $\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx$

3.103.1 Optimal result

Integrand size = 30, antiderivative size = 139

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=\frac{a^3c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{ac(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}$$

output
```
-1/2*a*c*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+a^3*c*
ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-
a^2*c*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.52

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{a^3c(-2\log(\cos(e+fx))+4\sec(e+fx)+\sec^2(e+fx))\tan(e+fx)}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-1/2*(a^3*c*(-2*Log[Cos[e + f*x]] + 4*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e
 + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.103.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4394, 3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4394}$$

$$a\int (\sec(e+fx)a+a)^{3/2}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4394}$$

$$a\left(a\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(a\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4393}$$

$$a\left(\frac{a^2c\tan(e+fx)\int-\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$\big\downarrow$ 25

$$a\left(-\frac{a^2c\tan(e+fx)\int\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$\big\downarrow$ 3042

$$a\left(-\frac{a^2c\tan(e+fx)\int\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$\big\downarrow$ 3956

$$a\left(\frac{a^2c\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-1/2*(a*c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f
*x]]) + a*((a^2*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*
x]]*Sqrt[c - c*Sec[e + f*x]]) - (a*c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x]
)/(f*Sqrt[c - c*Sec[e + f*x]]))
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

3.103.4 Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.98

method	result
default	$-\frac{a^{2}\sqrt{-c\left(\sec\left(fx+e\right)-1\right)}\,\sqrt{a\left(\sec\left(fx+e\right)+1\right)}\,\left(2\ln\left(-\cot\left(fx+e\right)+\csc\left(fx+e\right)-1\right)\cot\left(fx+e\right)+2\ln\left(-\cot\left(fx+e\right)+\csc\left(fx+e\right)+1\right)\cot(f \cdots}{2f}$
risch	$\frac{a^{2}\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\,\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\,\left(-i\mathrm{e}^{4i(fx+e)}\ln\left(1+\mathrm{e}^{2i(fx+e)}\right)-\mathrm{e}^{4i(fx+e)}fx-2i\mathrm{e}^{2i(fx+e)}\ln\left(1+\mathrm{e}^{2i(fx+e)}\right)-2\,\mathrm{e}^{4i(fx+e)}e-2\,\mathrm{e} \cdots}{\left(1+\mathrm{e}^{2i(fx+e)}\right)\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1 \cdots}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*a^2*(-c*(sec(f*x+e)-1))^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*(2*ln(-cot(f
*x+e)+csc(f*x+e)-1)*cot(f*x+e)+2*ln(-cot(f*x+e)+csc(f*x+e)+1)*cot(f*x+e)-2
*cot(f*x+e)*ln(2/(cos(f*x+e)+1))-3*cot(f*x+e)-4*csc(f*x+e)-sec(f*x+e)*csc(
f*x+e))
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 420, normalized size of antiderivative = 3.02

$$\int (a + a\sec(e + fx))^{5/2}\sqrt{c - c\sec(e + fx)}\, dx = \left[\frac{\left(5\, a^2 \cos\left(fx + e\right) + a^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(fx + e\right) + \left(a^2 \mathrm{co}\right.}{} \right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[1/2*((5*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + (a^2*cos(f*x + e)^2
+ a^2*cos(f*x + e))*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)
^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(
f*cos(f*x + e)^2 + f*cos(f*x + e)), 1/2*((5*a^2*cos(f*x + e) + a^2)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*
sin(f*x + e) + 2*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt(a*c)*arctan(
sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*
cos(f*x + e)^2 + f*cos(f*x + e))]
```

3.103.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{5/2}\sqrt{c - c\sec(e + fx)}\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.103.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(125) = 250$.

Time = 0.40 (sec) , antiderivative size = 710, normalized size of antiderivative = 5.11

$$\int (a + a\sec(e + fx))^{5/2}\sqrt{c - c\sec(e + fx)}\, dx =$$
$$-\frac{\left((fx+e)a^2\cos\left(4\,fx+4\,e\right)^2+4\left(fx+e\right)a^2\cos\left(2\,fx+2\,e\right)^2+\left(fx+e\right)a^2\sin\left(4\,fx+4\,e\right)^2+4\left(fx+e\right)a^2\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 +
(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*a^2*sin(2*f*x + 2*e)^2 + 4*
(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 + 2*a^2*sin(2*f*x + 2*e) -
(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + a^2*sin(4*f*x + 4*e)^
2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4
*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x +
 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*a^
2*cos(2*f*x + 2*e) + (f*x + e)*a^2 + a^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e
) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f
*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*
x + e)*a^2*sin(2*f*x + 2*e) - a^2*cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + 4*(
a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*
x + 2*e) + a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt
(a)*sqrt(c)/((2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*
e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(
2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

3.103.8 Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}\sqrt{-c\sec(fx+e)+c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2), x)
```

3.104 $\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

3.104.1 Optimal result

Integrand size = 30, antiderivative size = 152

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{a^3\sec(e+fx)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
a^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+4*a^3*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)+a^3*sec(f*x+e)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.45

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a^3(\log(\cos(e+fx))+4\log(1-\sec(e+fx))+\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(a^3*(Log[Cos[e + f*x]] + 4*Log[1 - Sec[e + f*x]] + Sec[e + f*x])*Tan[e +
f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.104.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{a^2\cos(e+fx)(\sec(e+fx)+1)^2}{c(1-\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)^2}{1-\sec(e+fx)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{a^3\tan(e+fx)\int \left(\cos(e+fx)-\frac{4}{\sec(e+fx)-1}-1\right)d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\tan(e+fx)(-\sec(e+fx)-4\log(1-\sec(e+fx))+\log(\sec(e+fx)))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-((a^3*(-4*Log[1 - Sec[e + f*x]] + Log[Sec[e + f*x]] - Sec[e + f*x])*Tan[e
 + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

3.104.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.104.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.02

method	result
default	$-\frac{a^2\sqrt{a(\sec(fx+e)+1)}\left(\ln\left(\frac{2}{\cos(fx+e)+1}\right)\sin(fx+e)-8\ln(-\cot(fx+e)+\csc(fx+e))\sin(fx+e)+3\ln(-\cot(fx+e)+\csc(fx+e)-1)\right.}{f(\cos(fx+e)+1)\sqrt{-c(\sec(fx+e)-1)}}$
risch	$-\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(-2ie^{i(fx+e)}+3ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+e^{3i(fx+e)}fx+8ie^{3i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+2ie^{2i(fx+e)}-3ie^{i(fx}\right.}{}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*a^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)/(-c*(sec(f*x+e)-1))^(1/2)
*(ln(2/(cos(f*x+e)+1))*sin(f*x+e)-8*ln(-cot(f*x+e)+csc(f*x+e))*sin(f*x+e)+
3*ln(-cot(f*x+e)+csc(f*x+e)-1)*sin(f*x+e)+3*ln(-cot(f*x+e)+csc(f*x+e)+1)*s
in(f*x+e)-sin(f*x+e)-tan(f*x+e))
```

3.104.5 Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
 e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.104.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.104.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(1/2), x)
```

3.105 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

3.105.1 Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{4a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^3\log(\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-4*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(c
os(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{a^3\left(\log(\cos(e+fx))+\frac{4}{-1+\sec(e+fx)}\right)\tan(e+fx)}{cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a^3*(Log[Cos[e + f*x]] + 4/(-1 + Sec[e + f*x]))*Tan[e + f*x])/(c*f*Sqrt[a
*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.105.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 4398, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4398}$$

$$\frac{a^2\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx}{c^2}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c^2}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4393}$$

$$\frac{a^3\tan(e+fx)\int -\tan(e+fx)dx}{c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^3\tan(e+fx)\int \tan(e+fx)dx}{c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^3\tan(e+fx)\int \tan(e+fx)dx}{c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{a^3\tan(e+fx)\log(\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(-4*a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/
2)) + (a^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

3.105.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(88) = 176$.

Time = 2.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.99

method	result
default	$\frac{a^2\left(\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)-1)-\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln\left(\frac{}{\cos(f}\right.\right.}{f\sqrt{-c(\sec(fx+e)-1)}\,c(\sec(fx}$
risch	$-\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+e^{2i(fx+e)}fx-2ie^{i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+2e^{2i(fx+e)}e-2e^{i(fx+e)}fx-8ie^{i(fx}\right.}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*a^2*(cos(f*x+e)*ln(2/(cos(f*x+e)+1))-cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x
+e)-1)-cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-ln(2/(cos(f*x+e)+1))+ln(-co
t(f*x+e)+csc(f*x+e)-1)+ln(-cot(f*x+e)+csc(f*x+e)+1)+2*cos(f*x+e)+2)*(a*(se
c(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/c/(sec(f*x+e)-1)/(cos(f*x+e)+
1)*tan(f*x+e)
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(88) = 176$.

Time = 0.33 (sec) , antiderivative size = 442, normalized size of antiderivative = 4.60

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \left[\frac{\left(a^2c\cos(fx+e)-a^2c\right)\sqrt{-\frac{a}{c}}\log\left(\frac{a\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-\frac{a}{c}}\sqrt{\frac{a\cos}{c\cos}}}{2\cos(fx+e)^2}\right.}{2}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
[1/2*((a^2*c*cos(f*x + e) - a^2*c)*sqrt(-a/c)*log(1/2*(a*cos(f*x + e)^4 -
(cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a/c)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a)/cos(f*x
 + e)^2)*sin(f*x + e) + 4*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt((a*
cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(
(c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), ((a^2*c*cos(f*x + e) - a^2*c)*
sqrt(a/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c
*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*cos(f*x + e)
^2 + a))*sin(f*x + e) + 2*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt((a*
cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(
(c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.105.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.105.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(3/2), x)
```

3.106 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

3.106.1 Optimal result

Integrand size = 30, antiderivative size = 100

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{2a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} + \frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-2*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(1
-cos(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2
)
```

3.106.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a^3\left(\log(\cos(e+fx))+\log(1-\sec(e+fx))-\frac{2}{(-1+\sec(e+fx))^2}\right)\tan(e+fx)}{c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
(a^3*(Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] - 2/(-1 + Sec[e + f*x])^2)
*Tan[e + f*x])/(c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.106.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4398, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4398}$$

$$\frac{a^2\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c^2}-\frac{2a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c^2}-\frac{2a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{a^3\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/
2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f
*x]]*Sqrt[c - c*Sec[e + f*x]])
```

3.106.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.106.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(92) = 184$.

Time = 2.01 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.40

method	result
default	$\frac{\sqrt{2}\,a^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(4\ln(-\cot(fx+e)+\csc(fx+e))(1-\cos(fx+e))^4\csc(fx+e)^4-2\ln\left((1-\cos(fx\right.\right.}{4f\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^2\left(\frac{c(1-\cos(fx+}{(1-\cos(fx+e)}\right.}$
risch	$\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(8ie^{i(fx+e)}-e^{4i(fx+e)}fx-8ie^{2i(fx+e)}-2\,e^{4i(fx+e)}e+4\,e^{3i(fx+e)}fx+8ie^{3i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+8\,e^{3i(fx+e)}e}{}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*2^(1/2)*a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f
*x+e))^2*csc(f*x+e)^2-1)^2/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e
)^2-1)*csc(f*x+e)^2)^(5/2)*(1-cos(f*x+e))*(4*ln(-cot(f*x+e)+csc(f*x+e))*(1
-cos(f*x+e))^4*csc(f*x+e)^4-2*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos(f
*x+e))^4*csc(f*x+e)^4+2*(1-cos(f*x+e))^2*csc(f*x+e)^2-1)*csc(f*x+e)
```

3.106.5 Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
 e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e
)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.39

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\frac{4\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{5}{2}}} - \frac{2\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{5}{2}}} - \frac{\left(\sqrt{-a}a^2\sqrt{c} - \frac{2\sqrt{-a}a^2\sqrt{c}\sin(fx+e}{(\cos(fx+e)+1)^2}\right.}{c^3\sin(fx+e)^4}}{2\,f}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
1/2*(4*sqrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(5/2) - 2*sqrt(
-a)*a^2*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) - (sqrt(-a)*a
^2*sqrt(c) - 2*sqrt(-a)*a^2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(
cos(f*x + e) + 1)^4/(c^3*sin(f*x + e)^4))/f
```

3.106.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(5/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(5/2), x)
```

3.107 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

3.107.1 Optimal result

Integrand size = 30, antiderivative size = 148

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a^3\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-4/3*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-a^3*ta
n(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(1-cos(
f*x+e))*tan(f*x+e)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a^3\left(-3\log(\cos(e+fx))-3\log(1-\sec(e+fx))+\frac{-4-3(-1+\sec(e+fx))^2}{(-1+\sec(e+fx))^3}\right)\tan(e+fx)}{3c^3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
-1/3*(a^3*(-3*Log[Cos[e + f*x]] - 3*Log[1 - Sec[e + f*x]] + (-4 - 3*(-1 +
Sec[e + f*x])^2)/(-1 + Sec[e + f*x])^3)*Tan[e + f*x])/(c^3*f*Sqrt[a*(1 + S
ec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.107.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4398, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4398}$$

$$\frac{a^2\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{3/2}}dx}{c^2}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4395}$$

$$\frac{a^2\left(\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\Big\downarrow$ 4399

$$\frac{a^2\left(\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\Big\downarrow$ 16

$$\frac{a^2\left(\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
(-4*a^3*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(
7/2)) + (a^2*(-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e
+ f*x])^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*S
ec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])))/c^2
```

3.107.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.107.4 Maple [A] (warning: unable to verify)

Time = 2.20 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.77

method	result
default	$\frac{\sqrt{2}\,a^{2}\sqrt{-\frac{2a}{(1-\cos(fx+e))^{2}\csc(fx+e)^{2}-1}}\,(1-\cos(fx+e))\left(12\ln(-\cot(fx+e)+\csc(fx+e))(1-\cos(fx+e))^{6}\csc(fx+e)^{6}-6\ln\left((1-\cos(f\right.\right.}{12f\left((1-\cos(fx+e))^{2}\csc(fx+e)^{2}-1\right.}$
risch	$\frac{a^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)x}{c^{3}\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}}-\frac{2a^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)(fx+e)}{c^{3}\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}\,f}+\frac{2ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\,\left(15\,e^{5i(fx+e)}-36\,e^{4}\right.}{3c^{3}\left(e^{i(fx+e)}+1\right)\left(e^{i(fx\right.}}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/f*2^(1/2)*a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(
f*x+e))^2*csc(f*x+e)^2-1)^3/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+
e)^2-1)*csc(f*x+e)^2)^(7/2)*(1-cos(f*x+e))*(12*ln(-cot(f*x+e)+csc(f*x+e))*
(1-cos(f*x+e))^6*csc(f*x+e)^6-6*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos
(f*x+e))^6*csc(f*x+e)^6+6*(1-cos(f*x+e))^4*csc(f*x+e)^4-3*(1-cos(f*x+e))^2
*csc(f*x+e)^2+1)*csc(f*x+e)
```

3.107.5 Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="fric
as")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x + e)^4 - 4*c^4*sec(f*x + e)
^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) + c^4), x)
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.107.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3738 vs. 2(134) = 268.

Time = 1.85 (sec) , antiderivative size = 3738, normalized size of antiderivative = 25.26

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxi
ma")
```

output
```
-1/3*(3*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 + 675*(f*x + e)*a^2*cos(4*f*x + 4
*e)^2 + 675*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*a^2*cos(5/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a^2*cos(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a^2*cos(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3*(f*x + e)*a^2*sin(6*f
*x + 6*e)^2 + 675*(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 675*(f*x + e)*a^2*sin
(2*f*x + 2*e)^2 + 108*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + 1200*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 108*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))^2 + 90*(f*x + e)*a^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a^2
- 72*a^2*sin(2*f*x + 2*e) - 6*(a^2*cos(6*f*x + 6*e)^2 + 225*a^2*cos(4*f*x
+ 4*e)^2 + 225*a^2*cos(2*f*x + 2*e)^2 + 36*a^2*cos(5/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e)))^2 + 400*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 36*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + a^2*sin(6*f*x + 6*e)^2 + 225*a^2*sin(4*f*x + 4*e)^2 + 450*a^2*s
in(4*f*x + 4*e)*sin(2*f*x + 2*e) + 225*a^2*sin(2*f*x + 2*e)^2 + 36*a^2*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a^2*sin(3/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*a^2*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 30*a^2*cos(2*f*x + 2*e) + a^2 + 2*(15*a
^2*cos(4*f*x + 4*e) + 15*a^2*cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) +...
```

3.107.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^{7/2}}\,dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c - \frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(7/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(7/2), x)
```

3.108 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

3.108.1 Optimal result

Integrand size = 30, antiderivative size = 194

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx &= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}}\\ &-\frac{a^3\tan(e+fx)}{2c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\ &-\frac{a^3\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\\ &+\frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^4f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output

```
-a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(9/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a^3*ta
n(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a^3*tan(f*x+e
)/c^3/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(1-cos(f*x+e))
*tan(f*x+e)/c^4/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 5.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.55

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a^3\left(-2\log(\cos(e+fx))-2\log(1-\sec(e+fx))+\frac{2+(-1+\sec(e+fx))^2-2(-1+\sec(e+fx))^3}{(-1+\sec(e+fx))^4}\right)\tan(e+fx)}{2c^4f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(9/2),x]
```

output
```
-1/2*(a^3*(-2*Log[Cos[e + f*x]] - 2*Log[1 - Sec[e + f*x]] + (2 + (-1 + Sec
[e + f*x])^2 - 2*(-1 + Sec[e + f*x])^3)/(-1 + Sec[e + f*x])^4)*Tan[e + f*x
])/(c^4*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.108.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4398, 3042, 4395, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{4398}$$

$$\frac{a^2\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{5/2}}dx}{c^2}-\frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{c^2}-\frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{4395}$$

$$\frac{a^2\left(\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c^2}-\frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c^2}-\frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{4395}$$

$$\frac{a^2\left(\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c^2}-\frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c^2}-\frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{a^2\left(\frac{\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c^2}-\frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

3.108. $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}dx$

$$\Big\downarrow \text{16}$$

$$\frac{a^2\left(\frac{\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c^2}-\frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(9/2),x]
```

output
```
-((a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(9/2
))) + (a^2*(-1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e
 + f*x])^(5/2)) + (-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*S
ec[e + f*x])^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a
+ a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))/c))/c^2
```

3.108.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.108.4 Maple [A] (warning: unable to verify)

Time = 2.45 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.46

method	result
default	$-\frac{\sqrt{2}\, a^{2} \sqrt{-\frac{2 a}{(1-\cos(fx+e))^{2} \csc(fx+e)^{2}-1}}\, (1-\cos(fx+e)) \left(16 \ln\left((1-\cos(fx+e))^{2} \csc(fx+e)^{2}+1\right) (1-\cos(fx+e))^{8} \csc(fx+e)^{8}-32 \ln \cdots\right.}{32 f \left((1-\cos(fx+ \cdots\right.}$
risch	$\frac{a^{2} \sqrt{\frac{a \left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\, \left(e^{i(fx+e)}-1\right) x}{c^{4} \left(e^{i(fx+e)}+1\right) \sqrt{\frac{c \left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}}-\frac{2 a^{2} \sqrt{\frac{a \left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\, \left(e^{i(fx+e)}-1\right) (fx+e)}{c^{4} \left(e^{i(fx+e)}+1\right) \sqrt{\frac{c \left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}\, f}+\frac{2 i a^{2} \sqrt{\frac{a \left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\, \left(6 e^{7i(fx+e)}-23 e^{6i} \cdots\right.}{c^{4} \left(e^{i(fx+ \cdots\right.}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/f*2^(1/2)*a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos
(f*x+e))^2*csc(f*x+e)^2-1)^4/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x
+e)^2-1)*csc(f*x+e)^2)^(9/2)*(1-cos(f*x+e))*(16*ln((1-cos(f*x+e))^2*csc(f*
x+e)^2+1)*(1-cos(f*x+e))^8*csc(f*x+e)^8-32*ln(-cot(f*x+e)+csc(f*x+e))*(1-c
os(f*x+e))^8*csc(f*x+e)^8-16*(1-cos(f*x+e))^6*csc(f*x+e)^6+8*(1-cos(f*x+e)
)^4*csc(f*x+e)^4-4*(1-cos(f*x+e))^2*csc(f*x+e)^2+1)*csc(f*x+e)
```

3.108.5 Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="fric
as")
```

output
```
integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
 e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^5*sec(f*x + e)^5 - 5*c^5*sec(f*x + e
)^4 + 10*c^5*sec(f*x + e)^3 - 10*c^5*sec(f*x + e)^2 + 5*c^5*sec(f*x + e) -
 c^5), x)
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

3.108.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6134 vs. $2(176) = 352$.

Time = 11.57 (sec) , antiderivative size = 6134, normalized size of antiderivative = 31.62

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a^2*cos(8*f*x + 8*e)^2 + 784*(f*x + e)*a^2*cos(6*f*x + 6*e)^2
+ 4900*(f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 784*(f*x + e)*a^2*cos(2*f*x + 2*
e)^2 + 64*(f*x + e)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 3136*(f*x + e)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 3136*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + 64*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + (f*x + e)*a^2*sin(8*f*x + 8*e)^2 + 784*(f*x + e)*a^2*sin(6*f*x
+ 6*e)^2 + 4900*(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 784*(f*x + e)*a^2*sin(2
*f*x + 2*e)^2 + 64*(f*x + e)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 3136*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 + 3136*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 64*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 56*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 - 46
*a^2*sin(2*f*x + 2*e) - 2*(a^2*cos(8*f*x + 8*e)^2 + 784*a^2*cos(6*f*x + 6*
e)^2 + 4900*a^2*cos(4*f*x + 4*e)^2 + 784*a^2*cos(2*f*x + 2*e)^2 + 64*a^2*c
os(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*cos(5/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*cos(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*a^2*cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(8*f*x + 8*e)^2 + 784*a^2*sin(6*f*x +
 6*e)^2 + 4900*a^2*sin(4*f*x + 4*e)^2 + 3920*a^2*sin(4*f*x + 4*e)*sin(2...
```

3.108.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(9/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(9/2), x)
```

3.109 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

3.109.1 Optimal result

Integrand size = 30, antiderivative size = 244

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx &= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}}\\ &- \frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}\\ &- \frac{a^3\tan(e+fx)}{2c^3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\ &- \frac{a^3\tan(e+fx)}{c^4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\\ &+ \frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^5f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output
```
-4/5*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(11/2)/(a+a*sec(f*x+e))^(1/2)-1/3*a
^3*tan(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a^3*
tan(f*x+e)/c^3/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a^3*tan(f*x
+e)/c^4/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(1-cos(f*x+e
))*tan(f*x+e)/c^5/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 5.51 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.49

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx =$$
$$-\frac{a^3\left(-30\log(\cos(e+fx))-30\log(1-\sec(e+fx))+\frac{-24-10(-1+\sec(e+fx))^2+15(-1+\sec(e+fx))^3-30(-1+\sec(e+fx))}{(-1+\sec(e+fx))^5}\right.}{30c^5f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(11/2),x]
```

output
```
-1/30*(a^3*(-30*Log[Cos[e + f*x]] - 30*Log[1 - Sec[e + f*x]] + (-24 - 10*(
-1 + Sec[e + f*x])^2 + 15*(-1 + Sec[e + f*x])^3 - 30*(-1 + Sec[e + f*x])^4
)/(-1 + Sec[e + f*x])^5)*Tan[e + f*x])/(c^5*f*Sqrt[a*(1 + Sec[e + f*x])]*S
qrt[c - c*Sec[e + f*x]])
```

3.109.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {3042, 4398, 3042, 4395, 3042, 4395, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{11/2}}dx$$

$$\Big\downarrow \text{4398}$$

$$\frac{a^2\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{7/2}}dx}{c^2}-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{c^2}-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$\Big\downarrow$ 4395

$$\frac{a^2\left(\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{5/2}}dx}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$\Big\downarrow$ 3042

$$\frac{a^2\left(\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$\Big\downarrow$ 4395

$$\frac{a^2\left(\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$\Big\downarrow$ 3042

$$\frac{a^2\left(\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$\Big\downarrow$ 4395

3.109. $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

$$\frac{a^2\left(\frac{\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}\right)}{c^2}$$

$$\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$\downarrow$ 3042

$$\frac{a^2\left(\frac{\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}\right)}{c^2}$$

$$\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$\downarrow$ 4399

$$\frac{a^2\left(\frac{\frac{\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}\right)}{c^2}$$

$$\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$\downarrow$ 16

$$\frac{a^2\left(\frac{\frac{\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}\right)}{c^2}$$

$$\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(11/2),x]
```

3.109. $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

output
```
(-4*a^3*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(
11/2)) + (a^2*(-1/3*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Se
c[e + f*x])^(7/2)) + (-1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c
 - c*Sec[e + f*x])^(5/2)) + (-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]
]*(c - c*Sec[e + f*x])^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c
*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))/c)/c))/c^2
```

3.109.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.109.4 Maple [A] (warning: unable to verify)

Time = 2.43 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.25

method	result
default	$\frac{\sqrt{2}\, a^2 \sqrt{-\frac{2a}{(1-\cos(fx+e))^2 \csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(240\ln(-\cot(fx+e)+\csc(fx+e))(1-\cos(fx+e))^{10}\csc(fx+e)^{10}-120\ln\left((1-\right.\right.}{}$
risch	$\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)x}{c^5\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}} - \frac{2a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)(fx+e)}{c^5\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\, f} + \frac{2ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(105\, e^{9i(fx+e)}-555\right.}{}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x,method=_RETURNVERBOSE
)
```

output
```
1/240/f*2^(1/2)*a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos
(f*x+e))^2*csc(f*x+e)^2-1)^5/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x
+e)^2-1)*csc(f*x+e)^2)^(11/2)*(1-cos(f*x+e))*(240*ln(-cot(f*x+e)+csc(f*x+e
))*(1-cos(f*x+e))^10*csc(f*x+e)^10-120*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)
*(1-cos(f*x+e))^10*csc(f*x+e)^10+120*(1-cos(f*x+e))^8*csc(f*x+e)^8-60*(1-c
os(f*x+e))^6*csc(f*x+e)^6+35*(1-cos(f*x+e))^4*csc(f*x+e)^4-15*(1-cos(f*x+e
))^2*csc(f*x+e)^2+3)*csc(f*x+e)
```

3.109.5 Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="fri
cas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^6*sec(f*x + e)^6 - 6*c^6*sec(f*x + e)
^5 + 15*c^6*sec(f*x + e)^4 - 20*c^6*sec(f*x + e)^3 + 15*c^6*sec(f*x + e)^2
 - 6*c^6*sec(f*x + e) + c^6), x)
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(11/2),x)
```

output
```
Timed out
```

3.109.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9150 vs. $2(218) = 436$.

Time = 70.10 (sec) , antiderivative size = 9150, normalized size of antiderivative = 37.50

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="max
ima")
```

output
```
-1/15*(15*(f*x + e)*a^2*cos(10*f*x + 10*e)^2 + 30375*(f*x + e)*a^2*cos(8*f
*x + 8*e)^2 + 661500*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 + 661500*(f*x + e)*a
^2*cos(4*f*x + 4*e)^2 + 30375*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + 1500*(f*x
 + e)*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*
(f*x + e)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 952
560*(f*x + e)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 216000*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + 1500*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 15*(f*x + e)*a^2*sin(10*f*x + 10*e)^2 + 30375*(f*x + e)*a^2*sin(8*f
*x + 8*e)^2 + 661500*(f*x + e)*a^2*sin(6*f*x + 6*e)^2 + 661500*(f*x + e)*a
^2*sin(4*f*x + 4*e)^2 + 30375*(f*x + e)*a^2*sin(2*f*x + 2*e)^2 + 1500*(f*x
 + e)*a^2*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*
(f*x + e)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 952
560*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 216000*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + 1500*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 1350*(f*x + e)*a^2*cos(2*f*x + 2*e) + 15*(f*x + e)*a^2 - 1110*a^2*s
in(2*f*x + 2*e) - 30*(a^2*cos(10*f*x + 10*e)^2 + 2025*a^2*cos(8*f*x + 8*e)
^2 + 44100*a^2*cos(6*f*x + 6*e)^2 + 44100*a^2*cos(4*f*x + 4*e)^2 + 2025*a^
2*cos(2*f*x + 2*e)^2 + 100*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*...
```

3.109.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="gia
c")
```

output
```
sage0*x
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{11/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(11/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(11/2), x)
```

3.110 $\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.110.1 Optimal result

Integrand size = 30, antiderivative size = 204

$$\begin{aligned}\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx &= \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &+ \frac{8c^4\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{4c^4\sec(e+fx)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &+ \frac{c^4\sec^2(e+fx)\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output
```
c^4*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+8*c^4*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)-4*c^4*sec(f*x+e)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(
f*x+e))^(1/2)+1/2*c^4*sec(f*x+e)^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-
c*sec(f*x+e))^(1/2)
```

3.110.2 Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.41

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^4(2(\log(\cos(e+fx))+8\log(1+\sec(e+fx)))-8\sec(e+fx)+\sec^2(e+f}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(7/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(c^4*(2*(Log[Cos[e + f*x]] + 8*Log[1 + Sec[e + f*x]]) - 8*Sec[e + f*x] + S
ec[e + f*x]^2)*Tan[e + f*x])/(2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Se
c[e + f*x]])
```

3.110.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{c^3\cos(e+fx)(1-\sec(e+fx))^3}{a(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\tan(e+fx)\int \frac{\cos(e+fx)(1-\sec(e+fx))^3}{\sec(e+fx)+1}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{c^4\tan(e+fx)\int \left(\cos(e+fx)-\sec(e+fx)-\frac{8}{\sec(e+fx)+1}+4\right)d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^4\tan(e+fx)\left(-\frac{1}{2}\sec^2(e+fx)+4\sec(e+fx)+\log(\sec(e+fx))-8\log(\sec(e+fx)+1)\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(7/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((c^4*(Log[Sec[e + f*x]] - 8*Log[1 + Sec[e + f*x]] + 4*Sec[e + f*x] - Sec
[e + f*x]^2/2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e
+ f*x]]))
```

3.110.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.110.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.81

method	result
default	$-\frac{\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^3c^3\sqrt{a(\sec(fx+e)+1)}\,\left(14\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e)+1)+14\cos(fx+e)^2\ln(-\right.}{2fa(\cos(fx+e)-}$
risch	$\frac{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,x}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)}-\frac{2c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(fx+e)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)f}+\frac{2ic^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(4e^{2i(fx+e)}-e^{i(fx+}\right.}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)}$

input
```
int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/a*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^3*c^3*(a*(sec(f*x+e)+1))
^(1/2)*(14*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+14*cos(f*x+e)^2*ln(-c
ot(f*x+e)+csc(f*x+e)-1)+2*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+9*cos(f*x+e)^2
+8*cos(f*x+e)-1)/(cos(f*x+e)-1)^3*cos(f*x+e)*cot(f*x+e)
```

3.110.5 Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{7}{2}}}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e)
- c^3)*sqrt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

3.110.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e+fx))^{7/2}}{\sqrt{a + a\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.110.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e+fx))^{7/2}}{\sqrt{a + a\sec(e+fx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.110.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e+fx))^{7/2}}{\sqrt{a + a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.111 $\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.111.1 Optimal result

Integrand size = 30, antiderivative size = 151

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{c^3\sec(e+fx)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
c^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+4*c^3*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)-c^3*sec(f*x+e)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.46

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^3(\log(\cos(e+fx))+4\log(1+\sec(e+fx))-\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(5/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(c^3*(Log[Cos[e + f*x]] + 4*Log[1 + Sec[e + f*x]] - Sec[e + f*x])*Tan[e +
f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.111.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{c^2\cos(e+fx)(1-\sec(e+fx))^2}{a(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\tan(e+fx)\int \frac{\cos(e+fx)(1-\sec(e+fx))^2}{\sec(e+fx)+1}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{c^3\tan(e+fx)\int \left(\cos(e+fx)-\frac{4}{\sec(e+fx)+1}+1\right)d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^3\tan(e+fx)(\sec(e+fx)+\log(\sec(e+fx))-4\log(\sec(e+fx)+1))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(5/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((c^3*(Log[Sec[e + f*x]] - 4*Log[1 + Sec[e + f*x]] + Sec[e + f*x])*Tan[e
+ f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

3.111.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.111.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

method	result
default	$\frac{\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^2c^2\sqrt{a(\sec(fx+e)+1)}\left(\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+3\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)-\right.}{fa(\cos(fx+e)-1)^2}$
risch	$-\frac{c^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(-3ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)-2ie^{i(fx+e)}+e^{3i(fx+e)}fx-3ie^{i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+8ie^{2i(fx+e)}\ln\left(e^{i(fx\right.\right.}}{}$

input
```
int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/a*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^2*c^2*(a*(sec(f*x+e)+1))^(1
/2)*(cos(f*x+e)*ln(2/(cos(f*x+e)+1))+3*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e
)-1)+3*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+cos(f*x+e)+1)/(cos(f*x+e)-1
)^2*cos(f*x+e)*cot(f*x+e)
```

3.111.5 Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(-c\sec(fx+e)+c)^{\frac{5}{2}}}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral((c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2)*sqrt(-c*sec(f*x +
 e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.111.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.111.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.112 $\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.112.1 Optimal result

Integrand size = 30, antiderivative size = 102

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2c^2\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
c^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+2*c^2*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)
```

3.112.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^2(\log(\cos(e+fx))+2\log(1+\sec(e+fx)))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(3/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(c^2*(Log[Cos[e + f*x]] + 2*Log[1 + Sec[e + f*x]])*Tan[e + f*x])/(f*Sqrt[a
*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.112.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4400, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{c\cos(e+fx)(1-\sec(e+fx))}{a(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\tan(e+fx)\int \frac{\cos(e+fx)(1-\sec(e+fx))}{\sec(e+fx)+1}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{c^2\tan(e+fx)\int \left(\cos(e+fx)-\frac{2}{\sec(e+fx)+1}\right)d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2\tan(e+fx)(\log(\sec(e+fx))-2\log(\sec(e+fx)+1))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(3/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((c^2*(Log[Sec[e + f*x]] - 2*Log[1 + Sec[e + f*x]])*Tan[e + f*x])/(f*Sqrt
[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.112.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

method	result
default	$-\frac{\sqrt{-c(\sec(fx+e)-1)}\, c\sqrt{a(\sec(fx+e)+1)}\, \ln\left(-\frac{4\cos(fx+e)}{(\cos(fx+e)+1)^2}\right)(\cot(fx+e)-\cos(fx+e)\cot(fx+e))}{fa(\cos(fx+e)-1)}$
risch	$\frac{c(e^{i(fx+e)}+1)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\, x}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)} - \frac{2c(e^{i(fx+e)}+1)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(fx+e)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)f} - \frac{4ic(e^{i(fx+e)}+1)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\, \ln(e^{i(fx+e}}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)f}$

input
```
int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a*(-c*(sec(f*x+e)-1))^(1/2)*c*(a*(sec(f*x+e)+1))^(1/2)*ln(-4*cos(f*x+
e)/(cos(f*x+e)+1)^2)/(cos(f*x+e)-1)*(cot(f*x+e)-cos(f*x+e)*cot(f*x+e))
```

3.112.5 Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c\sec\left(fx+e\right)+c)^{\frac{3}{2}}}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral((-c*sec(f*x + e) + c)^(3/2)/sqrt(a*sec(f*x + e) + a), x)
```

3.112.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c(\sec\left(e+fx\right)-1))^{\frac{3}{2}}}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**(3/2)/sqrt(a*(sec(e + f*x) + 1)), x)
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=$$
$$-\frac{((fx+e)c+c\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)-4\,c\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)+1\right))}{\sqrt{a}f}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*c + c*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*c*ar
ctan2(sin(f*x + e), cos(f*x + e) + 1))*sqrt(c)/(sqrt(a)*f)
```

3.112.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.113 $\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.113.1 Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c\log(1+\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
c*ln(1+cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

3.113.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c(\log(\cos(e+fx))+\log(1+\sec(e+fx)))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[c - c*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(c*(Log[Cos[e + f*x]] + Log[1 + Sec[e + f*x]])*Tan[e + f*x])/(f*Sqrt[a*(1
+ Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.113.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4399}$$

$$\frac{ac\tan(e+fx)\int \frac{1}{\cos(e+fx)a+a}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sqrt[c - c*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
- c*Sec[e + f*x]])
```

3.113.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.113.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

method	result
default	$\frac{\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{a(\sec(fx+e)+1)}\,\ln\left(\frac{2}{\cos(fx+e)+1}\right)\cot(fx+e)}{fa}$
risch	$\frac{\left(\mathrm{e}^{i(fx+e)}+1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\,x}{\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\left(\mathrm{e}^{i(fx+e)}-1\right)}-\frac{2\left(\mathrm{e}^{i(fx+e)}+1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\,(fx+e)}{\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\left(\mathrm{e}^{i(fx+e)}-1\right)f}-\frac{2i\left(\mathrm{e}^{i(fx+e)}+1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\,\ln\left(\mathrm{e}^{i(fx+e)}+\right.}{\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\left(\mathrm{e}^{i(fx+e)}-1\right)f}$

input
```
int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/a*(-c*(sec(f*x+e)-1))^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*ln(2/(cos(f*x+e)+
1))*cot(f*x+e)
```

3.113.5 Fricas [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\sqrt{-c\sec\left(fx+e\right)+c}}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

3.113.6 Sympy [F]

$$\int \frac{\sqrt{c - c\sec(e+fx)}}{\sqrt{a + a\sec(e+fx)}}\,dx = \int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(-c*(sec(e + f*x) - 1))/sqrt(a*(sec(e + f*x) + 1)), x)
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{c - c\sec(e+fx)}}{\sqrt{a + a\sec(e+fx)}}\,dx = \frac{\sqrt{c}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{-a}f}$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
sqrt(c)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*f)
```

3.113.8 Giac [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{c - c\sec(e+fx)}}{\sqrt{a + a\sec(e+fx)}}\,dx = -\frac{\sqrt{-ac}c\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{af|c|}$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
-sqrt(-a*c)*c*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a*f*abs(c))
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.114 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

3.114.1 Optimal result

Integrand size = 30, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx=\frac{\log(\sin(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
ln(sin(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx =\frac{(\log(\cos(e+fx))+\log(\tan(e+fx)))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
((Log[Cos[e + f*x]] + Log[Tan[e + f*x]])*Tan[e + f*x])/(f*Sqrt[a*(1 + Sec[
e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.114.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4393, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4393}$$

$$-\frac{\tan(e+fx)\int -\cot(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(e+fx)\int \cot(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(e+fx)\int -\tan\left(e+fx+\frac{\pi}{2}\right)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(e+fx)\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tan(e+fx)\log(-\sin(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
(Log[-Sin[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*S
ec[e + f*x]])
```

3.114.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

3.114.4 Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(\ln(-\cot(fx+e)+\csc(fx+e))-\ln\left(\frac{2}{\cos(fx+e)+1}\right)\right)(\cot(fx+e)-\csc(fx+e))}{fa\sqrt{-c(\sec(fx+e)-1)}}$
risch	$\frac{(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)x}{\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\,(1+e^{2i(fx+e)})\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}}-\frac{2(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)(fx+e)}{\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\,(1+e^{2i(fx+e)})\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}\,f}-\frac{i(e^{i(fx+e)}+1}{\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}}$

input
```
int(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/f/a*(a*(sec(f*x+e)+1))^(1/2)*(ln(-cot(f*x+e)+csc(f*x+e))-ln(2/(cos(f*x+
e)+1)))/(-c*(sec(f*x+e)-1))^(1/2)*(cot(f*x+e)-csc(f*x+e))
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(42) = 84$.

Time = 0.43 (sec) , antiderivative size = 272, normalized size of antiderivative = 5.91

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$$
$$= \left[-\frac{\sqrt{-ac}\log\left(-\frac{8\left(\left(256\cos(fx+e)^5-512\cos(fx+e)^3+175\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}-\left(256\,ac\cos(fx+e)^4-51\right.\right.}{\left(\cos(fx+e)^2-1\right)\sin(fx+e)}\right.}{2\,acf}\right.$$
$$\left.-\frac{\sqrt{ac}\arctan\left(\frac{\left(16\cos(fx+e)^3-7\cos(fx+e)\right)\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{\left(16\,ac\cos(fx+e)^2-25\,ac\right)\sin(fx+e)}\right)}{acf}\right]$$

input
```
integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
[-1/2*sqrt(-a*c)*log(-8*((256*cos(f*x + e)^5 - 512*cos(f*x + e)^3 + 175*co
s(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e)) - (256*a*c*cos(f*x + e)^4 - 512*a*c*cos(f*x +
 e)^2 + 337*a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))/(a*c*f
), -sqrt(a*c)*arctan((16*cos(f*x + e)^3 - 7*cos(f*x + e))*sqrt(a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/
((16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))/(a*c*f)]
```

3.114.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$$
$$= \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

input
```
integrate(1/(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1))), x)
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{fx+e-\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)-1\right)}{\sqrt{a}\sqrt{c}f}$$

input
```
integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
-(f*x + e - arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) - 1))/(sqrt(a)*sqrt
(c)*f)
```

3.114.8 Giac [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\frac{\sqrt{-ac}\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{a|c|} - \frac{2\,\sqrt{-ac}\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c\right|\right)}{a|c|}}{2\,f}$$

input
```
integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
1/2*(sqrt(-a*c)*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(a*abs(c)) - 2*sqrt(-a*
c)*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a*abs(c)))/f
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.115 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx$

3.115.1 Optimal result

Integrand size = 30, antiderivative size = 168

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{2cf(1-\cos(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+ \cdots}}$$
$$+\frac{3\log(1-\cos(e+fx))\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{\log(1+\cos(e+fx))\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
1/2*tan(f*x+e)/c/f/(1-cos(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)+3/4*ln(1-cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(
f*x+e))^(1/2)+1/4*ln(1+cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(
c-c*sec(f*x+e))^(1/2)
```

3.115.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\left(4\log(\cos(e+fx))+3\log(1-\sec(e+fx))+\log(1+\mathrm{se}\cdots\right.}{4cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-\cdots}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
((4*Log[Cos[e + f*x]] + 3*Log[1 - Sec[e + f*x]] + Log[1 + Sec[e + f*x]] +
2/(-1 + Sec[e + f*x]))*Tan[e + f*x])/(4*c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqr
t[c - c*Sec[e + f*x]])
```

3.115.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.57, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)}{ac^2(1-\sec(e+fx))^2(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(1-\sec(e+fx))^2(\sec(e+fx)+1)}d\sec(e+fx)}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\tan(e+fx)\int \left(\cos(e+fx)-\frac{3}{4(\sec(e+fx)-1)}-\frac{1}{4(\sec(e+fx)+1)}+\frac{1}{2(\sec(e+fx)-1)^2}\right)d\sec(e+fx)}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(\frac{1}{2(1-\sec(e+fx))}-\frac{3}{4}\log(1-\sec(e+fx))+\log(\sec(e+fx))-\frac{1}{4}\log(\sec(e+fx)+1)\right)}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
-((((-3*Log[1 - Sec[e + f*x]])/4 + Log[Sec[e + f*x]] - Log[1 + Sec[e + f*x
]]/4 + 1/(2*(1 - Sec[e + f*x])))*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]]))
```

3.115.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.115.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\left(6\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e))-4\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-\cos(fx+e)-6\ln(-\cot(fx+e)+\csc(fx+e))+4\ln\left(\frac{}{\cos}\right.\right.}{4fa\sqrt{-c(\sec(fx+e)-1)}\,c(\sec(fx+e)-1)(\cos(fx+e)+1)}$
risch	$\frac{2ie^{i(fx+e)}-3ie^{3i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)-2\,e^{3i(fx+e)}fx+3ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+3ie^{i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)-4\,e^{3i(fx+e)}e+2\,e^{2}}{2c}$

input
```
int(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/4/f/a*(6*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e))-4*cos(f*x+e)*ln(2/(cos(f
*x+e)+1))-cos(f*x+e)-6*ln(-cot(f*x+e)+csc(f*x+e))+4*ln(2/(cos(f*x+e)+1))-1
)*(a*(sec(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/c/(sec(f*x+e)-1)/(cos
(f*x+e)+1)*tan(f*x+e)
```

3.115.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{\sqrt{a\sec(fx+e)+a}(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a*c^2*sec(f*x
 + e)^3 - a*c^2*sec(f*x + e)^2 - a*c^2*sec(f*x + e) + a*c^2), x)
```

3.115.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{\sqrt{a(\sec(e+fx)+1)}\,(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2)), x)
```

3.115.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(148) = 296$.

Time = 0.40 (sec) , antiderivative size = 818, normalized size of antiderivative = 4.87

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
-1/2*(2*(f*x + e)*cos(2*f*x + 2*e)^2 + 8*(f*x + e)*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(f*x + e)*sin(2*f*x + 2*e)^2 + 8*(f*x
+ e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*f*x - (cos
(2*f*x + 2*e)^2 - 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + sin(2*f*x + 2*e)^2 - 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 1) - 3*(cos(2*f*x + 2*e)^2 - 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 - 4*sin(2*f*x + 2*e)*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 1) + 4*(f*x + e)*cos(2*f*x + 2*e) - 2*(4*f*x +
4*(f*x + e)*cos(2*f*x + 2*e) + 4*e + sin(2*f*x + 2*e))*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(4*(f*x + e)*sin(2*f*x + 2*e) - cos(
2*f*x + 2*e) - 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2
*e)/((c*cos(2*f*x + 2*e)^2 + 4*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(...
```

3.115.8 Giac [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx =$$
$$-\frac{\frac{3\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}|c|}-\frac{4\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{\sqrt{-ac}|c|}-\frac{3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{\sqrt{-ac}c|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}}{4f}$$

input
```
integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
-1/4*(3*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*abs(c)) - 4*log(abs
(c*tan(1/2*f*x + 1/2*e)^2 + c))/(sqrt(-a*c)*abs(c)) - (3*c*tan(1/2*f*x + 1
/2*e)^2 - c)/(sqrt(-a*c)*c*abs(c)*tan(1/2*f*x + 1/2*e)^2))/f
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.116 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx$

3.116.1 Optimal result

Integrand size = 30, antiderivative size = 274

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\log(\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{7\log(1-\sec(e+fx))\tan(e+fx)}{8c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(1+\sec(e+fx))\tan(e+fx)}{8c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{4c^2f(1-\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{3\tan(e+fx)}{4c^2f(1-\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
ln(cos(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+7/8*ln(1-sec(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(
f*x+e))^(1/2)+1/8*ln(1+sec(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)
/(c-c*sec(f*x+e))^(1/2)-1/4*tan(f*x+e)/c^2/f/(1-sec(f*x+e))^2/(a+a*sec(f*x
+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-3/4*tan(f*x+e)/c^2/f/(1-sec(f*x+e))/(a+a
*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\left(8\log(\cos(e+fx))+7\log(1-\sec(e+fx))+\log(1+\text{se}\right.}{8c^2f\sqrt{a(1+\sec(e+f}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
((8*Log[Cos[e + f*x]] + 7*Log[1 - Sec[e + f*x]] + Log[1 + Sec[e + f*x]] -
2/(-1 + Sec[e + f*x])^2 + 6/(-1 + Sec[e + f*x]))*Tan[e + f*x])/(8*c^2*f*Sq
rt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.116.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.41, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)}{ac^3(1-\sec(e+fx))^3(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(1-\sec(e+fx))^3(\sec(e+fx)+1)}d\sec(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\tan(e+fx)\int\left(\cos(e+fx)-\frac{7}{8(\sec(e+fx)-1)}-\frac{1}{8(\sec(e+fx)+1)}+\frac{3}{4(\sec(e+fx)-1)^2}-\frac{1}{2(\sec(e+fx)-1)^3}\right)d\sec(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(\frac{3}{4(1-\sec(e+fx))}+\frac{1}{4(1-\sec(e+fx))^2}-\frac{7}{8}\log(1-\sec(e+fx))+\log(\sec(e+fx))-\frac{1}{8}\log(\sec(e+fx)+\right.}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
-((((-7*Log[1 - Sec[e + f*x]])/8 + Log[Sec[e + f*x]] - Log[1 + Sec[e + f*x
]]/8 + 1/(4*(1 - Sec[e + f*x])^2) + 3/(4*(1 - Sec[e + f*x])))*Tan[e + f*x]
)/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

3.116.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.116.4 Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\sqrt{2}\,\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(16\ln\left((1-\cos(fx+e))^2\csc(fx+e)^2+1\right)(1-\cos(fx+e))^4\csc(fx+e)^4-28\ln(-\cdots\right.}{32fa\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^2\left(\frac{c(1-\cos(\cdots}{(1-\cos(f\cdots}\right.}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(1+e^{2i(fx+e)}\right)}-\frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(1+e^{2i(fx+e)}\right)f}+\frac{i(\cdots}{2c^2\sqrt{\frac{a\left(e^{i(f\cdots}{1+e^{2\cdots}}}}$

input
```
int(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/32/f*2^(1/2)/a*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f
*x+e))^2*csc(f*x+e)^2-1)^2/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e
)^2-1)*csc(f*x+e)^2)^(5/2)*(1-cos(f*x+e))*(16*ln((1-cos(f*x+e))^2*csc(f*x+
e)^2+1)*(1-cos(f*x+e))^4*csc(f*x+e)^4-28*ln(-cot(f*x+e)+csc(f*x+e))*(1-cos
(f*x+e))^4*csc(f*x+e)^4-8*(1-cos(f*x+e))^2*csc(f*x+e)^2+1)*csc(f*x+e)
```

3.116.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{1}{\sqrt{a\sec(fx+e)+a}(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a*c^3*sec(f*
x + e)^4 - 2*a*c^3*sec(f*x + e)^3 + 2*a*c^3*sec(f*x + e) - a*c^3), x)
```

3.116.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(5/2)), x)
```

3.116.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2206 vs. $2(242) = 484$.

Time = 0.52 (sec) , antiderivative size = 2206, normalized size of antiderivative = 8.05

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
-1/4*(4*(f*x + e)*cos(4*f*x + 4*e)^2 + 144*(f*x + e)*cos(2*f*x + 2*e)^2 +
64*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(
f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(f*x +
 e)*sin(4*f*x + 4*e)^2 + 144*(f*x + e)*sin(2*f*x + 2*e)^2 + 64*(f*x + e)*s
in(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*f*x - (2*(6*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e)^
2 - 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) - 4*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 -
8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 3
6*sin(2*f*x + 2*e)^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e) - 4*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), ...
```

3.116.8 Giac [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\frac{14\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}c|c|} - \frac{16\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{\sqrt{-ac}c|c|} - \frac{21\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2+34\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c+14c^2}{\sqrt{-ac}c^3|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}}{16f}$$

input
```
integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
-1/16*(14*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*c*abs(c)) - 16*lo
g(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(sqrt(-a*c)*c*abs(c)) - (21*(c*tan(1/
2*f*x + 1/2*e)^2 - c)^2 + 34*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 14*c^2)/(s
qrt(-a*c)*c^3*abs(c)*tan(1/2*f*x + 1/2*e)^4))/f
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.117 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.117.1 Optimal result

Integrand size = 30, antiderivative size = 215

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{c^4\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{4c^4\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{c^4\sec(e+fx)\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{8c^4\tan(e+fx)}{af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
c^4*ln(cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)-4*c^4*ln(1+sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*se
c(f*x+e))^(1/2)+c^4*sec(f*x+e)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*
sec(f*x+e))^(1/2)-8*c^4*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/
2)/(c-c*sec(f*x+e))^(1/2)
```

3.117.2 Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.45

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\left(-c^3\log(\cos(e+fx))+4c^3\log(1+\sec(e+fx))-c^3\sec(e+fx)+\frac{8c^3}{1+\sec(e+fx)}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c*(-(c^3*Log[Cos[e + f*x]]) + 4*c^3*Log[1 + Sec[e + f*x]] - c^3*Sec[e +
 f*x] + (8*c^3)/(1 + Sec[e + f*x]))*Tan[e + f*x])/(a*f*Sqrt[a*(1 + Sec[e +
 f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

3.117.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{c^3\cos(e+fx)(1-\sec(e+fx))^3}{a^2(\sec(e+fx)+1)^2}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^4\tan(e+fx)\int \frac{\cos(e+fx)(1-\sec(e+fx))^3}{(\sec(e+fx)+1)^2}d\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 99$$

$$-\frac{c^4 \tan(e+fx) \int \left(\cos(e+fx)+\frac{4}{\sec(e+fx)+1}-\frac{8}{(\sec(e+fx)+1)^2}-1\right)d\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{c^4 \tan(e+fx) \left(-\sec(e+fx)+\frac{8}{\sec(e+fx)+1}+\log(\sec(e+fx))+4\log(\sec(e+fx)+1)\right)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c^4*(Log[Sec[e + f*x]] + 4*Log[1 + Sec[e + f*x]] - Sec[e + f*x] + 8/(1
+ Sec[e + f*x]))*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Se
c[e + f*x]]))
```

3.117.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.117.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.08

method	result
default	$-\frac{\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^3 c^3 \sqrt{a(\sec(fx+e)+1)}\left(5\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e)-1)+5\cos(fx+e)^2\ln(-\cot\right.}{}$
risch	$\frac{c^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(8i\ln\left(e^{i(fx+e)}+1\right)e^{4i(fx+e)}-18ie^{i(fx+e)}-fx-2\,e^{3i(fx+e)}fx-2e-2\,e^{4i(fx+e)}e-4\,e^{3i(fx+e)}e-4\,e^{2i(fx+e)}e-2}{}$

input
```
int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a^2*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^3*c^3*(a*(sec(f*x+e)+1))
^(1/2)*(5*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)-1)+5*cos(f*x+e)^2*ln(-cot
(f*x+e)+csc(f*x+e)+1)-cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-3*cos(f*x+e)^2+5*c
os(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)-1)+5*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x
+e)+1)-cos(f*x+e)*ln(2/(cos(f*x+e)+1))+6*cos(f*x+e)+1)/(cos(f*x+e)-1)^2*co
t(f*x+e)^3
```

3.117.5 Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{7}{2}}}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e)
- c^3)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e
)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.117.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2393 vs. $2(199) = 398$.

Time = 0.53 (sec) , antiderivative size = 2393, normalized size of antiderivative = 11.13

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*c^3*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*c^3*cos(2*f*x + 2*e)^2 +
(f*x + e)*c^3*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*c^3*sin(2*f*x + 2*e)^2 + 4*
(f*x + e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3 - 4*c^3*sin(2*f*x + 2*e) +
4*((f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*
cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*((f*x + e)*c^3
- 5*c^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*((f*x + e)*c^3 - 5*c^3*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 + 4*((f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 - 5*(c^3*cos(4*f*x + 4*e)^2 + 4*c^3*cos(2*f*x + 2*e)^2 + c^3*sin(4*f*x
 + 4*e)^2 + 4*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^3*sin(2*f*x + 2*
e)^2 + 4*c^3*cos(2*f*x + 2*e) + c^3 + 2*(2*c^3*cos(2*f*x + 2*e) + c^3)*cos
(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 8*(c^3*co
s(4*f*x + 4*e)^2 + 4*c^3*cos(2*f*x + 2*e)^2 + 4*c^3*cos(3/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 + c^3*sin(4*f*x + 4*e)^2 + 4*c^3*sin(4*f*x + 4*e)*si
n(2*f*x + 2*e) + 4*c^3*sin(2*f*x + 2*e)^2 + 4*c^3*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))^2 + 4*c^3*cos(2*f*x + 2*e) + c^3 + 2*(2*c^3*cos(2*f*x...
```

3.117.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^{7/2}}{(a + a\sec(e + fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(3/2), x)
```

3.118 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.118.1 Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{4c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^3\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-4*c^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c^3*ln(c
os(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c^3(-4+\log(\cos(e+fx))+\log(\cos(e+fx))\sec(e+fx))\tan(e+fx)}{f(a(1+\sec(e+fx)))^{3/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(c^3*(-4 + Log[Cos[e + f*x]] + Log[Cos[e + f*x]]*Sec[e + f*x])*Tan[e + f*x
])/(f*(a*(1 + Sec[e + f*x]))^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

3.118.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 4398, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4398}$$

$$\frac{c^2\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx}{a^2}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4393}$$

$$\frac{c^3\tan(e+fx)\int -\tan(e+fx)dx}{a\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^3\tan(e+fx)\int \tan(e+fx)dx}{a\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{c^3\tan(e+fx)\int \tan(e+fx)dx}{a\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{c^3\tan(e+fx)\log(\cos(e+fx))}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(-4*c^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x
]]) + (c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

3.118.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

3.118.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(88) = 176$.

Time = 1.96 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.28

method	result
default	$\frac{\sqrt{2}\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^3\left(\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\right)^{\frac{5}{2}}\sin(fx+e)^5\left(2(1-\cos(fx\right.}{2f\,a^2(1-\cos(f}$
risch	$-\frac{c^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+e^{2i(fx+e)}fx+2\,e^{2i(fx+e)}e+2ie^{i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+2\,e^{i(fx+e)}fx+8ie^{i(fx\right.}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)f}$

input
```
int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*2^(1/2)/a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f
*x+e))^2*csc(f*x+e)^2-1)^3*(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e
)^2-1)*csc(f*x+e)^2)^(5/2)/(1-cos(f*x+e))^5*sin(f*x+e)^5*(2*(1-cos(f*x+e))
^2*csc(f*x+e)^2+ln(-cot(f*x+e)+csc(f*x+e)+1)-ln((1-cos(f*x+e))^2*csc(f*x+e
)^2+1)+ln(-cot(f*x+e)+csc(f*x+e)-1))
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(88) = 176$.

Time = 0.34 (sec) , antiderivative size = 453, normalized size of antiderivative = 4.72

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{4\,c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(ac^2\cos\left(fx+\right.\right.}{2\,(a}\right.$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
[-1/2*(4*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (a*c^2*cos(f*x + e)^2 + 2*a
*c^2*cos(f*x + e) + a*c^2)*sqrt(-c/a)*log(1/2*(c*cos(f*x + e)^4 - (cos(f*x
 + e)^3 + cos(f*x + e))*sqrt(-c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + c)/cos(f*x + e)^2)
)/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -(2*c^2*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos
(f*x + e)*sin(f*x + e) - (a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*
c^2)*sqrt(c/a)*arctan(sqrt(c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(c*cos(f*x
 + e)^2 + c))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

3.118.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e+fx))^{5/2}}{(a + a\sec(e+fx))^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.118.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e+fx))^{5/2}}{(a + a\sec(e+fx))^{3/2}}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.118.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^{5/2}}{(a + a\sec(e + fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^{5/2}}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(3/2), x)
```

3.119 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.119.1 Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^2\log(1+\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-2*c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c^2*ln(1
+cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\left(-c\log(\cos(e+fx)) - c\log(1+\sec(e+fx)) + \frac{2c}{1+\sec(e+fx)}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c*(-(c*Log[Cos[e + f*x]]) - c*Log[1 + Sec[e + f*x]] + (2*c)/(1 + Sec[e
+ f*x]))*Tan[e + f*x])/(a*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e +
f*x]]))
```

3.119.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4396, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4396}$$

$$\frac{c\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx}{a}-\frac{2c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{2c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{c^2\tan(e+fx)\int \frac{1}{\cos(e+fx)a+a}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$\frac{c^2\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*c^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x
]]) + (c^2*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

3.119.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4396
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a/c   Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.119.4 Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

method	result
default	$\frac{\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)c\sqrt{a(\sec(fx+e)+1)}\,\left(\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\cos(fx+e)+\ln\left(\frac{2}{\cos(fx+e)+1}\right)-1\right)\cot(fx+e}{f\,a^2}$
risch	$-\frac{c\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,\left(2ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)+e^{2i(fx+e)}fx+2\,e^{2i(fx+e)}e+4ie^{i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)+2\,e^{i(fx+e)}fx+4ie^{i(fx+e}}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)f}$

input
```
int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/a^2*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)*c*(a*(sec(f*x+e)+1))^(1/2
)*(cos(f*x+e)*ln(2/(cos(f*x+e)+1))+cos(f*x+e)+ln(2/(cos(f*x+e)+1))-1)*cot(
f*x+e)^2*csc(f*x+e)
```

3.119.5 Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{3}{2}}}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)/(a^2*sec(f*x
 + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

3.119.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**(3/2)/(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{-a}a} - \frac{c^{\frac{3}{2}}\sin(fx+e)^2}{\sqrt{-a}a(\cos(fx+e)+1)^2}}{f}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
(c^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*a) - c^(3/
2)*sin(f*x + e)^2/(sqrt(-a)*a*(cos(f*x + e) + 1)^2))/f
```

3.119.8 Giac [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{\frac{\sqrt{-ac}c^2\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^2|c|} - \frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}c}{a^2|c|}}{f}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
-(sqrt(-a*c)*c^2*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^2*abs(c)) - (c*
tan(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*c/(a^2*abs(c)))/f
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(3/2), x)
```

3.120 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.120.1 Optimal result

Integrand size = 30, antiderivative size = 94

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c\log(1+\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c*ln(1+cos(f
*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\left(-\log(\cos(e+fx))-\log(1+\sec(e+fx))+\frac{1}{1+\sec(e+fx)}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c*(-Log[Cos[e + f*x]] - Log[1 + Sec[e + f*x]] + (1 + Sec[e + f*x])^(-1)
)*Tan[e + f*x])/(a*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

3.120.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4395}$$

$$\frac{\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx}{a}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{c\tan(e+fx)\int \frac{1}{\cos(e+fx)a+a}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]])
) + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

3.120.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.120.4 Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

method	result
default	$\frac{\left(2\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+2\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\cos(fx+e)-1\right)\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-c(\sec(fx+e)-1)}\,\cot(fx+e)}{2f\,a^2(\cos(fx+e)+1)}$
risch	$-\frac{\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,\left(2i\mathrm{e}^{2i(fx+e)}\ln\left(\mathrm{e}^{i(fx+e)}+1\right)+\mathrm{e}^{2i(fx+e)}fx+4i\mathrm{e}^{i(fx+e)}\ln\left(\mathrm{e}^{i(fx+e)}+1\right)+2\,\mathrm{e}^{2i(fx+e)}e+2\,\mathrm{e}^{i(fx+e)}fx+2i\ln\left(\mathrm{e}^{i(fx}\right.\right.}{a\left(\mathrm{e}^{i(fx+e)}+1\right)\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,\left(\mathrm{e}^{i(fx+e)}-1\right)f}$

input
```
int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/a^2*(2*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+2*ln(2/(cos(f*x+e)+1))+cos(f*
x+e)-1)*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2)/(cos(f*x+e)+1)*
cot(f*x+e)
```

3.120.5 Fricas [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{-c\sec\left(fx+e\right)+c}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x +
 e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

3.120.6 Sympy [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(-c*(sec(e + f*x) - 1))/(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.120.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(86) = 172$.

Time = 0.37 (sec) , antiderivative size = 395, normalized size of antiderivative = 4.20

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx =$$
$$-\frac{\left((fx+e)\cos\left(2\,fx+2\,e\right)^2+4\,(fx+e)\cos\left(fx+e\right)^2+(fx+e)\sin\left(2\,fx+2\,e\right)^2+4\,(fx+e)\sin\left(fx+e\right.\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*cos(2*f*x + 2*e)^2 + 4*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*si
n(2*f*x + 2*e)^2 + 4*(f*x + e)*sin(f*x + e)^2 + f*x - 2*(2*(2*cos(f*x + e)
 + 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos(f*x + e)^2 + sin(2*f*x
 + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(f*x + e)^2 + 4*cos(f*x
 + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*(f*x + 2*(f*x + e)*
cos(f*x + e) + e - sin(f*x + e))*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x +
e) + 2*(2*(f*x + e)*sin(f*x + e) + cos(f*x + e))*sin(2*f*x + 2*e) + e - 2*
sin(f*x + e))*sqrt(a)*sqrt(c)/((a^2*cos(2*f*x + 2*e)^2 + 4*a^2*cos(f*x + e
)^2 + a^2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a^2
*sin(f*x + e)^2 + 4*a^2*cos(f*x + e) + a^2 + 2*(2*a^2*cos(f*x + e) + a^2)*
cos(2*f*x + 2*e))*f)
```

3.120.8 Giac [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{2\sqrt{2}\sqrt{-ac}c\log\left(\left|-2\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,c\right|\right)}{a^2|c|}-\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)\sqrt{-ac}}{a^2|c|}\right)}{4\,f}$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
-1/4*sqrt(2)*(2*sqrt(2)*sqrt(-a*c)*c*log(abs(-2*c*tan(1/2*f*x + 1/2*e)^2 -
 2*c))/(a^2*abs(c)) - sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)/(a
^2*abs(c)))/f
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{\sqrt{c - \frac{c}{\cos(e+f\,x)}}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(3/2), x)
```

3.121 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx$

3.121.1 Optimal result

Integrand size = 30, antiderivative size = 215

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(1-\sec(e+fx))\tan(e+fx)}{4af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{3\log(1+\sec(e+fx))\tan(e+fx)}{4af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
ln(cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2
)+1/4*ln(1-sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+
e))^(1/2)+3/4*ln(1+sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*
sec(f*x+e))^(1/2)-1/2*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)
/(c-c*sec(f*x+e))^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{c\left(-\frac{\log(\cos(e+fx))}{c}-\frac{\log(1-\sec(e+fx))}{4c}-\frac{3\log(1+\sec(e+fx))}{4c}+\frac{1}{2c(1+\sec(e+fx))}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
-((c*(-(Log[Cos[e + f*x]]/c) - Log[1 - Sec[e + f*x]]/(4*c) - (3*Log[1 + Se
c[e + f*x]])/(4*c) + 1/(2*c*(1 + Sec[e + f*x])))*Tan[e + f*x])/(a*f*Sqrt[a
*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

3.121.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.43, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)}{a^2c(1-\sec(e+fx))(\sec(e+fx)+1)^2}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(1-\sec(e+fx))(\sec(e+fx)+1)^2}d\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\tan(e+fx)\int \left(\cos(e+fx)-\frac{1}{4(\sec(e+fx)-1)}-\frac{3}{4(\sec(e+fx)+1)}-\frac{1}{2(\sec(e+fx)+1)^2}\right)d\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(\frac{1}{2(\sec(e+fx)+1)}-\frac{1}{4}\log(1-\sec(e+fx))+\log(\sec(e+fx))-\frac{3}{4}\log(\sec(e+fx)+1)\right)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
-(((-1/4*Log[1 - Sec[e + f*x]] + Log[Sec[e + f*x]] - (3*Log[1 + Sec[e + f*
x]])/4 + 1/(2*(1 + Sec[e + f*x])))*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f
*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

3.121.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.121.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.63

method	result
default	$-\frac{\sin(fx+e)\left(4\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e))+4\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\ln(-\cot(fx+e)+\csc}{4f\,a^2(\cos(fx+e)+1)^2\sqrt{-c(\sec(fx+e)-1)}}$
risch	$-\frac{-ie^{i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)-2ie^{i(fx+e)}+2\,e^{3i(fx+e)}fx+4\,e^{3i(fx+e)}e+3i\ln\left(e^{i(fx+e)}+1\right)e^{3i(fx+e)}+2ie^{2i(fx+e)}+2\,e^{2i(fx+e)}fx+4}{}$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/4/f/a^2*sin(f*x+e)*(4*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)*ln(-
cot(f*x+e)+csc(f*x+e))+4*ln(2/(cos(f*x+e)+1))-2*ln(-cot(f*x+e)+csc(f*x+e))
+cos(f*x+e)-1)*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)^2/(-c*(sec(f*x+e)-1
))^(1/2)
```

3.121.5 Fricas [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*c*sec(f*
x + e)^3 + a^2*c*sec(f*x + e)^2 - a^2*c*sec(f*x + e) - a^2*c), x)
```

3.121.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{3}{2}}\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1))), x)
```

3.121.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(193) = 386$.

Time = 0.39 (sec) , antiderivative size = 818, normalized size of antiderivative = 3.80

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
-1/2*(2*(f*x + e)*cos(2*f*x + 2*e)^2 + 8*(f*x + e)*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(f*x + e)*sin(2*f*x + 2*e)^2 + 8*(f*x
+ e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*f*x - 3*(c
os(2*f*x + 2*e)^2 + 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 1) - (cos(2*f*x + 2*e)^2 + 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 1) + 4*(f*x + e)*cos(2*f*x + 2*e) + 2*(4*f*x +
4*(f*x + e)*cos(2*f*x + 2*e) + 4*e + sin(2*f*x + 2*e))*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(4*(f*x + e)*sin(2*f*x + 2*e) - cos(
2*f*x + 2*e) - 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2
*e)/((a*cos(2*f*x + 2*e)^2 + 4*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(...
```

3.121.8 Giac [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.46

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{\sqrt{-ac}a|c|}-\frac{\sqrt{-ac}\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{a^2|c|}+\frac{4\sqrt{-ac}\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^2|c|}}{4f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
-1/4*((c*tan(1/2*f*x + 1/2*e)^2 - c)/(sqrt(-a*c)*a*abs(c)) - sqrt(-a*c)*lo
g(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(a^2*abs(c)) + 4*sqrt(-a*c)*log(abs(c*tan
(1/2*f*x + 1/2*e)^2 + c))/(a^2*abs(c)))/f
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.122 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx$

3.122.1 Optimal result

Integrand size = 30, antiderivative size = 101

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\cot(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(\sin(e+fx))\tan(e+fx)}{acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
1/2*cot(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+ln(sin(
f*x+e))*tan(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\cot(e+fx)+2(\log(\cos(e+fx))+\log(\tan(e+fx)))\,t}{2acf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+f}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
(Cot[e + f*x] + 2*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]])*Tan[e + f*x])/(2
*a*c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.122.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4393, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4393}$$

$$\frac{\tan(e+fx)\int -\cot^3(e+fx)dx}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(e+fx)\int \cot^3(e+fx)dx}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tan(e+fx)\int -\tan\left(e+fx+\frac{\pi}{2}\right)^3dx}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(e+fx)\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3dx}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan(e+fx)\left(\frac{\cot^2(e+fx)}{2f}-\int -\cot(e+fx)dx\right)}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(e+fx)\left(\int \cot(e+fx)dx+\frac{\cot^2(e+fx)}{2f}\right)}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(e+fx)\left(\int -\tan\left(e+fx+\frac{\pi}{2}\right)dx+\frac{\cot^2(e+fx)}{2f}\right)}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(e+fx)\left(\frac{\cot^2(e+fx)}{2f}-\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)dx\right)}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tan(e+fx)\left(\frac{\cot^2(e+fx)}{2f}+\frac{\log(-\sin(e+fx))}{f}\right)}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
((Cot[e + f*x]^2/(2*f) + Log[-Sin[e + f*x]]/f)*Tan[e + f*x])/(a*c*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

3.122.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

3.122.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.55

method	result
default	$-\frac{\left(4\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e))-4\cos(fx+e)^2\ln\left(\frac{2}{\cos(fx+e)+1}\right)-\cos(fx+e)^2-4\ln(-\cot(fx+e)+\csc(fx+e))+4\ln\left(\right.}{4f\,a^2\sqrt{-c(\sec(fx+e)-1)}\,c(\sec(fx+e)-1)(\cos(fx+e)+1)^2}$
risch	$-\frac{ie^{4i(fx+e)}\ln(e^{2i(fx+e)}-1)+e^{4i(fx+e)}fx+2\,e^{4i(fx+e)}e-2ie^{2i(fx+e)}\ln(e^{2i(fx+e)}-1)-2\,e^{2i(fx+e)}fx-4\,e^{2i(fx+e)}e-2ie^{2i(fx+e)}+}{ac(e^{i(fx+e)}+1)\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\,(1+e^{2i(fx+e)})\,(e^{i(fx+e)}-1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}\,f}$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/4/f/a^2*(4*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e))-4*cos(f*x+e)^2*ln(2/
(cos(f*x+e)+1))-cos(f*x+e)^2-4*ln(-cot(f*x+e)+csc(f*x+e))+4*ln(2/(cos(f*x+
e)+1))-1)*(a*(sec(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/c/(sec(f*x+e)
-1)/(cos(f*x+e)+1)^2*tan(f*x+e)
```

3.122.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(91) = 182.

Time = 0.49 (sec) , antiderivative size = 492, normalized size of antiderivative = 4.87

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \left[-\frac{9\sqrt{-ac}\left(\cos(fx+e)^2-1\right)\log\left(-\frac{8\left(\left(256\cos(fx+e)^5-\right.\right.}{}\right.}{}\right.$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fr
icas")
```

output
```
[-1/18*(9*sqrt(-a*c)*(cos(f*x + e)^2 - 1)*log(-8*((256*cos(f*x + e)^5 - 51
2*cos(f*x + e)^3 + 175*cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) - (256*a*c*cos(f*x +
 e)^4 - 512*a*c*cos(f*x + e)^2 + 337*a*c)*sin(f*x + e))/((cos(f*x + e)^2 -
 1)*sin(f*x + e)))*sin(f*x + e) + (16*cos(f*x + e)^3 - 25*cos(f*x + e))*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e)), -1/18*(18*sqrt
(a*c)*(cos(f*x + e)^2 - 1)*arctan((16*cos(f*x + e)^3 - 7*cos(f*x + e))*sqr
t(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e))/((16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))*sin(f*x + e)
 + (16*cos(f*x + e)^3 - 25*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2
 - a^2*c^2*f)*sin(f*x + e))]
```

3.122.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{(a\,(\sec{(e+fx)}+1))^{\frac{3}{2}}\,(-c\,(\sec{(e+fx)}-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(-c*(sec(e + f*x) - 1))**(3/2)),
 x)
```

3.122.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(91) = 182$.

Time = 0.38 (sec) , antiderivative size = 486, normalized size of antiderivative = 4.81

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx=$$
$$-\frac{\left((fx+e)\cos{(4\,fx+4\,e)}^2+4\,(fx+e)\cos{(2\,fx+2\,e)}^2+(fx+e)\sin{(4\,fx+4\,e)}^2+4\,(fx+e)\sin{(2\,f}\right.}{}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="ma
xima")
```

output
```
-((f*x + e)*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*cos(2*f*x + 2*e)^2 + (f*x + e
)*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*sin(2*f*x + 2*e)^2 + f*x + (2*(2*cos(2*
f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4*cos(2*f*x + 2*e)
^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - 4*sin(2*f*
x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) - 1) + 2*(f*x - 2*(f*x + e)*cos(2*f*x + 2*e) + e + sin(2*f*x + 2*e))
*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) - 2*(2*(f*x + e)*sin(2*f*
x + 2*e) + cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + e + 2*sin(2*f*x + 2*e))*sq
rt(a)*sqrt(c)/((a^2*c^2*cos(4*f*x + 4*e)^2 + 4*a^2*c^2*cos(2*f*x + 2*e)^2
+ a^2*c^2*sin(4*f*x + 4*e)^2 - 4*a^2*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + 4*a^2*c^2*sin(2*f*x + 2*e)^2 - 4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2 - 2
*(2*a^2*c^2*cos(2*f*x + 2*e) - a^2*c^2)*cos(4*f*x + 4*e))*f)
```

3.122.8 Giac [A] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx =$$
$$-\frac{\frac{4\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}a|c|}-\frac{8\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{\sqrt{-ac}a|c|}+\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{\sqrt{-ac}ac|c|}-\frac{4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{\sqrt{-ac}ac|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}}{8f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="gi
ac")
```

output
```
-1/8*(4*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a*abs(c)) - 8*log(a
bs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(sqrt(-a*c)*a*abs(c)) + (c*tan(1/2*f*x +
 1/2*e)^2 - c)/(sqrt(-a*c)*a*c*abs(c)) - (4*c*tan(1/2*f*x + 1/2*e)^2 - c)/
(sqrt(-a*c)*a*c*abs(c)*tan(1/2*f*x + 1/2*e)^2))/f
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.123 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx$

3.123.1 Optimal result

Integrand size = 30, antiderivative size = 347

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\log(\cos(e+fx))\tan(e+fx)}{ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{11\log(1-\sec(e+fx))\tan(e+fx)}{16ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{5\log(1+\sec(e+fx))\tan(e+fx)}{16ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8ac^2f(1-\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{2ac^2f(1-\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8ac^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
ln(cos(f*x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)+11/16*ln(1-sec(f*x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-
c*sec(f*x+e))^(1/2)+5/16*ln(1+sec(f*x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+
e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a/c^2/f/(1-sec(f*x+e))^2/(
a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/2*tan(f*x+e)/a/c^2/f/(1-sec
(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a/c^
2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\left(16\log(\cos(e+fx))+11\log(1-\sec(e+fx))+5\log\right.}{16ac^2f\sqrt{a(1}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
((16*Log[Cos[e + f*x]] + 11*Log[1 - Sec[e + f*x]] + 5*Log[1 + Sec[e + f*x]
] - 2/(-1 + Sec[e + f*x])^2 + 8/(-1 + Sec[e + f*x]) - 2/(1 + Sec[e + f*x])
)*Tan[e + f*x])/(16*a*c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e +
f*x]])
```

3.123.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.37, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)}{a^2c^3(1-\sec(e+fx))^3(\sec(e+fx)+1)^2}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(1-\sec(e+fx))^3(\sec(e+fx)+1)^2}d\sec(e+fx)}{ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\tan(e+fx)\int\left(\cos(e+fx)-\frac{11}{16(\sec(e+fx)-1)}-\frac{5}{16(\sec(e+fx)+1)}+\frac{1}{2(\sec(e+fx)-1)^2}-\frac{1}{8(\sec(e+fx)+1)^2}-\frac{1}{4(\sec(e+fx)-1)}\right.}{ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(\frac{1}{2(1-\sec(e+fx))}+\frac{1}{8(\sec(e+fx)+1)}+\frac{1}{8(1-\sec(e+fx))^2}-\frac{11}{16}\log(1-\sec(e+fx))+\log(\sec(e+fx))-\frac{5}{16}\right.}{ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
-((((-11*Log[1 - Sec[e + f*x]])/16 + Log[Sec[e + f*x]] - (5*Log[1 + Sec[e
+ f*x]])/16 + 1/(8*(1 - Sec[e + f*x])^2) + 1/(2*(1 - Sec[e + f*x])) + 1/(8
*(1 + Sec[e + f*x])))*Tan[e + f*x])/(a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]))
```

3.123.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.123.4 Maple [A] (warning: unable to verify)

Time = 2.00 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\sqrt{2}\,\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(-2(1-\cos(fx+e))^6\csc(fx+e)^6+32\ln\left((1-\cos(fx+e))^2\csc(fx+e)^2+1\right)(1-\right.}{64f\,a^2\left((1-\cos(fx+e))^2\csc(fx+e\right.}$
risch	$\frac{\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)x}{a\,c^2\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\left(1+\mathrm{e}^{2i(fx+e)}\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}}-\frac{2\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)(fx+e)}{a\,c^2\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\left(1+\mathrm{e}^{2i(fx+e)}\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,f}+\frac{}{4a\,c^2\left(\mathrm{e}^{i(f\right.}}$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/64/f*2^(1/2)/a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos
(f*x+e))^2*csc(f*x+e)^2-1)^2/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x
+e)^2-1)*csc(f*x+e)^2)^(5/2)*(1-cos(f*x+e))*(-2*(1-cos(f*x+e))^6*csc(f*x+e
)^6+32*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos(f*x+e))^4*csc(f*x+e)^4-4
4*ln(-cot(f*x+e)+csc(f*x+e))*(1-cos(f*x+e))^4*csc(f*x+e)^4-10*(1-cos(f*x+e
))^2*csc(f*x+e)^2+1)*csc(f*x+e)
```

3.123.5 Fricas [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fr
icas")
```

output
```
integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*c^3*sec(
f*x + e)^5 - a^2*c^3*sec(f*x + e)^4 - 2*a^2*c^3*sec(f*x + e)^3 + 2*a^2*c^3
*sec(f*x + e)^2 + a^2*c^3*sec(f*x + e) - a^2*c^3), x)
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.123.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4272 vs. $2(309) = 618$.

Time = 1.88 (sec) , antiderivative size = 4272, normalized size of antiderivative = 12.31

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="ma
xima")
```

output
```
-1/8*(8*(f*x + e)*cos(6*f*x + 6*e)^2 + 8*(f*x + e)*cos(4*f*x + 4*e)^2 + 8*
(f*x + e)*cos(2*f*x + 2*e)^2 + 32*(f*x + e)*cos(5/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 8*(f*x + e)*sin(6*f*x + 6*e)^2 + 8*(f*x + e)*sin(4*f*x
 + 4*e)^2 + 8*(f*x + e)*sin(2*f*x + 2*e)^2 + 32*(f*x + e)*sin(5/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*sin(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*f*x + 5*(2*(cos(4*f*x + 4*e) + cos(2*f
*x + 2*e) - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e)
 - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 + 4*(cos(
6*f*x + 6*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 4*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 4*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(cos(6*f*x
 + 6*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) - 2*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 4*(cos(6*f*x + 6*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*...
```

3.123.8 Giac [A] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.53

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{\frac{22\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}ac|c|}-\frac{32\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{\sqrt{-ac}ac|c|}+\frac{2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{\sqrt{-ac}ac^2|c|}-\frac{33\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2+56\left(c\tan\left(\frac{1}{2}fx+\right.\right.}{\sqrt{-ac}ac^3|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}}{32f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="gi
ac")
```

output
```
-1/32*(22*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a*c*abs(c)) - 32*
log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(sqrt(-a*c)*a*c*abs(c)) + 2*(c*tan(
1/2*f*x + 1/2*e)^2 - c)/(sqrt(-a*c)*a*c^2*abs(c)) - (33*(c*tan(1/2*f*x + 1
/2*e)^2 - c)^2 + 56*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 24*c^2)/(sqrt(-a*c)
*a*c^3*abs(c)*tan(1/2*f*x + 1/2*e)^4))/f
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.124 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.124.1 Optimal result

Integrand size = 30, antiderivative size = 220

$$\begin{aligned}\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx &= \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &+ \frac{2c^4\log(1+\sec(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &- \frac{4c^4\tan(e+fx)}{a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &+ \frac{4c^4\tan(e+fx)}{a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output
```
c^4*ln(cos(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)+2*c^4*ln(1+sec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-
c*sec(f*x+e))^(1/2)-4*c^4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e
))^(1/2)/(c-c*sec(f*x+e))^(1/2)+4*c^4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a
*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.124.2 Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.39

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^4\left(-\log(\cos(e+fx))-2\log(1+\sec(e+fx))-\frac{4\sec(e+fx)}{(1+\sec(e+fx))^2}\right)\tan(e+fx)}{a^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((c^4*(-Log[Cos[e + f*x]] - 2*Log[1 + Sec[e + f*x]] - (4*Sec[e + f*x])/(1
 + Sec[e + f*x])^2)*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
 - c*Sec[e + f*x]]))
```

3.124.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{c^3\cos(e+fx)(1-\sec(e+fx))^3}{a^3(\sec(e+fx)+1)^3}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\tan(e+fx)\int \frac{\cos(e+fx)(1-\sec(e+fx))^3}{(\sec(e+fx)+1)^3}d\sec(e+fx)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{c^4 \tan(e+fx) \int \left(\cos(e+fx) - \frac{2}{\sec(e+fx)+1} + \frac{4}{(\sec(e+fx)+1)^2} - \frac{8}{(\sec(e+fx)+1)^3}\right) d\sec(e+fx)}{a^2 f \sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^4 \tan(e+fx) \left(-\frac{4}{\sec(e+fx)+1} + \frac{4}{(\sec(e+fx)+1)^2} + \log(\sec(e+fx)) - 2\log(\sec(e+fx)+1)\right)}{a^2 f \sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((c^4*(Log[Sec[e + f*x]] - 2*Log[1 + Sec[e + f*x]] + 4/(1 + Sec[e + f*x])
^2 - 4/(1 + Sec[e + f*x]))*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]]))
```

3.124.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.124.4 Maple [A] (warning: unable to verify)

Time = 2.36 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98

method	result
default	$-\frac{\sqrt{2}\,\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^4\left(\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\right)^{\frac{7}{2}}\sin(fx+e)^7\left((1-\cos(f\ldots\right.}{2f\,a^3(1-\cos\ldots}$
risch	$-\frac{c^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(-4i\ln\left(1+e^{2i(fx+e)}\right)e^{3i(fx+e)}+8ie^{i(fx+e)}+e^{4i(fx+e)}fx+2\,e^{4i(fx+e)}e-4ie^{i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+16i\ln(\ldots\right.}{\ldots}$

input
```
int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*2^(1/2)/a^3*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(
f*x+e))^2*csc(f*x+e)^2-1)^4*(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+
e)^2-1)*csc(f*x+e)^2)^(7/2)/(1-cos(f*x+e))^7*sin(f*x+e)^7*((1-cos(f*x+e))^
4*csc(f*x+e)^4+ln(-cot(f*x+e)+csc(f*x+e)+1)+ln((1-cos(f*x+e))^2*csc(f*x+e)
^2+1)+ln(-cot(f*x+e)+csc(f*x+e)-1))
```

3.124.5 Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{(-c\sec(fx+e)+c)^{\frac{7}{2}}}{(a\sec(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e)
- c^3)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e
)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

3.124.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e+fx))^{7/2}}{(a + a\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.124.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e+fx))^{7/2}}{(a + a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.124.8 Giac [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.42

$$\int \frac{(c - c\sec(e+fx))^{7/2}}{(a + a\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}a^2|c|+2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}a^2c|c|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3\right.}{a^5f}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
-((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^2*abs(c) + 2*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*sqrt(-a*c)*a^2*c*abs(c))*sgn(tan(1/2*f*x + 1/2*e)^3 + ta
n(1/2*f*x + 1/2*e))/(a^5*f)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^{7/2}}{(a + a\sec(e + fx))^{5/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^{7/2}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(5/2), x)
```

3.125 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.125.1 Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{2c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^3\log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-2*c^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)+c^3*ln(1
+cos(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2
)
```

3.125.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^3\left(-\log(\cos(e+fx))-\log(1+\sec(e+fx))+\frac{2}{(1+\sec(e+fx))^2}\right)\tan(e+fx)}{a^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((c^3*(-Log[Cos[e + f*x]] - Log[1 + Sec[e + f*x]] + 2/(1 + Sec[e + f*x])^
2)*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]
]))
```

3.125.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4398, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4398}$$

$$\frac{c^2\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx}{a^2}-\frac{2c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a^2}-\frac{2c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{c^3\tan(e+fx)\int \frac{1}{\cos(e+fx)a+a}d\cos(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$\frac{c^3\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*c^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x
]]) + (c^3*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f
*x]]*Sqrt[c - c*Sec[e + f*x]])
```

3.125.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.125.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.45

method	result
default	$\frac{\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^2 c^2 \sqrt{a(\sec(fx+e)+1)}\,\left(2\cos(fx+e)^2\ln\left(\frac{2}{\cos(fx+e)+1}\right)+3\cos(fx+e)^2+4\cos(fx+e)\ln\left(\frac{2}{\cos(fx+\cdots}\right.\right.}{2f\,a^3}$
risch	$-\frac{c^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,\left(8ie^{i(fx+e)}+e^{4i(fx+e)}fx+8ie^{i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)+2e^{4i(fx+e)}e+4e^{3i(fx+e)}fx+8ie^{2i(fx+e)}+8e^{3i(fx+e)}\cdots\right.}{\cdots}$

3.125. $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

input
```
int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/a^3*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^2*c^2*(a*(sec(f*x+e)+1)
)^(1/2)*(2*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+3*cos(f*x+e)^2+4*cos(f*x+e)*l
n(2/(cos(f*x+e)+1))-2*cos(f*x+e)+2*ln(2/(cos(f*x+e)+1))-1)*cot(f*x+e)^3*cs
c(f*x+e)^2
```

3.125.5 Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{5}{2}}}{(a\sec(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral((c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)
^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

3.125.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\frac{2c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{-a}a^2} + \frac{\frac{2\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}{a^3}}{2f}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
1/2*(2*c^(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*a^2)
 + (2*sqrt(-a)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - sqrt(-a)*c^(5
/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/a^3)/f
```

3.125.8 Giac [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\frac{2\sqrt{-ac}c^3\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^3|c|} + \frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}|c|}{a^3c}}{2f}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
-1/2*(2*sqrt(-a*c)*c^3*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^3*abs(c))
 + (c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*abs(c)/(a^3*c))/f
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+fx)}\right)^{5/2}}{\left(a+\frac{a}{\cos(e+fx)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(5/2), x)
```

3.126 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.126.1 Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} - \frac{c^2\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^2\log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)-c^2*tan(f*
x+e)/a/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c^2*ln(1+cos(f*x+e)
)*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^2\left(-\log(\cos(e+fx))-\log(1+\sec(e+fx))+\frac{2+\sec(e+fx)}{(1+\sec(e+fx))^2}\right)\tan(e+fx)}{a^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((c^2*(-Log[Cos[e + f*x]] - Log[1 + Sec[e + f*x]] + (2 + Sec[e + f*x])/(1
 + Sec[e + f*x])^2)*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
 - c*Sec[e + f*x]]))
```

3.126.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4396, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4396}$$

$$\frac{c\int \frac{\sqrt{c-c\sec(e+fx)}}{(\sec(e+fx)a+a)^{3/2}}dx}{a}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{a}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4395}$$

$$\frac{c\left(\frac{\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx}{a}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}\right)}{a}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\left(\frac{\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}\right)}{a}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 4399$$

$$\frac{c\left(\frac{c\tan(e+fx)\int \frac{1}{\cos(e+fx)a+a}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}\right)}{a}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 16$$

$$\frac{c\left(\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}\right)}{a}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((c^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]
])) + (c*(-((c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[
e + f*x]])) + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])))/a
```

3.126.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4396
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a/c   Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.126.4 Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07

method	result
default	$-\frac{(\sec(fx+e)-1)\sqrt{-c(\sec(fx+e)-1)}\,c\sqrt{a(\sec(fx+e)+1)}\left(4\cos(fx+e)^2\ln\left(\frac{2}{\cos(fx+e)+1}\right)+8\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+5\cos(f\right.}{4f\,a^3(\cos(fx+e)+1)^2(\cos(fx+e)-1)}$
risch	$-\frac{c\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(6ie^{i(fx+e)}+e^{4i(fx+e)}fx+2i\ln(e^{i(fx+e)}+1)e^{4i(fx+e)}+2e^{4i(fx+e)}e+4e^{3i(fx+e)}fx+8ie^{2i(fx+e)}+8e^{3i(fx+e)}\right.}{}$

input
```
int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/a^3*(sec(f*x+e)-1)*(-c*(sec(f*x+e)-1))^(1/2)*c*(a*(sec(f*x+e)+1))^(
1/2)*(4*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+8*cos(f*x+e)*ln(2/(cos(f*x+e)+1)
)+5*cos(f*x+e)^2+4*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)-3)/(cos(f*x+e)+1)^2/(
cos(f*x+e)-1)*cos(f*x+e)*cot(f*x+e)
```

3.126.5 Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{3}{2}}}{(a\sec(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)/(a^3*sec(f*x
 + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

3.126.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}{(a(\sec(e+fx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**(3/2)/(a*(sec(e + f*x) + 1))**(5/2), x)
```

3.126.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1786 vs. $2(132) = 264$.

Time = 0.51 (sec) , antiderivative size = 1786, normalized size of antiderivative = 12.40

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*c*cos(4*f*x + 4*e)^2 + 36*(f*x + e)*c*cos(2*f*x + 2*e)^2 + 16*
(f*x + e)*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f
*x + e)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + (f*x +
e)*c*sin(4*f*x + 4*e)^2 + 36*(f*x + e)*c*sin(2*f*x + 2*e)^2 + 16*(f*x + e)
*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*c
*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*(f*x + e)*c*c
os(2*f*x + 2*e) + (f*x + e)*c - 2*(c*cos(4*f*x + 4*e)^2 + 36*c*cos(2*f*x +
 2*e)^2 + 16*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16
*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*sin(4*f*x +
4*e)^2 + 12*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*c*sin(2*f*x + 2*e)^2
+ 16*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(6*c*cos(2*f*x + 2*e
) + c)*cos(4*f*x + 4*e) + 12*c*cos(2*f*x + 2*e) + 8*(c*cos(4*f*x + 4*e) +
6*c*cos(2*f*x + 2*e) + 4*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + c)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(c*cos
(4*f*x + 4*e) + 6*c*cos(2*f*x + 2*e) + c)*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 8*(c*sin(4*f*x + 4*e) + 6*c*sin(2*f*x + 2*e) + 4*c*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(c*sin(4*f*x + 4*e) + 6*c*sin(2*f*x +
 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c)*arctan...
```

3.126.8 Giac [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{\frac{4\sqrt{-ac}c^2\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^3|c|}+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}a^3|c|-2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}a^3c|c|}{a^6c^2}}{4f}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
-1/4*(4*sqrt(-a*c)*c^2*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^3*abs(c))
 + ((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^3*abs(c) - 2*(c*tan(1/2*
f*x + 1/2*e)^2 - c)*sqrt(-a*c)*a^3*c*abs(c))/(a^6*c^2))/f
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(5/2), x)
```

3.127 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.127.1 Optimal result

Integrand size = 30, antiderivative size = 140

$$\begin{aligned}\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx =& -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\\ &-\frac{c\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\\ &+\frac{c\log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output
```
-1/2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)-c*tan(f*
x+e)/a/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c*ln(1+cos(f*x+e))*
tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c\left(-\log(\cos(e+fx))-\log(1+\sec(e+fx))+\frac{1}{2(1+\sec(e+fx))^2}+\frac{1}{1+\sec(e+fx)}\right)\tan(e+fx)}{a^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((c*(-Log[Cos[e + f*x]] - Log[1 + Sec[e + f*x]] + 1/(2*(1 + Sec[e + f*x])
^2) + (1 + Sec[e + f*x])^(-1))*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*
x])]*Sqrt[c - c*Sec[e + f*x]]))
```

3.127.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4395, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4395}$$

$$\frac{\int \frac{\sqrt{c-c\sec(e+fx)}}{(\sec(e+fx)a+a)^{3/2}}dx}{a}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{a}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4395}$$

$$\frac{\frac{\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx}{a}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}}{a}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}}{a}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 4399}$$

$$\frac{\frac{c\tan(e+fx)\int \frac{1}{\cos(e+fx)a+a}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}}{a}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}}{a}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-1/2*(c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x
]]) + (-((c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e +
 f*x]])) + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]]))/a
```

3.127.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

3.127.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

method	result
default	$\frac{\left(8\cos(fx+e)^2\ln\left(\frac{2}{\cos(fx+e)+1}\right)+16\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+7\cos(fx+e)^2+8\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\cos(fx+e)-5\right)\sqrt{a(\sec(fx+}}{8f\,a^3(\cos(fx+e)+1)^2}$
risch	$-\frac{\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(2i\ln(e^{i(fx+e)}+1)e^{4i(fx+e)}+e^{4i(fx+e)}fx+8ie^{i(fx+e)}\ln(e^{i(fx+e)}+1)+2\,e^{4i(fx+e)}e+4\,e^{3i(fx+e)}fx+4ie^{i(fx+}}{}$

input
```
int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/a^3*(8*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+16*cos(f*x+e)*ln(2/(cos(f*x
+e)+1))+7*cos(f*x+e)^2+8*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)-5)*(a*(sec(f*x+
e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2)/(cos(f*x+e)+1)^2*cot(f*x+e)
```

3.127.5 Fricas [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{-c\sec\left(fx+e\right)+c}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x +
 e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

3.127.6 Sympy [F]

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{(a + a\sec(e + fx))^{5/2}}\,dx = \int \frac{\sqrt{-c(\sec(e + fx) - 1)}}{(a(\sec(e + fx) + 1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(-c*(sec(e + f*x) - 1))/(a*(sec(e + f*x) + 1))**(5/2), x)
```

3.127.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. $2(126) = 252$.

Time = 0.49 (sec) , antiderivative size = 1165, normalized size of antiderivative = 8.32

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(3*f*x + 3*e)^2 + 36*(f*x
 + e)*cos(2*f*x + 2*e)^2 + 16*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin(4*f
*x + 4*e)^2 + 16*(f*x + e)*sin(3*f*x + 3*e)^2 + 36*(f*x + e)*sin(2*f*x + 2
*e)^2 + 16*(f*x + e)*sin(f*x + e)^2 + f*x - 2*(2*(4*cos(3*f*x + 3*e) + 6*c
os(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^
2 + 8*(6*cos(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + 16*cos(
3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + 36*cos(2*f*x +
 2*e)^2 + 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x + 3*e) + 3*sin(2*f*x + 2*e) +
 2*sin(f*x + e))*sin(4*f*x + 4*e) + sin(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x +
 2*e) + 2*sin(f*x + e))*sin(3*f*x + 3*e) + 16*sin(3*f*x + 3*e)^2 + 36*sin(
2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e) + 16*sin(f*x + e)^2 + 8*
cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*(f*x + 4*(f*
x + e)*cos(3*f*x + 3*e) + 6*(f*x + e)*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f
*x + e) + e - 2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*co
s(4*f*x + 4*e) + 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f
*x + e) + e)*cos(3*f*x + 3*e) + 12*(f*x + 4*(f*x + e)*cos(f*x + e) + e)*co
s(2*f*x + 2*e) + 8*(f*x + e)*cos(f*x + e) + 2*(4*(f*x + e)*sin(3*f*x + 3*e
) + 6*(f*x + e)*sin(2*f*x + 2*e) + 4*(f*x + e)*sin(f*x + e) + 2*cos(3*f*x
+ 3*e) + 3*cos(2*f*x + 2*e) + 2*cos(f*x + e))*sin(4*f*x + 4*e) + 4*(12*(f*
x + e)*sin(2*f*x + 2*e) + 8*(f*x + e)*sin(f*x + e) - 1)*sin(3*f*x + 3*e...
```

3.127.8 Giac [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{8\sqrt{2}\sqrt{-ac}\,c\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^3|c|}+\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}\,a^3c|c|-4\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}\,a^3c^2|c|}{a^6c^4}\right)}{16f}$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
-1/16*sqrt(2)*(8*sqrt(2)*sqrt(-a*c)*c*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c
))/(a^3*abs(c)) + (sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^3
*c*abs(c) - 4*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*a^3*c^2*ab
s(c))/(a^6*c^4))/f
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(5/2), x)
```

3.128 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx$

3.128.1 Optimal result

Integrand size = 30, antiderivative size = 270

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx &= \frac{\log(\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &+\frac{\log(1-\sec(e+fx))\tan(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{7\log(1+\sec(e+fx))\tan(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &-\frac{\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &-\frac{3\tan(e+fx)}{4a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output
```
ln(cos(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+1/8*ln(1-sec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(
f*x+e))^(1/2)+7/8*ln(1+sec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)
/(c-c*sec(f*x+e))^(1/2)-1/4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x
+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-3/4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a
*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.128.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.37

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\left(8\log(\cos(e+fx))+\log(1-\sec(e+fx))+7\log(1+\mathrm{se}\right.}{8a^2f\sqrt{a(1+\sec(e+fx}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
((8*Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] + 7*Log[1 + Sec[e + f*x]] -
2/(1 + Sec[e + f*x])^2 - 6/(1 + Sec[e + f*x]))*Tan[e + f*x])/(8*a^2*f*Sqrt
[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

3.128.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4400$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)}{a^3c(1-\sec(e+fx))(\sec(e+fx)+1)^3}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(1-\sec(e+fx))(\sec(e+fx)+1)^3}d\sec(e+fx)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 93$$

$$-\frac{\tan(e+fx)\int\left(\cos(e+fx)-\frac{1}{8(\sec(e+fx)-1)}-\frac{7}{8(\sec(e+fx)+1)}-\frac{3}{4(\sec(e+fx)+1)^2}-\frac{1}{2(\sec(e+fx)+1)^3}\right)d\sec(e+fx)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(\frac{3}{4(\sec(e+fx)+1)}+\frac{1}{4(\sec(e+fx)+1)^2}-\frac{1}{8}\log(1-\sec(e+fx))+\log(\sec(e+fx))-\frac{7}{8}\log(\sec(e+fx)+\right.}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
-(((-1/8*Log[1 - Sec[e + f*x]] + Log[Sec[e + f*x]] - (7*Log[1 + Sec[e + f*
x]])/8 + 1/(4*(1 + Sec[e + f*x])^2) + 3/(4*(1 + Sec[e + f*x])))*Tan[e + f*
x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.128.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.72

method	result
default	$-\frac{\sqrt{2}\,\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left((1-\cos(fx+e))^4\csc(fx+e)^4-8(1-\cos(fx+e))^2\csc(fx+e)^2+16\ln\left((1-\cos(\right.\right.}{32f\,a^3\sqrt{\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}}-\frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}-\frac{i(}{2a^2\left(e^{i(fx+e}\right.}$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/32/f*2^(1/2)/a^3*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/(c*(1-c
os(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*csc(f*x+e)^2)^(1/2)*(1-cos(
f*x+e))*((1-cos(f*x+e))^4*csc(f*x+e)^4-8*(1-cos(f*x+e))^2*csc(f*x+e)^2+16*
ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)-4*ln(-cot(f*x+e)+csc(f*x+e)))*csc(f*x+
e)
```

3.128.5 Fricas [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{1}{(a\sec(fx+e)+a)^{\frac{5}{2}}\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*c*sec(f*
x + e)^4 + 2*a^3*c*sec(f*x + e)^3 - 2*a^3*c*sec(f*x + e) - a^3*c), x)
```

3.128.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{(a\left(\sec\left(e+fx\right)+1\right))^{\frac{5}{2}}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*sqrt(-c*(sec(e + f*x) - 1))), x)
```

3.128.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2206 vs. 2(242) = 484.

Time = 0.51 (sec) , antiderivative size = 2206, normalized size of antiderivative = 8.17

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
-1/4*(4*(f*x + e)*cos(4*f*x + 4*e)^2 + 144*(f*x + e)*cos(2*f*x + 2*e)^2 +
64*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(
f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(f*x +
 e)*sin(4*f*x + 4*e)^2 + 144*(f*x + e)*sin(2*f*x + 2*e)^2 + 64*(f*x + e)*s
in(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*f*x - 7*(2*(6*cos(2*
f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e
)^2 + 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 4*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
+ 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 36*sin(2*f*x + 2*e)^2 + 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e) + 4*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e)...
```

3.128.8 Giac [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}a^2c|c|-6\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}a^2c^2|c|}{16\,a^5c^5f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
-1/16*((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^2*c*abs(c) - 6*(c*tan
(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*a^2*c^2*abs(c))/(a^5*c^5*f)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.129 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx$

3.129.1 Optimal result

Integrand size = 30, antiderivative size = 345

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\log(\cos(e+fx))\tan(e+fx)}{a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{5\log(1-\sec(e+fx))\tan(e+fx)}{16a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{11\log(1+\sec(e+fx))\tan(e+fx)}{16a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8a^2cf(1-\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8a^2cf(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{2a^2cf(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
ln(cos(f*x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)+5/16*ln(1-sec(f*x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c
*sec(f*x+e))^(1/2)+11/16*ln(1+sec(f*x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+
e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a^2/c/f/(1-sec(f*x+e))/(a+
a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a^2/c/f/(1+sec(f
*x+e))^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/2*tan(f*x+e)/a^2/
c/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\left(16\log(\cos(e+fx))+5\log(1-\sec(e+fx))+11\log\right.}{16a^2cf\sqrt{a(1-}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
((16*Log[Cos[e + f*x]] + 5*Log[1 - Sec[e + f*x]] + 11*Log[1 + Sec[e + f*x]
] + 2/(-1 + Sec[e + f*x]) - 2/(1 + Sec[e + f*x])^2 - 8/(1 + Sec[e + f*x]))
*Tan[e + f*x])/(16*a^2*c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f
*x]])
```

3.129.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.37, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)}{a^3c^2(1-\sec(e+fx))^2(\sec(e+fx)+1)^3}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(1-\sec(e+fx))^2(\sec(e+fx)+1)^3}d\sec(e+fx)}{a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\tan(e+fx)\int\left(\cos(e+fx)-\frac{5}{16(\sec(e+fx)-1)}-\frac{11}{16(\sec(e+fx)+1)}+\frac{1}{8(\sec(e+fx)-1)^2}-\frac{1}{2(\sec(e+fx)+1)^2}-\frac{1}{4(\sec(e+fx)+1)}\right.}{a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\tan(e+fx)\left(\frac{1}{8(1-\sec(e+fx))}+\frac{1}{2(\sec(e+fx)+1)}+\frac{1}{8(\sec(e+fx)+1)^2}-\frac{5}{16}\log(1-\sec(e+fx))+\log(\sec(e+fx))-\frac{11}{16}\right.}{a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
-((((-5*Log[1 - Sec[e + f*x]])/16 + Log[Sec[e + f*x]] - (11*Log[1 + Sec[e
+ f*x]])/16 + 1/(8*(1 - Sec[e + f*x])) + 1/(8*(1 + Sec[e + f*x])^2) + 1/(2
*(1 + Sec[e + f*x])))*Tan[e + f*x])/(a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]))
```

3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.129.4 Maple [A] (warning: unable to verify)

Time = 2.26 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.76

method	result
default	$\frac{\sqrt{2}\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(-(1-\cos(fx+e))^6\csc(fx+e)^6+10(1-\cos(fx+e))^4\csc(fx+e)^4+20\ln(-\cot(f\right.}{64f\,a^3\left((1-\cos(fx+e))^2\csc(fx+e)^2\right.}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{a^2c\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}}-\frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{a^2c\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}-\frac{}{4a^2c(1+e^2}$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/64/f*2^(1/2)/a^3*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(
f*x+e))^2*csc(f*x+e)^2-1)/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)
^2-1)*csc(f*x+e)^2)^(3/2)*(1-cos(f*x+e))*(-(1-cos(f*x+e))^6*csc(f*x+e)^6+1
0*(1-cos(f*x+e))^4*csc(f*x+e)^4+20*ln(-cot(f*x+e)+csc(f*x+e))*(1-cos(f*x+e
))^2*csc(f*x+e)^2-32*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos(f*x+e))^2*
csc(f*x+e)^2+2)*csc(f*x+e)
```

3.129.5 Fricas [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{(a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*c^2*sec(f
*x + e)^5 + a^3*c^2*sec(f*x + e)^4 - 2*a^3*c^2*sec(f*x + e)^3 - 2*a^3*c^2*
sec(f*x + e)^2 + a^3*c^2*sec(f*x + e) + a^3*c^2), x)
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.129.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4272 vs. $2(309) = 618$.

Time = 1.90 (sec) , antiderivative size = 4272, normalized size of antiderivative = 12.38

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="ma
xima")
```

output
```
-1/8*(8*(f*x + e)*cos(6*f*x + 6*e)^2 + 8*(f*x + e)*cos(4*f*x + 4*e)^2 + 8*
(f*x + e)*cos(2*f*x + 2*e)^2 + 32*(f*x + e)*cos(5/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 8*(f*x + e)*sin(6*f*x + 6*e)^2 + 8*(f*x + e)*sin(4*f*x
 + 4*e)^2 + 8*(f*x + e)*sin(2*f*x + 2*e)^2 + 32*(f*x + e)*sin(5/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*sin(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*f*x + 11*(2*(cos(4*f*x + 4*e) + cos(2*
f*x + 2*e) - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e
) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 - 4*(cos
(6*f*x + 6*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) - 4*cos(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - 4*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*(cos(6*f*
x + 6*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 2*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) - 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
- 4*(cos(6*f*x + 6*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f...
```

3.129.8 Giac [A] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\frac{10\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}a^2|c|} - \frac{32\log\left(\left|-c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right|\right)}{\sqrt{-ac}a^2|c|} - \frac{2\left(5c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{\sqrt{-ac}a^2c|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2} + \frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}a^2c^2|c|-8}{a^5c^7}}{32f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="gi
ac")
```

output
```
-1/32*(10*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a^2*abs(c)) - 32*
log(abs(-c*tan(1/2*f*x + 1/2*e)^2 - c))/(sqrt(-a*c)*a^2*abs(c)) - 2*(5*c*t
an(1/2*f*x + 1/2*e)^2 - c)/(sqrt(-a*c)*a^2*c*abs(c)*tan(1/2*f*x + 1/2*e)^2
) + ((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^2*c^2*abs(c) - 8*(c*tan
(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*a^2*c^3*abs(c))/(a^5*c^7))/f
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.130 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx$

3.130.1 Optimal result

Integrand size = 30, antiderivative size = 151

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\cot(e+fx)}{2a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(\sin(e+fx))\tan(e+fx)}{a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
1/2*cot(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/4
*cot(f*x+e)^3/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+ln(s
in(f*x+e))*tan(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{2\cot(e+fx)-\cot^3(e+fx)+4(\log(\cos(e+fx))+\mathrm{lo}}{4a^2c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\,\mathrm{s}}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
(2*Cot[e + f*x] - Cot[e + f*x]^3 + 4*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]
])*Tan[e + f*x])/(4*a^2*c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e
+ f*x]])
```

3.130.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.58, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 4393, 25, 3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4393}$$

$$-\frac{\tan(e+fx)\int -\cot^5(e+fx)dx}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(e+fx)\int \cot^5(e+fx)dx}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(e+fx)\int -\tan\left(e+fx+\frac{\pi}{2}\right)^5dx}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(e+fx)\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)^5dx}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{\tan(e+fx)\left(\frac{\cot^4(e+fx)}{4f}-\int -\cot^3(e+fx)dx\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(e+fx)\left(\int \cot^3(e+fx)dx+\frac{\cot^4(e+fx)}{4f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\tan(e+fx)\left(\int -\tan\left(e+fx+\frac{\pi}{2}\right)^3dx+\frac{\cot^4(e+fx)}{4f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\tan(e+fx)\left(\frac{\cot^4(e+fx)}{4f}-\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3dx\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 3954$$

$$-\frac{\tan(e+fx)\left(\int -\cot(e+fx)dx+\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\tan(e+fx)\left(-\int \cot(e+fx)dx+\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\tan(e+fx)\left(-\int -\tan\left(e+fx+\frac{\pi}{2}\right)dx+\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\tan(e+fx)\left(\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)dx+\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 3956$$

$$-\frac{\tan(e+fx)\left(\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}-\frac{\log(-\sin(e+fx))}{f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
-(((-1/2*Cot[e + f*x]^2/f + Cot[e + f*x]^4/(4*f) - Log[-Sin[e + f*x]]/f)*T
an[e + f*x])/(a^2*c^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

3.130. $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx$

3.130.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

3.130.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.47

method	result
default	$\frac{\left(32\cos(fx+e)^4\ln(-\cot(fx+e)+\csc(fx+e))-32\cos(fx+e)^4\ln\left(\frac{2}{\cos(fx+e)+1}\right)-13\cos(fx+e)^4-64\cos(fx+e)^2\ln(-\cot(fx+e)+\mathrm{cs}\right.}{32f\,a^3\sqrt{-c(\sec(}}$
risch	$-\frac{-4i\mathrm{e}^{2i(fx+e)}\ln\left(\mathrm{e}^{2i(fx+e)}-1\right)+\mathrm{e}^{8i(fx+e)}fx+2\,\mathrm{e}^{8i(fx+e)}e+4i\mathrm{e}^{4i(fx+e)}-4\,\mathrm{e}^{6i(fx+e)}fx-8\,\mathrm{e}^{6i(fx+e)}e+i\mathrm{e}^{8i(fx+e)}\ln\left(\mathrm{e}^{2i(fx+e)}-1\right)}{a^2c^2\left(1+\mathrm{e}^{2i(fx+e)}\right)(\mathrm{e}}$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
1/32/f/a^3*(32*cos(f*x+e)^4*ln(-cot(f*x+e)+csc(f*x+e))-32*cos(f*x+e)^4*ln(
2/(cos(f*x+e)+1))-13*cos(f*x+e)^4-64*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e
))+64*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-6*cos(f*x+e)^2+32*ln(-cot(f*x+e)+c
sc(f*x+e))-32*ln(2/(cos(f*x+e)+1))+11)*(a*(sec(f*x+e)+1))^(1/2)/(-c*(sec(f
*x+e)-1))^(1/2)/(sec(f*x+e)-1)^2/c^2/(cos(f*x+e)+1)^3*tan(f*x+e)*sec(f*x+e
)
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 564, normalized size of antiderivative = 3.74

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{162\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\sqrt{-ac}\log}{}\right.$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fr
icas")
```

output
```
[-1/324*(162*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-a*c)*log(-8*((2
56*cos(f*x + e)^5 - 512*cos(f*x + e)^3 + 175*cos(f*x + e))*sqrt(-a*c)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)
) - (256*a*c*cos(f*x + e)^4 - 512*a*c*cos(f*x + e)^2 + 337*a*c)*sin(f*x +
e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) + (832*cos(f*x + e)^
5 - 1988*cos(f*x + e)^3 + 1075*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x +
e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e)), -1/324*(324*
(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(a*c)*arctan((16*cos(f*x + e)^
3 - 7*cos(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) - c)/cos(f*x + e))/((16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(
f*x + e)))*sin(f*x + e) + (832*cos(f*x + e)^5 - 1988*cos(f*x + e)^3 + 1075
*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e
)^2 + a^3*c^3*f)*sin(f*x + e))]
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.130.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(135) = 270$.

Time = 0.50 (sec) , antiderivative size = 1386, normalized size of antiderivative = 9.18

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="ma
xima")
```

output
```
-((f*x + e)*cos(8*f*x + 8*e)^2 + 16*(f*x + e)*cos(6*f*x + 6*e)^2 + 36*(f*x
 + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(2*f*x + 2*e)^2 + (f*x + e)*sin
(8*f*x + 8*e)^2 + 16*(f*x + e)*sin(6*f*x + 6*e)^2 + 36*(f*x + e)*sin(4*f*x
 + 4*e)^2 + 16*(f*x + e)*sin(2*f*x + 2*e)^2 + f*x + (2*(4*cos(6*f*x + 6*e)
 - 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*cos(8*f*x + 8*e) - cos(8*f
*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x +
6*e) - 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e
) - 36*cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e)
- 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - sin(8*f*x +
8*e)^2 + 16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 1
6*sin(6*f*x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f
*x + 2*e) - 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) - 1)*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) - 1) + 2*(f*x - 4*(f*x + e)*cos(6*f*x + 6*e)
+ 6*(f*x + e)*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) + e + 2*sin(
6*f*x + 6*e) - 2*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(8*f*x + 8*e) -
 8*(f*x + 6*(f*x + e)*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) + e
+ sin(4*f*x + 4*e))*cos(6*f*x + 6*e) + 4*(3*f*x - 12*(f*x + e)*cos(2*f*x +
 2*e) + 3*e + 2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) - 8*(f*x + e)*cos(2*f*x
 + 2*e) - 4*(2*(f*x + e)*sin(6*f*x + 6*e) - 3*(f*x + e)*sin(4*f*x + 4*e) +
 2*(f*x + e)*sin(2*f*x + 2*e) + cos(6*f*x + 6*e) - cos(4*f*x + 4*e) + c...
```

3.130.8 Giac [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{\frac{32\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}a^2c|c|}-\frac{64\log\left(\left|-c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right|\right)}{\sqrt{-ac}a^2c|c|}-\frac{48\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2+84\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c+37c^2}{\sqrt{-ac}a^2c^3|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}+\frac{(c\tan}{}}{64f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="gi
ac")
```

output
```
-1/64*(32*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a^2*c*abs(c)) - 6
4*log(abs(-c*tan(1/2*f*x + 1/2*e)^2 - c))/(sqrt(-a*c)*a^2*c*abs(c)) - (48*
(c*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 84*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 3
7*c^2)/(sqrt(-a*c)*a^2*c^3*abs(c)*tan(1/2*f*x + 1/2*e)^4) + ((c*tan(1/2*f*
x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^2*c^3*abs(c) - 10*(c*tan(1/2*f*x + 1/2*e)
^2 - c)*sqrt(-a*c)*a^2*c^4*abs(c))/(a^5*c^9))/f
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.131 $\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$

3.131.1 Optimal result

Integrand size = 24, antiderivative size = 92

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2}+n,\frac{1}{2}-m,1,\frac{3}{2}+n,\frac{1}{2}(1-\sec(e+fx)),1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}$$

output
```
2^(1/2+m)*AppellF1(1/2+n,1,1/2-m,3/2+n,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(c
-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)
```

3.131.2 Mathematica [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$

input
```
Integrate[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

output
```
Integrate[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]
```

3.131.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 4400, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^m(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \; 4400$$

$$-\frac{c\tan(e+fx)\int \cos(e+fx)(\sec(e+fx)+1)^{m-\frac{1}{2}}(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{\sec(e+fx)+1}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 153$$

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))^n\operatorname{AppellF1}\left(n+\frac{1}{2},\frac{1}{2}-m,1,n+\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2^(1/2 + m)*AppellF1[1/2 + n, 1/2 - m, 1, 3/2 + n, (1 - Sec[e + f*x])/2,
1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1
 + Sec[e + f*x]])
```

3.131.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.131.4 Maple [F]

$$\int (\sec (fx+e)+1)^m (c-c\sec (fx+e))^n\,dx$$

input
```
int((sec(f*x+e)+1)^m*(c-c*sec(f*x+e))^n,x)
```

output
```
int((sec(f*x+e)+1)^m*(c-c*sec(f*x+e))^n,x)
```

3.131.5 Fricas [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (-c\sec (fx+e)+c)^n(\sec (fx+e)+1)^m\,dx$$

input
```
integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

3.131.6 Sympy [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (-c(\sec (e+fx)-1))^n (\sec (e+fx)+1)^m\,dx$$

input
```
integrate((1+sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**n*(sec(e + f*x) + 1)**m, x)
```

3.131.7 Maxima [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (-c\sec(fx+e)+c)^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

3.131.8 Giac [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (-c\sec(fx+e)+c)^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int \left(\frac{1}{\cos(e+f\,x)}+1\right)^m \left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^m*(c - c/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^m*(c - c/cos(e + f*x))^n, x)
```

3.132 $\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$

3.132.1 Optimal result

Integrand size = 26, antiderivative size = 109

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$= \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2}-n,1,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a}{f(1+2m)}$$

output
```
2^(1/2+n)*c*AppellF1(1/2+m,1,1/2-n,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*
(1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+
e)/f/(1+2*m)
```

3.132.2 Mathematica [F]

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

output
```
Integrate[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]
```

3.132.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 4400, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^m(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int\cos(e+fx)(\sec(e+fx)a+a)^{m-\frac{1}{2}}(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{154}$$

$$-\frac{ac2^{n-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{n-1}\int\cos(e+fx)\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{n-\frac{1}{2}}(\sec(e+
}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{153}$$

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{n-1}\text{AppellF1}\left(m+\frac{1}{2},\frac{1}{2}-n,1,m
}{f(2m+1)}$$

input
```
Int[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2^(1/2 + n)*c*AppellF1[1/2 + m, 1/2 - n, 1, 3/2 + m, (1 + Sec[e + f*x])/2
, 1 + Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^m*(c
 - c*Sec[e + f*x])^(-1 + n)*Tan[e + f*x])/(f*(1 + 2*m))
```

3.132.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.132.4 Maple [F]

$$\int (a+a\sec(fx+e))^m (c-c\sec(fx+e))^n dx$$

input
```
int((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

3.132.5 Fricas [F]

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

3.132.6 Sympy [F]

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (a(\sec(e+fx)+1))^m\,(-c(\sec(e+fx)-1))^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**n, x)
```

3.132.7 Maxima [F]

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

3.132.8 Giac [F]

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^m \left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n, x)
```

3.133 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx$

3.133.1 Optimal result

Integrand size = 26, antiderivative size = 101

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx$$
$$=\frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{7}{2},\frac{1}{2}-n,1,\frac{9}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+}{7f}$$

output
```
1/7*2^(1/2+n)*c*AppellF1(7/2,1,1/2-n,9/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*
(1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+
e)/f
```

3.133.2 Mathematica [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx=\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n,x]
```

output
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n, x]
```

3.133.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 4400, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \; 4400$$

$$-\frac{ac\tan(e+fx)\int \cos(e+fx)(\sec(e+fx)a+a)^{5/2}(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 154$$

$$-\frac{ac2^{n-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{n-1}\int \cos(e+fx)\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{n-\frac{1}{2}}(\sec(e+f$$
$$f\sqrt{a\sec(e+fx)+a}$$

$$\Big\downarrow \; 153$$

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)^3(1-\sec(e+fx))^{\frac{1}{2}-n}\operatorname{AppellF1}\left(\frac{7}{2},\frac{1}{2}-n,1,\frac{9}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+$$
$$7f$$

input
```
Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2^(1/2 + n)*c*AppellF1[7/2, 1/2 - n, 1, 9/2, (1 + Sec[e + f*x])/2, 1 + Se
c[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^3*(c - c*Sec
[e + f*x])^(-1 + n)*Tan[e + f*x])/(7*f)
```

3.133.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.133.4 Maple [F]

$$\int (a + a\sec(fx + e))^3 (c - c\sec(fx + e))^n \, dx$$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x)
```

3.133.5 Fricas [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^3(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
 a^3)*(-c*sec(f*x + e) + c)^n, x)
```

3.133.6 Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx = a^3\left(\int 3(-c\sec(e+fx)+c)^n\sec(e+fx)\,dx + \int 3(-c\sec(e+fx)+c)^n\sec^2(e+fx)\,dx + \int (-c\sec(e+fx)+c)^n\sec^3(e+fx)\,dx + \int (-c\sec(e+fx)+c)^n\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**n,x)
```

output
```
a**3*(Integral(3*(-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral(3*(-
c*sec(e + f*x) + c)**n*sec(e + f*x)**2, x) + Integral((-c*sec(e + f*x) + c
)**n*sec(e + f*x)**3, x) + Integral((-c*sec(e + f*x) + c)**n, x))
```

3.133.7 Maxima [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx=\int (a\sec{(fx+e)}+a)^3(-c\sec{(fx+e)}+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^n, x)
```

3.133.8 Giac [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx=\int (a\sec{(fx+e)}+a)^3(-c\sec{(fx+e)}+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^n, x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx = \int \left(a+\frac{a}{\cos{(e+f\,x)}}\right)^3\left(c-\frac{c}{\cos{(e+f\,x)}}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^n, x)
```

3.134 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$

3.134.1 Optimal result

Integrand size = 26, antiderivative size = 101

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{5}{2},\frac{1}{2}-n,1,\frac{7}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+f}{5f}$$

output
```
1/5*2^(1/2+n)*c*AppellF1(5/2,1,1/2-n,7/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*
(1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+
e)/f
```

3.134.2 Mathematica [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = \int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n,x]
```

output
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n, x]
```

3.134.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 4400, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \cos(e+fx)(\sec(e+fx)a+a)^{3/2}(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{154}$$

$$-\frac{ac2^{n-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{n-1}\int \cos(e+fx)\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{n-\frac{1}{2}}(\sec(e+\ldots}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{153}$$

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)^2(1-\sec(e+fx))^{\frac{1}{2}-n}\operatorname{AppellF1}\left(\frac{5}{2},\frac{1}{2}-n,1,\frac{7}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+\ldots}{5f}$$

input
```
Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2^(1/2 + n)*c*AppellF1[5/2, 1/2 - n, 1, 7/2, (1 + Sec[e + f*x])/2, 1 + Se
c[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^2*(c - c*Sec
[e + f*x])^(-1 + n)*Tan[e + f*x])/(5*f)
```

3.134.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.134.4 Maple [F]

$$\int (a+a\sec(fx+e))^2 (c-c\sec(fx+e))^n\,dx$$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x)
```

3.134.5 Fricas [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^2(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*(-c*sec(f*x + e)
+ c)^n, x)
```

3.134.6 Sympy [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = a^2\left(\int 2(-c\sec(e+fx)+c)^n\sec(e+fx)\,dx + \int (-c\sec(e+fx)+c)^n\sec^2(e+fx)\,dx + \int (-c\sec(e+fx)+c)^n\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**n,x)
```

output
```
a**2*(Integral(2*(-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral((-c*
sec(e + f*x) + c)**n*sec(e + f*x)**2, x) + Integral((-c*sec(e + f*x) + c)*
*n, x))
```

3.134.7 Maxima [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^2(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^n, x)
```

3.134.8 Giac [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^2(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^n, x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^2\left(c-\frac{c}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^n, x)
```

3.135 $\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$

3.135.1 Optimal result

Integrand size = 24, antiderivative size = 99

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{2}-n,1,\frac{5}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+f}{3f}$$

output
```
1/3*2^(1/2+n)*c*AppellF1(3/2,1,1/2-n,5/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*
(1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)
/f
```

3.135.2 Mathematica [F]

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = \int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n,x]
```

output
```
Integrate[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n, x]
```

3.135.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4400, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \; 4400$$

$$-\frac{ac\tan(e+fx)\int \cos(e+fx)\sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 154$$

$$-\frac{ac2^{n-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{n-1}\int \cos(e+fx)\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{n-\frac{1}{2}}\sqrt{\sec(e+}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 153$$

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)(1-\sec(e+fx))^{\frac{1}{2}-n}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{2}-n,1,\frac{5}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+f\right.}{3f}$$

input
```
Int[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2^(1/2 + n)*c*AppellF1[3/2, 1/2 - n, 1, 5/2, (1 + Sec[e + f*x])/2, 1 + Se
c[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])*(c - c*Sec[e
 + f*x])^(-1 + n)*Tan[e + f*x])/(3*f)
```

3.135.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.135.4 Maple [F]

$$\int (a + a\sec(fx + e))(c - c\sec(fx + e))^n dx$$

input
```
int((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)
```

3.135.5 Fricas [F]

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

3.135.6 Sympy [F]

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = a\left(\int (-c\sec(e+fx)+c)^n\sec(e+fx)\,dx + \int (-c\sec(e+fx)+c)^n\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))**n,x)
```

output
```
a*(Integral((-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral((-c*sec(e
 + f*x) + c)**n, x))
```

3.135.7 Maxima [F]

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

3.135.8 Giac [F]

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)\left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^n, x)
```

3.136 $\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx$

3.136.1 Optimal result

Integrand size = 26, antiderivative size = 99

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx = -\frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(-\frac{1}{2},\frac{1}{2}-n,1,\frac{1}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e}{f(a+a\sec(e+fx))}$$

output
```
-2^(1/2+n)*c*AppellF1(-1/2,1,1/2-n,1/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(1
-sec(f*x+e))^(1/2-n)*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

3.136.2 Mathematica [F]

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx = \int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx$$

input
```
Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

output
```
Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]), x]
```

3.136.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 4400, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^n}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}}{(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{154}$$

$$-\frac{ac2^{n-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{n-1}\int \frac{\cos(e+fx)\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{n-\frac{1}{2}}}{(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{153}$$

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}\text{AppellF1}\left(-\frac{1}{2},\frac{1}{2}-n,1,\frac{1}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec(e+fx))^{n-1}}{f(a\sec(e+fx)+a)}$$

input
```
Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

output
```
-((2^(1/2 + n)*c*AppellF1[-1/2, 1/2 - n, 1, 1/2, (1 + Sec[e + f*x])/2, 1 +
 Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(c - c*Sec[e + f*x])^(-1 + n)*
Tan[e + f*x])/(f*(a + a*Sec[e + f*x])))
```

3.136.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.136.4 Maple [F]

$$\int \frac{(c - c\sec(fx + e))^n}{a + a\sec(fx + e)} dx$$

input
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

output
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

3.136.5 Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx = \int \frac{(-c\sec(fx+e)+c)^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

3.136.6 Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{(-c\sec(e+fx)+c)^n}{\sec(e+fx)+1}\,dx}{a}$$

input
```
integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e)),x)
```

output
```
Integral((-c*sec(e + f*x) + c)**n/(sec(e + f*x) + 1), x)/a
```

3.136.7 Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx = \int \frac{(-c\sec(fx+e)+c)^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

3.136.8 Giac [F]

$$\int \frac{(c - c\sec(e + fx))^n}{a + a\sec(e + fx)}\,dx = \int \frac{(-c\sec(fx + e) + c)^n}{a\sec(fx + e) + a}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^n}{a + a\sec(e + fx)}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^n}{a + \frac{a}{\cos(e+f\,x)}}\,dx$$

input
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x)),x)
```

output
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x)), x)
```

3.137 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$

3.137.1 Optimal result

Integrand size = 26, antiderivative size = 101

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = -\frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(-\frac{3}{2},\frac{1}{2}-n,1,-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec}{3f(a+a\sec(e+fx))^2}$$

output
```
-1/3*2^(1/2+n)*c*AppellF1(-3/2,1,1/2-n,-1/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e
))*(1-sec(f*x+e))^(1/2-n)*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f/(a+a*sec(f*
x+e))^2
```

3.137.2 Mathematica [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$$

input
```
Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2, x]
```

3.137.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 4400, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^n}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \ 4400$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}}{(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 154$$

$$-\frac{ac2^{n-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{n-1}\int \frac{\cos(e+fx)\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{n-\frac{1}{2}}}{(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 153$$

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}\operatorname{AppellF1}\left(-\frac{3}{2},\frac{1}{2}-n,1,-\frac{1}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c}{3f(a\sec(e+fx)+a)^2}$$

input
```
Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
-1/3*(2^(1/2 + n)*c*AppellF1[-3/2, 1/2 - n, 1, -1/2, (1 + Sec[e + f*x])/2,
 1 + Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(c - c*Sec[e + f*x])^(-1 +
 n)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^2)
```

3.137.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.137.4 Maple [F]

$$\int \frac{(c - c\sec(fx+e))^n}{(a + a\sec(fx+e))^2} dx$$

input
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

output
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

3.137.5 Fricas [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^2}\, dx = \int \frac{(-c\sec(fx + e) + c)^n}{(a\sec(fx + e) + a)^2}\, dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((-c*sec(f*x + e) + c)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e)
+ a^2), x)
```

3.137.6 Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^2}\, dx = \frac{\int \frac{(-c\sec(e+fx)+c)^n}{\sec^2(e+fx)+2\sec(e+fx)+1}\, dx}{a^2}$$

input
```
integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**2,x)
```

output
```
Integral((-c*sec(e + f*x) + c)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),
x)/a**2
```

3.137.7 Maxima [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^2}\, dx = \int \frac{(-c\sec(fx + e) + c)^n}{(a\sec(fx + e) + a)^2}\, dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^2, x)
```

3.137.8 Giac [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^2} \, dx = \int \frac{(-c\sec(fx + e) + c)^n}{(a\sec(fx + e) + a)^2} \, dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^2, x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^2} \, dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^n}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^2} \, dx$$

input
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^2,x)
```

output
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^2, x)
```

3.138 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx$

3.138.1 Optimal result

Integrand size = 28, antiderivative size = 172

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\frac{6a^3(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{2a^3\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{2a^3(c-c\sec(e+fx))^{1+n}\tan(e+fx)}{cf(3+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
6*a^3*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^3
*hypergeom([1, 1/2+n],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/
f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)-2*a^3*(c-c*sec(f*x+e))^(1+n)*tan(f*x+e)/c
/f/(3+2*n)/(a+a*sec(f*x+e))^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.59

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\frac{2a^3(c-c\sec(e+fx))^n\left(4(2+n)+(3+2n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\right.\right.}{f(1+2n)(3+2n)\sqrt{a(1+\sec(e+f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2*a^3*(c - c*Sec[e + f*x])^n*(4*(2 + n) + (3 + 2*n)*Hypergeometric2F1[1,
1/2 + n, 3/2 + n, 1 - Sec[e + f*x]] + (1 + 2*n)*Sec[e + f*x])*Tan[e + f*x]
)/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[a*(1 + Sec[e + f*x])])
```

3.138.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3042, 4400, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int a^2\cos(e+fx)(\sec(e+fx)+1)^2(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3c\tan(e+fx)\int \cos(e+fx)(\sec(e+fx)+1)^2(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^3c\tan(e+fx)\int \left(\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}+3(c-c\sec(e+fx))^{n-\frac{1}{2}}-\frac{(c-c\sec(e+fx))^{n+\frac{1}{2}}}{c}\right)d\sec(e+$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3c\tan(e+fx)\left(\frac{2(c-c\sec(e+fx))^{n+\frac{3}{2}}}{c^2(2n+3)}-\frac{2(c-c\sec(e+fx))^{n+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{c(2n+1)}-\frac{6(c-c\sec(e+fx))}{c(2n+1)}\right.}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n,x]
```

output
```
-((a^3*c*((-6*(c - c*Sec[e + f*x])^(1/2 + n))/(c*(1 + 2*n)) - (2*Hypergeom
etric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^(1/2
+ n))/(c*(1 + 2*n)) + (2*(c - c*Sec[e + f*x])^(3/2 + n))/(c^2*(3 + 2*n)))*
Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

3.138.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.138.4 Maple [F]

$$\int (a+a\sec (fx+e))^{\frac{5}{2}} (c-c\sec (fx+e))^{n} dx$$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x)
```

3.138.5 Fricas [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx = \int (a\sec (fx+e)+a)^{\frac{5}{2}}(-c\sec (fx+e)+c)^{n}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

3.138.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**n,x)
```

output
```
Timed out
```

3.138.7 Maxima [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(5/2)*(-c*sec(f*x + e) + c)^n, x)
```

3.138.8 Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
sage0*x
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^n, x)
```

3.139 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx$

3.139.1 Optimal result

Integrand size = 28, antiderivative size = 119

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx=\frac{2a^2(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2a^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^2*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2
*hypergeom([1, 1/2+n],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/
f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx=-\frac{2a^2\left(-6-4n+(1+2n)\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}+n,\frac{5}{2}+n,1-\sec(e+fx)\right)(-1+\sec(e+fx))\right)(c-}{f(1+2n)(3+2n)\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n,x]
```

output
```
(-2*a^2*(-6 - 4*n + (1 + 2*n)*Hypergeometric2F1[1, 3/2 + n, 5/2 + n, 1 - S
ec[e + f*x]]*(-1 + Sec[e + f*x]))*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*
(1 + 2*n)*(3 + 2*n)*Sqrt[a*(1 + Sec[e + f*x])])
```

3.139.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3042, 4397, 3042, 4400, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4397}$$

$$a\int \sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^n dx+\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx+\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4400}$$

$$\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{a^2c\tan(e+fx)\int\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2*a^2*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e
+ f*x]]) + (2*a^2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]
*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]
])
```

3.139.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4397
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-2*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a   Int[Sqrt[a +
b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.139.4 Maple [F]

$$\int (a + a \sec (fx + e))^{\frac{3}{2}} (c - c \sec (fx + e))^n \, dx$$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x)
```

3.139.5 Fricas [F]

$$\int (a+a\sec(e+fx))^{3/2}(c - c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**n,x)
```

output
```
Timed out
```

3.139.7 Maxima [F]

$$\int (a+a\sec(e+fx))^{3/2}(c - c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)
```

3.139.8 Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
sage0*x
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^n, x)
```

3.140 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$

3.140.1 Optimal result

Integrand size = 28, antiderivative size = 68

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1, 1/2+n],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+
e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)\sqrt{a(1+\sec(e+fx))}(c-c\sec(e+fx))^n\tan\left(\frac{1}{2}(\right.}{f+2fn}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*Sqrt[a*(1 + Se
c[e + f*x])]*(c - c*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f + 2*f*n)
```

3.140.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 4400, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \ 4400$$

$$-\frac{ac\tan(e+fx)\int \cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 75$$

$$\frac{2a\tan(e+fx)(c-c\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2*a*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e
 + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

3.140.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.140.4 Maple [F]

$$\int \sqrt{a + a\sec(fx+e)}\,(c - c\sec(fx+e))^n\,dx$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x)
```

3.140.5 Fricas [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx = \int \sqrt{a\sec(fx+e)+a}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

3.140.6 Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx = \int \sqrt{a(\sec(e+fx)+1)}(-c(\sec(e+fx)-1))^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**n,x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**n, x)
```

3.140.7 Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx = \int \sqrt{a\sec(fx+e)+a}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

3.140.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx = \int \sqrt{a\sec(fx+e)+a}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
sage0*x
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^n, x)
```

3.141 $\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.141.1 Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,\frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
-hypergeom([1, 1/2+n],[3/2+n],1/2-1/2*sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f
*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*hypergeom([1, 1/2+n],[3/2+n],1-se
c(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\left(\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,\frac{1}{2}(1-\sec(e+fx))\right)-2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1\right.\right.}{(f+2fn)\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-(((Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2] - 2*Hyper
geometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]])*(c - c*Sec[e + f*x])^
n*Tan[e + f*x])/((f + 2*f*n)*Sqrt[a*(1 + Sec[e + f*x])]))
```

3.141.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4400, 27, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}}{a(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\tan(e+fx)\int \frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}}{\sec(e+fx)+1}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{97}$$

$$-\frac{c\tan(e+fx)\left(\int \cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)-\int \frac{(c-c\sec(e+fx))^{n-\frac{1}{2}}}{\sec(e+fx)+1}d\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{75}$$

$$-\frac{c\tan(e+fx)\left(-\int \frac{(c-c\sec(e+fx))^{n-\frac{1}{2}}}{\sec(e+fx)+1}d\sec(e+fx)-\frac{2(c-c\sec(e+fx))^{n+\frac{1}{2}}\,\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{c(2n+1)}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 78$$

$$-\frac{c\tan(e+fx)\left(\frac{(c-c\sec(e+fx))^{n+\frac{1}{2}}\,\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)}{c(2n+1)}-\frac{2(c-c\sec(e+fx))^{n+\frac{1}{2}}\,\text{Hypergeometric2F1}(1}{c(2n+1)}\right.}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((c*((Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2]*(c - c
*Sec[e + f*x])^(1/2 + n))/(c*(1 + 2*n)) - (2*Hypergeometric2F1[1, 1/2 + n,
 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^(1/2 + n))/(c*(1 + 2*n)))
*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

3.141.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.141.4 Maple [F]

$$\int \frac{(c - c\sec(fx + e))^n}{\sqrt{a + a\sec(fx + e)}}dx$$

input
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

3.141.5 Fricas [F]

$$\int \frac{(c - c\sec(e + fx))^n}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{(-c\sec(fx + e) + c)^n}{\sqrt{a\sec(fx + e) + a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

3.141.6 Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^n}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{(-c(\sec(e + fx) - 1))^n}{\sqrt{a(\sec(e + fx) + 1)}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

3.141.7 Maxima [F]

$$\int \frac{(c - c\sec(e + fx))^n}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{(-c\sec(fx + e) + c)^n}{\sqrt{a\sec(fx + e) + a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

3.141.8 Giac [F]

$$\int \frac{(c - c\sec(e + fx))^n}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{(-c\sec(fx + e) + c)^n}{\sqrt{a\sec(fx + e) + a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^n}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

3.142 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.142.1 Optimal result

Integrand size = 28, antiderivative size = 205

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{(5-2n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,\frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(e+fx)}{4af(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{af(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{(c-c\sec(e+fx))^n\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$

output
```
-1/4*(5-2*n)*hypergeom([1, 1/2+n],[3/2+n],1/2-1/2*sec(f*x+e))*(c-c*sec(f*x
+e))^n*tan(f*x+e)/a/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*hypergeom([1, 1/2+n
],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/a/f/(1+2*n)/(a+a*sec
(f*x+e))^(1/2)-1/2*(c-c*sec(f*x+e))^n*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*s
ec(f*x+e))^(1/2)
```

3.142.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{(c-c\sec(e+fx))^n\left(-2-4n+(-5+2n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}\right.\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
((c - c*Sec[e + f*x])^n*(-2 - 4*n + (-5 + 2*n)*Hypergeometric2F1[1, 1/2 +
n, 3/2 + n, (1 - Sec[e + f*x])/2]*(1 + Sec[e + f*x]) + 8*Hypergeometric2F1
[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(1 + Sec[e + f*x]))*Tan[e + f*x])/
(4*(f + 2*f*n)*(a*(1 + Sec[e + f*x]))^(3/2))
```

3.142.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4400, 27, 114, 27, 174, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^n}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}}{a^2(\sec(e+fx)+1)^2}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\tan(e+fx)\int \frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}}{(\sec(e+fx)+1)^2}d\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{114}$$

$$-\frac{c\tan(e+fx)\left(\frac{\int \frac{c\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}(4-(1-2n)\sec(e+fx))}{2(\sec(e+fx)+1)}d\sec(e+fx)}{2c}+\frac{(c-c\sec(e+fx))^{n+\frac{1}{2}}}{2c(\sec(e+fx)+1)}\right)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c\tan(e+fx)\left(\frac{1}{4}\int \frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}(4-(1-2n)\sec(e+fx))}{\sec(e+fx)+1}d\sec(e+fx)+\frac{(c-c\sec(e+fx))^{n+\frac{1}{2}}}{2c(\sec(e+fx)+1)}\right)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 174$$

$$-\frac{c\tan(e+fx)\left(\frac{1}{4}\left(4\int\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)-(5-2n)\int\frac{(c-c\sec(e+fx))^{n-\frac{1}{2}}}{\sec(e+fx)+1}d\sec(e+f\right.\right.}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 75$$

$$-\frac{c\tan(e+fx)\left(\frac{1}{4}\left(-(5-2n)\int\frac{(c-c\sec(e+fx))^{n-\frac{1}{2}}}{\sec(e+fx)+1}d\sec(e+fx)-\frac{8(c-c\sec(e+fx))^{n+\frac{1}{2}}\text{Hypergeometric2F1}(1,n+\frac{1}{2},n+\frac{3}{2},1-\text{s}}{c(2n+1)}\right.\right.}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 78$$

$$-\frac{c\tan(e+fx)\left(\frac{1}{4}\left(\frac{(5-2n)(c-c\sec(e+fx))^{n+\frac{1}{2}}\text{Hypergeometric2F1}(1,n+\frac{1}{2},n+\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)))}{c(2n+1)}-\frac{8(c-c\sec(e+fx))^{n+\frac{1}{2}}\text{Hypergeon}}{c(}\right.\right.}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c*((c - c*Sec[e + f*x])^(1/2 + n)/(2*c*(1 + Sec[e + f*x])) + (((5 - 2*n
)*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2]*(c - c*Sec[
e + f*x])^(1/2 + n))/(c*(1 + 2*n)) - (8*Hypergeometric2F1[1, 1/2 + n, 3/2
+ n, 1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^(1/2 + n))/(c*(1 + 2*n)))/4)*T
an[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

3.142. $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.142.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.142. $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.142.4 Maple [F]

$$\int \frac{(c - c\sec(fx + e))^n}{(a + a\sec(fx + e))^{\frac{3}{2}}}dx$$

input
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

3.142.5 Fricas [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^{3/2}} dx = \int \frac{(-c\sec(fx + e) + c)^n}{(a\sec(fx + e) + a)^{\frac{3}{2}}} dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n/(a^2*sec(f*x + e
)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

3.142.6 Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^{3/2}} dx = \int \frac{(-c(\sec(e + fx) - 1))^n}{(a(\sec(e + fx) + 1))^{\frac{3}{2}}} dx$$

input
```
integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.142.7 Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

3.142.8 Giac [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

3.143 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

3.143.1 Optimal result

Integrand size = 27, antiderivative size = 91

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}-\frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{cf}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c/f-arctan(1/2
*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)*a^(1/2)/c/f
```

3.143.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=$$
$$-\frac{i\sqrt{1+e^{2i(e+fx)}}\left(\operatorname{arcsinh}\left(e^{i(e+fx)}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{-1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)-\operatorname{arctanh}\left(\sqrt{1+e^{2i(e+fx)}}\right)\right)\sqrt{a(1+\text{s}}}{c\left(1+e^{i(e+fx)}\right)f}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + c*Sec[e + f*x]),x]
```

output
```
((-I)*Sqrt[1 + E^((2*I)*(e + f*x))]*(ArcSinh[E^(I*(e + f*x))] - Sqrt[2]*Ar
cTanh[(-1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])] - Ar
cTanh[Sqrt[1 + E^((2*I)*(e + f*x))]])*Sqrt[a*(1 + Sec[e + f*x])])/(c*(1 +
E^(I*(e + f*x)))*f)
```

3.143.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2011, 3042, 4263, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{c\sec(e+fx)+c}dx$$

$$\Big\downarrow \text{2011}$$

$$\frac{a\int \frac{1}{\sqrt{\sec(e+fx)a+a}}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{1}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{c}$$

$$\Big\downarrow \text{4263}$$

$$\frac{a\left(\frac{\int \sqrt{\sec(e+fx)a+a}dx}{a}-\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx\right)}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{a}-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx\right)}{c}$$

$$\Big\downarrow \text{4261}$$

$$\frac{a\left(-\frac{2\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx\right)}{c}$$

$$\Big\downarrow \text{216}$$

$$\frac{a\left(\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx\right)}{c}$$

$\Big\downarrow$ 4282

$$\frac{a\left(\frac{2\int\frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+2a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}+\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}\right)}{c}$$

$\Big\downarrow$ 216

$$\frac{a\left(\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}\right)}{c}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c + c*Sec[e + f*x]),x]
```

output
```
(a*((2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f
) - (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x
]])])/(Sqrt[a]*f)))/c
```

3.143.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.143. $\int\frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4263
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[1/a   I
nt[Sqrt[a + b*Csc[c + d*x]], x], x] - Simp[b/a   Int[Csc[c + d*x]/Sqrt[a +
b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.143.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.49

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)-\right.}{cf}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/c/f*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2^(1/2
)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e
)^2-1)^(1/2))-2*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)))
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \left[\frac{\sqrt{2}\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+3\,a\cos(fx+e)^2+2\,a\cos(fx+e)-a}{\cos(fx+e)^2+2\,\cos(fx+e)+1}\right)+2\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2\cdots}{}\right.}{2\,cf}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x
+ e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-a)*log((2*a*cos
(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
 e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), (sqrt(2
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
 e)/(sqrt(a)*sin(f*x + e))) - 2*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*f)]
```

3.143.6 Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c + c\sec(e + fx)}\,dx = \frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)+1}\,dx}{c}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

output
```
Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) + 1), x)/c
```

3.143.7 Maxima [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c + c\sec(e + fx)}\,dx = \int \frac{\sqrt{a\sec(fx + e) + a}}{c\sec(fx + e) + c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) + c), x)
```

3.143.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{c\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="giac")
```

output
```
sage0*x
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{c+\frac{c}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c + c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c + c/cos(e + f*x)), x)
```

3.144 $\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx$

3.144.1 Optimal result

Integrand size = 27, antiderivative size = 231

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx =$$

$$-\frac{2c\cot(e+fx)\,\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(1+\sec(e+fx))}{c+d\sec(e+fx)}}(c+d\sec(e+}{a\sqrt{c+d}f}$$

$$-\frac{(c-d)E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}}$$

output
```
-2*c*cot(f*x+e)*EllipticPi((c+d)^(1/2)/(c+d*sec(f*x+e))^(1/2),c/(c+d),((c-
d)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(-d*(1-sec(f*x+e))/(c+d*sec(f*x+e)))^(1/
2)*(d*(1+sec(f*x+e))/(c+d*sec(f*x+e)))^(1/2)/a/f/(c+d)^(1/2)-(c-d)*Ellipti
cE(tan(f*x+e)/(1+sec(f*x+e)),((c-d)/(c+d))^(1/2))*(1/(1+sec(f*x+e)))^(1/2)
*(c+d*sec(f*x+e))^(1/2)/a/f/((c+d*sec(f*x+e))/(c+d)/(1+sec(f*x+e)))^(1/2)
```

3.144.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 810 vs. $2(231) = 462$.

Time = 34.07 (sec) , antiderivative size = 810, normalized size of antiderivative = 3.51

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)(c+d\sec(e+fx))^{3/2}\left(2\sec\left(\frac{1}{2}(e+fx)\right)\left(-c\sin\left(\frac{1}{2}(e+fx)\right)+\right.\right.}{f(d+c\cos(e+fx))(a+a\sec(e+}$$

$$+\frac{2\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)(c+d\sec(e+fx))^{3/2}\left(c^2\tan\left(\frac{1}{2}(e+fx)\right)-d^2\tan\left(\frac{1}{2}(e+fx)\right)-2c^2\tan^3\left(\frac{1}{2}(e+fx)\right)+\right.}{}$$

input
```
Integrate[(c + d*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x]),x]
```

output
```
(Cos[e/2 + (f*x)/2]^2*(c + d*Sec[e + f*x])^(3/2)*(2*Sec[(e + f*x)/2]*(-(c*
Sin[(e + f*x)/2]) + d*Sin[(e + f*x)/2]) - 2*(-c + d)*Sin[e + f*x]))/(f*(d
+ c*Cos[e + f*x])*(a + a*Sec[e + f*x])) + (2*Cos[e/2 + (f*x)/2]^2*(c + d*S
ec[e + f*x])^(3/2)*(c^2*Tan[(e + f*x)/2] - d^2*Tan[(e + f*x)/2] - 2*c^2*Ta
n[(e + f*x)/2]^3 + 2*c*d*Tan[(e + f*x)/2]^3 + c^2*Tan[(e + f*x)/2]^5 - 2*c
*d*Tan[(e + f*x)/2]^5 + d^2*Tan[(e + f*x)/2]^5 - 4*c^2*EllipticPi[-1, ArcS
in[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(
c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)] - 4*c^2*Elli
pticPi[-1, ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Tan[(e + f*x)/2]^2*S
qrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e
+ f*x)/2]^2)/(c + d)] + (c^2 - d^2)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c
 - d)/(c + d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[
(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)] + 2*c*(c -
d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Sqrt[1 - Tan[(e +
f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d
*Tan[(e + f*x)/2]^2)/(c + d)]))/(f*(d + c*Cos[e + f*x])^(3/2)*Sqrt[Sec[e +
 f*x]]*(a + a*Sec[e + f*x])*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)]*(-1 + Tan[
(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)^(3/2)*Sqrt[(c + d - c*Tan[(e + f*
x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)])
```

3.144.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4415, 3042, 4267, 4456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4415}$$

$$\frac{c\int \sqrt{c+d\sec(e+fx)}dx}{a}-(c-d)\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sec(e+fx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\int \sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a}-(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4267}$$

$$-(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-\frac{2c\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)}{af\sqrt{c+d}}$$

$$\Big\downarrow \text{4456}$$

$$-\frac{2c\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)}{af\sqrt{c+d}} - \frac{(c-d)\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(\sec(e+fx)+1)}}}$$

input
```
Int[(c + d*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x]),x]
```

output
```
(-2*c*Cot[e + f*x]*EllipticPi[c/(c + d), ArcSin[Sqrt[c + d]/Sqrt[c + d*Sec
[e + f*x]]], (c - d)/(c + d)]*Sqrt[-((d*(1 - Sec[e + f*x]))/(c + d*Sec[e +
 f*x]))]*Sqrt[(d*(1 + Sec[e + f*x]))/(c + d*Sec[e + f*x])]*(c + d*Sec[e +
f*x]))/(a*Sqrt[c + d]*f) - ((c - d)*EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec
[e + f*x])], (c - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec
[e + f*x]])/(a*f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

3.144.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4267
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

rule 4415
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Simp[a/c   Int[Sqrt[a + b*Csc[e + f*x]], x], x]
+ Simp[(b*c - a*d)/c   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Cs
c[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

rule 4456
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x])))]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

3.144.4 Maple [A] (verified)

Time = 8.25 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.11

method	result
default	$\frac{(\cos(fx+e)+1)\left(2\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)c^{2}-2\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)cd+\mathrm{EllipticE}\left(\cot(fx+e\right.\right.}{}$

input
```
int((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/a/f*(cos(f*x+e)+1)*(2*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2
))*c^2-2*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*c*d+Elliptic
E(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*c^2-EllipticE(cot(f*x+e)-csc(
f*x+e),((c-d)/(c+d))^(1/2))*d^2-4*c^2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,
((c-d)/(c+d))^(1/2)))*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(d+c*cos(f*x+e))
```

3.144.5 Fricas [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx=\int \frac{(d\sec{(fx+e)}+c)^{\frac{3}{2}}}{a\sec{(fx+e)}+a}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

3.144.6 Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx=\frac{\int \frac{c\sqrt{c+d\sec{(e+fx)}}}{\sec{(e+fx)}+1}\,dx+\int \frac{d\sqrt{c+d\sec{(e+fx)}}\sec{(e+fx)}}{\sec{(e+fx)}+1}\,dx}{a}$$

input
```
integrate((c+d*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e)),x)
```

output
```
(Integral(c*sqrt(c + d*sec(e + f*x))/(sec(e + f*x) + 1), x) + Integral(d*s
qrt(c + d*sec(e + f*x))*sec(e + f*x)/(sec(e + f*x) + 1), x))/a
```

3.144.7 Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx=\int \frac{(d\sec(fx+e)+c)^{\frac{3}{2}}}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

3.144.8 Giac [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx=\int \frac{(d\sec(fx+e)+c)^{\frac{3}{2}}}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx=\int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

input
```
int((c + d/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x)),x)
```

output
```
int((c + d/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x)), x)
```

3.145 $\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$

3.145.1 Optimal result

Integrand size = 27, antiderivative size = 225

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\,\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(1+\sec(e+fx))}{c+d\sec(e+fx)}}(c+d\sec(e}{a\sqrt{c+d}f}$$
$$-\frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)\middle|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}}$$

output
```
-2*cot(f*x+e)*EllipticPi((c+d)^(1/2)/(c+d*sec(f*x+e))^(1/2),c/(c+d),((c-d)
/(c+d))^(1/2))*(c+d*sec(f*x+e))*(-d*(1-sec(f*x+e))/(c+d*sec(f*x+e)))^(1/2)
*(d*(1+sec(f*x+e))/(c+d*sec(f*x+e)))^(1/2)/a/f/(c+d)^(1/2)-EllipticE(tan(f
*x+e)/(1+sec(f*x+e)),((c-d)/(c+d))^(1/2))*(1/(1+sec(f*x+e)))^(1/2)*(c+d*se
c(f*x+e))^(1/2)/a/f/((c+d*sec(f*x+e))/(c+d)/(1+sec(f*x+e)))^(1/2)
```

3.145.2 Mathematica [A] (verified)

Time = 7.53 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = -\frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\left((c+d)E\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{c-d}{c+d}\right)+2(c-d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+f\right.\right.\right.\right.}{a(c+d)f(1+\cos(e+fx)}$$

input
```
Integrate[Sqrt[c + d*Sec[e + f*x]]/(a + a*Sec[e + f*x]),x]
```

output
```
(-4*Cos[(e + f*x)/2]^4*((c + d)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d
)/(c + d)] + 2*(c - d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)
] - 4*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)])*Sqrt[(1
 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*(c + d)*f*(1 + Cos[e +
 f*x])^2*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x]))])
```

3.145.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4413, 3042, 4267, 4456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4413}$$

$$\frac{\int \sqrt{c+d\sec(e+fx)}dx}{a}-\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sec(e+fx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a}-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4267}$$

$$-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-\frac{2\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\operatorname{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)}{af\sqrt{c+d}}$$

$$\Big\downarrow \text{4456}$$

$$-\frac{2\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\operatorname{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)}{af\sqrt{c+d}}-\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(\sec(e+fx)+1)}}}$$

input
```
Int[Sqrt[c + d*Sec[e + f*x]]/(a + a*Sec[e + f*x]),x]
```

output
```
(-2*Cot[e + f*x]*EllipticPi[c/(c + d), ArcSin[Sqrt[c + d]/Sqrt[c + d*Sec[e
 + f*x]]], (c - d)/(c + d)]*Sqrt[-((d*(1 - Sec[e + f*x]))/(c + d*Sec[e + f
*x]))]*Sqrt[(d*(1 + Sec[e + f*x]))/(c + d*Sec[e + f*x])]*(c + d*Sec[e + f*
x]))/(a*Sqrt[c + d]*f) - (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])
], (c - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]]
)/(a*f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

3.145.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4267
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

rule 4413
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[1/c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[d/c   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 -
 b^2, 0] || EqQ[c^2 - d^2, 0])
```

rule 4456
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x]))])]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

3.145.4 Maple [A] (verified)

Time = 5.05 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.10

method	result
default	$\frac{(\cos(fx+e)+1)\left(2\,\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)c-2\,\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)d+c\,\text{EllipticE}\left(\cot(fx+e\right.\right.}{}$

input
```
int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/a/f*(cos(f*x+e)+1)*(2*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2
))*c-2*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*d+c*EllipticE(
cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))+d*EllipticE(cot(f*x+e)-csc(f*x+
e),((c-d)/(c+d))^(1/2))-4*c*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((c-d)/(c+
d))^(1/2)))*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(d+c*cos(f*x+e))
```

3.145.5 Fricas [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{a\sec\left(fx+e\right)+a}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

3.145.6 Sympy [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{\sqrt{c+d\sec\left(e+fx\right)}}{\sec\left(e+fx\right)+1}\,dx}{a}$$

input
```
integrate((c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e)),x)
```

output
```
Integral(sqrt(c + d*sec(e + f*x))/(sec(e + f*x) + 1), x)/a
```

3.145.7 Maxima [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{a\sec\left(fx+e\right)+a}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

3.145.8 Giac [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=\int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{a\sec\left(fx+e\right)+a}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=\int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

input
```
int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x)),x)
```

output
```
int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x)), x)
```

3.146 $\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$

3.146.1 Optimal result

Integrand size = 27, antiderivative size = 319

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sec(e+fx))}{c-d}}}{a(c-d)f} - \frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{c+d}{c},\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sec(e+fx))}{c-d}}}{acf} - \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{a(c-d)f\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}}$$

output
```
2*cot(f*x+e)*EllipticF((c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2),((c+d)/(c-d))^(1
/2))*(c+d)^(1/2)*(d*(1-sec(f*x+e))/(c+d))^(1/2)*(-d*(1+sec(f*x+e))/(c-d))^
(1/2)/a/(c-d)/f-2*cot(f*x+e)*EllipticPi((c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)
,(c+d)/c,((c+d)/(c-d))^(1/2))*(c+d)^(1/2)*(d*(1-sec(f*x+e))/(c+d))^(1/2)*(
-d*(1+sec(f*x+e))/(c-d))^(1/2)/a/c/f-EllipticE(tan(f*x+e)/(1+sec(f*x+e)),(
(c-d)/(c+d))^(1/2))*(1/(1+sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))^(1/2)/a/(c-d
)/f/((c+d*sec(f*x+e))/(c+d)/(1+sec(f*x+e)))^(1/2)
```

3.146.2 Mathematica [A] (verified)

Time = 8.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx =$$

$$-\frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{d+c\cos(e+fx)}{(c+d)(1+\cos(e+fx))}}\left((c+d)E\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{c-d}{c+d}\right)+2(c-2d)\,\text{EllipticF}\,(a}{a(c-}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
(-4*Cos[(e + f*x)/2]^4*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x
]))]*((c + d)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)] + 2*(c
- 2*d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)] + 4*(-c + d)*E
llipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)])*Sec[e + f*x]^2*(
(1 + Sec[e + f*x])^(-1))^(3/2))/(a*(c - d)*f*Sqrt[c + d*Sec[e + f*x]])
```

3.146.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4417, 25, 3042, 4409, 3042, 4271, 4319, 4456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4417}$$

$$-\frac{\int -\frac{a(c-d)+ad\sec(e+fx)}{\sqrt{c+d\sec(e+fx)}}dx}{a^2(c-d)}-\frac{\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sec(e+fx)a+a}dx}{c-d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(c-d)+ad\sec(e+fx)}{\sqrt{c+d\sec(e+fx)}}dx}{a^2(c-d)}-\frac{\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sec(e+fx)a+a}dx}{c-d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{a(c-d)+ad\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a^2(c-d)}-\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c-d}$$

$$\Big\downarrow \text{ 4409}$$

$$\frac{a(c-d)\int \frac{1}{\sqrt{c+d\sec(e+fx)}}dx+ad\int \frac{\sec(e+fx)}{\sqrt{c+d\sec(e+fx)}}dx}{a^2(c-d)}-\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c-d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(c-d)\int \frac{1}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+ad\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a^2(c-d)}-\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c-d}$$

$$\Big\downarrow \text{ 4271}$$

$$\frac{ad\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2a(c-d)\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(\sec(e+fx)+1)}{c-d}}\text{EllipticPi}\left(\frac{c+d}{c},\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)}{cf}}{a^2(c-d)}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c-d}$$

$$\Big\downarrow \text{ 4319}$$

$$\frac{\frac{2a\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(\sec(e+fx)+1)}{c-d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)}{f}-\frac{2a(c-d)\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}}{}}{a^2(c-d)}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c-d}$$

$$\Big\downarrow \text{ 4456}$$

$$\frac{\frac{2a\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(\sec(e+fx)+1)}{c-d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)}{f}-\frac{2a(c-d)\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}}{}}{a^2(c-d)}$$

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{c-d}{c+d}\right)}{af(c-d)\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(\sec(e+fx)+1)}}}$$

3.146. $\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$

input
```
Int[1/((a + a*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
-((EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (c - d)/(c + d)]*Sqr
t[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*(c - d)*f*Sqrt[(c
+ d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])) + ((2*a*Sqrt[c + d]*Cot[
e + f*x]*EllipticF[ArcSin[Sqrt[c + d*Sec[e + f*x]]/Sqrt[c + d]], (c + d)/(
c - d)]*Sqrt[(d*(1 - Sec[e + f*x]))/(c + d)]*Sqrt[-((d*(1 + Sec[e + f*x]))
/(c - d))])/f - (2*a*(c - d)*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(c + d)/c
, ArcSin[Sqrt[c + d*Sec[e + f*x]]/Sqrt[c + d]], (c + d)/(c - d)]*Sqrt[(d*(
1 - Sec[e + f*x]))/(c + d)]*Sqrt[-((d*(1 + Sec[e + f*x]))/(c - d))])/(c*f)
)/(a^2*(c - d))
```

3.146.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4417
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Simp[1/(c*(b*c - a*d))   Int[(b*c - a*d - b*d*
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[d^2/(c*(b*c - a*d))
 Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] |
| EqQ[c^2 - d^2, 0])
```

rule 4456
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x])))]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

3.146.4 Maple [A] (verified)

Time = 7.30 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.90

method	result
default	$\frac{(\cos(fx+e)+1)\left(2\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)c-4\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)d+c\,\mathrm{EllipticE}\left(\cot(fx+e)\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/f/(c-d)*(cos(f*x+e)+1)*(2*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d)
)^(1/2))*c-4*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*d+c*Elli
pticE(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))+d*EllipticE(cot(f*x+e)-cs
c(f*x+e),((c-d)/(c+d))^(1/2))-4*c*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((c-
d)/(c+d))^(1/2))+4*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((c-d)/(c+d))^(1/2)
)*d)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+
e)+1))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(d+c*cos(f*x+e))
```

3.146.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.146.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{1}{\sqrt{c+d\sec(e+fx)}\sec(e+fx)+\sqrt{c+d\sec(e+fx)}}\,dx}{a}$$

input
```
integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(c + d*sec(e + f*x))*sec(e + f*x) + sqrt(c + d*sec(e + f*x
))), x)/a
```

3.146.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{(a\sec(fx+e)+a)\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

3.146.8 Giac [F]

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{(a\sec{(fx+e)}+a)\sqrt{d\sec{(fx+e)}+c}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)), x)
```

3.147 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$

3.147.1 Optimal result

Integrand size = 27, antiderivative size = 271

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx = \frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{3/2}c^4\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2d^2\left(6c^2+8cd+3d^2\right)(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3(4c+3d)(a-a\sec(e+fx))^2\tan(e+fx)}{5af\sqrt{a+a\sec(e+fx)}} - \frac{2d^4(a-a\sec(e+fx))^3\tan(e+fx)}{7a^2f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a*d*(2*c+d)*(2*c^2+2*c*d+d^2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*d^
2*(6*c^2+8*c*d+3*d^2)*(a-a*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
+2/5*d^3*(4*c+3*d)*(a-a*sec(f*x+e))^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2
)-2/7*d^4*(a-a*sec(f*x+e))^3*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)+2*a^(
3/2)*c^4*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x
+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.147.2 Mathematica [A] (warning: unable to verify)

Time = 8.41 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.85

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$$
$$= \frac{2\sqrt{a(1+\sec(e+fx))}(c+d\sec(e+fx))^4\left(105c^4\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\sqrt{\frac{\sec(e+fx)}{(1+\sec(e+fx))^2}}\sqrt{1+\sec(e+fx)}\right.}{}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4,x]
```

output
```
(2*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x])^4*(105*c^4*ArcTan[Tan[(
e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]]*Sqrt[Sec[e + f*x]/(1 +
Sec[e + f*x])^2]*Sqrt[1 + Sec[e + f*x]] + d*Sqrt[Sec[e + f*x]]*(420*c^3 +
420*c^2*d + 224*c*d^2 + 48*d^3 + 2*d*(105*c^2 + 56*c*d + 12*d^2)*Sec[e + f
*x] + 6*d^2*(14*c + 3*d)*Sec[e + f*x]^2 + 15*d^3*Sec[e + f*x]^3)*Tan[(e +
f*x)/2]))/(105*f*(d + c*Cos[e + f*x])^4*Sec[e + f*x]^(9/2))
```

3.147.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 4428, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^4}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^2\tan(e+fx)\int\left(\frac{\cos(e+fx)c^4}{\sqrt{a-a\sec(e+fx)}}-\frac{d^4(a-a\sec(e+fx))^{5/2}}{a^3}+\frac{d^3(4c+3d)(a-a\sec(e+fx))^{3/2}}{a^2}-\frac{d^2(6c^2+8dc+3d^2)\sqrt{a-a\sec(e+fx}}{a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 2009

$$-\frac{a^2\tan(e+fx)\left(\frac{2d^4(a-a\sec(e+fx))^{7/2}}{7a^4}-\frac{2d^3(4c+3d)(a-a\sec(e+fx))^{5/2}}{5a^3}+\frac{2d^2(6c^2+8cd+3d^2)(a-a\sec(e+fx))^{3/2}}{3a^2}-\frac{2c^4\text{arctanh}}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4,x]
```

output
```
-((a^2*((-2*c^4*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*d*
(2*c + d)*(2*c^2 + 2*c*d + d^2)*Sqrt[a - a*Sec[e + f*x]])/a + (2*d^2*(6*c^
2 + 8*c*d + 3*d^2)*(a - a*Sec[e + f*x])^(3/2))/(3*a^2) - (2*d^3*(4*c + 3*d
)*(a - a*Sec[e + f*x])^(5/2))/(5*a^3) + (2*d^4*(a - a*Sec[e + f*x])^(7/2))
/(7*a^4))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x
]]))
```

3.147.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.147.4 Maple [A] (verified)

Time = 6.83 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.11

method	result
parts	$\frac{2c^4\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}+\frac{2d^4\left(16\cos(fx+e)^3+8\cos(fx+e)^2+6\cos(fx\cdots\right.}{35f(\text{co}\cdots}$
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(105\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,c^4\cos(fx+e)+105\,\operatorname{arctanh}\left(\frac{\sin(fx\cdots}{(\cos(fx+e)+1)\sqrt{-\cdots}}\right.\right.}{\cdots}$

input
```
int((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*c^4/f*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctan
h(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+2/35*d^4/f
*(16*cos(f*x+e)^3+8*cos(f*x+e)^2+6*cos(f*x+e)+5)*(a*(sec(f*x+e)+1))^(1/2)/
(cos(f*x+e)+1)*tan(f*x+e)*sec(f*x+e)^2-8*c^3*d/f*(a*(sec(f*x+e)+1))^(1/2)*
(cot(f*x+e)-csc(f*x+e))+4*c^2*d^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1
)*(2*sin(f*x+e)+tan(f*x+e))+8/15*c*d^3/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x
+e)+1)*(8*sin(f*x+e)+4*tan(f*x+e)+3*sec(f*x+e)*tan(f*x+e))
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.74

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$$
$$= \left[\frac{105\left(c^4\cos(fx+e)^4+c^4\cos(fx+e)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+\right.}{\cos(fx+e)+1}\right.}{}\right.$$
$$-\frac{2\left(105\left(c^4\cos(fx+e)^4+c^4\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(15\,d^4+4\left(105\,c^3d+\right.\right.\right.}{}$$

input
```
integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(-a)*log((2*a*co
s(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*d^4 +
4*(105*c^3*d + 105*c^2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^3 + 2*(105*c^
2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^2 + 6*(14*c*d^3 + 3*d^4)*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)
^4 + f*cos(f*x + e)^3), -2/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)
^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) - (15*d^4 + 4*(105*c^3*d + 105*c^2*d^2 + 56*c*d^3 +
12*d^4)*cos(f*x + e)^3 + 2*(105*c^2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^
2 + 6*(14*c*d^3 + 3*d^4)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]
```

3.147.6 Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$$
$$= \int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^4\,dx$$

input
```
integrate((c+d*sec(f*x+e))**4*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**4, x)
```

3.147.7 Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx = \int \sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)^4\,dx$$

input
```
integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/210*(16*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*(7*(15*c^3*d*sin(6*f*x + 6*e) + 5*(9*c^3*d + 3*c^2*d^2 + 4*c*d^3
)*sin(4*f*x + 4*e) + (45*c^3*d + 30*c^2*d^2 + 28*c*d^3 + 6*d^4)*sin(2*f*x
+ 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - (105*c^
3*d*cos(6*f*x + 6*e) + 105*c^3*d + 105*c^2*d^2 + 56*c*d^3 + 12*d^4 + 35*(9
*c^3*d + 3*c^2*d^2 + 4*c*d^3)*cos(4*f*x + 4*e) + 7*(45*c^3*d + 30*c^2*d^2
+ 28*c*d^3 + 6*d^4)*cos(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)))*sqrt(a) + 105*((c^4*cos(2*f*x + 2*e)^4 + c^4*sin(2*f
*x + 2*e)^4 + 4*c^4*cos(2*f*x + 2*e)^3 + 6*c^4*cos(2*f*x + 2*e)^2 + 4*c^4*
cos(2*f*x + 2*e) + c^4 + 2*(c^4*cos(2*f*x + 2*e)^2 + 2*c^4*cos(2*f*x + 2*e
) + c^4)*sin(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
) + 1) - (c^4*cos(2*f*x + 2*e)^4 + c^4*sin(2*f*x + 2*e)^4 + 4*c^4*cos(2*f*
x + 2*e)^3 + 6*c^4*cos(2*f*x + 2*e)^2 + 4*c^4*cos(2*f*x + 2*e) + c^4 + 2*(
c^4*cos(2*f*x + 2*e)^2 + 2*c^4*cos(2*f*x + 2*e) + c^4)*sin(2*f*x + 2*e)^2)
*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/...
```

3.147.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx = \int \sqrt{a\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)^4\,dx$$

input
```
integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx = \int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)^4\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^4,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^4, x)
```

3.148 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$

3.148.1 Optimal result

Integrand size = 27, antiderivative size = 205

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx = \frac{2ad(3c^2+3cd+d^2)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{3/2}c^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(3c+2d)(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3(a-a\sec(e+fx))^2\tan(e+fx)}{5af\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a*d*(3*c^2+3*c*d+d^2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*d^2*(3*c+2
*d)*(a-a*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/5*d^3*(a-a*sec(
f*x+e))^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2)*c^3*arctanh((a-a
*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f
*x+e))^(1/2)
```

3.148.2 Mathematica [A] (verified)

Time = 9.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.94

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx = \frac{2\sqrt{a(1+\sec(e+fx))}(c+d\sec(e+fx))^3\left(15c^3\arctan\left(\frac{\tan(\frac{1}{2}(e+fx))}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\sqrt{\frac{\sec(e+fx)}{(1+\sec(e+fx))^2}}\sqrt{1+\sec(e+fx)}}{15f(d+c\cos(e+f}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3,x]
```

output
```
(2*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x])^3*(15*c^3*ArcTan[Tan[(e
 + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]]*Sqrt[Sec[e + f*x]/(1 + S
ec[e + f*x])^2]*Sqrt[1 + Sec[e + f*x]] + d*Sqrt[Sec[e + f*x]]*(45*c^2 + 30
*c*d + 8*d^2 + d*(15*c + 4*d)*Sec[e + f*x] + 3*d^2*Sec[e + f*x]^2)*Tan[(e
+ f*x)/2]))/(15*f*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^(7/2))
```

3.148.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3042, 4428, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^2\tan(e+fx)\int \left(\frac{\cos(e+fx)c^3}{\sqrt{a-a\sec(e+fx)}}+\frac{d^3(a-a\sec(e+fx))^{3/2}}{a^2}-\frac{d^2(3c+2d)\sqrt{a-a\sec(e+fx)}}{a}+\frac{d(3c^2+3dc+d^2)}{\sqrt{a-a\sec(e+fx)}}\right)d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{2d^3(a-a\sec(e+fx))^{5/2}}{5a^3}+\frac{2d^2(3c+2d)(a-a\sec(e+fx))^{3/2}}{3a^2}-\frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d(3c^2+3cd+d^2)}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^2*((-2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*d*
(3*c^2 + 3*c*d + d^2)*Sqrt[a - a*Sec[e + f*x]])/a + (2*d^2*(3*c + 2*d)*(a
- a*Sec[e + f*x])^(3/2))/(3*a^2) - (2*d^3*(a - a*Sec[e + f*x])^(5/2))/(5*a
^3))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.148.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.148.4 Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.09

method	result
parts	$\frac{2c^3\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}+\frac{2d^3\sqrt{a(\sec(fx+e)+1)}\,(8\sin(fx+e)+4\tan(f}{15f(\cos(fx+e)+}$
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(15\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,c^3\cos(fx+e)+15\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-}}\right.\right.}{}$

input
```
int((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*c^3/f*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctan
h(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+2/15*d^3/f
*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(8*sin(f*x+e)+4*tan(f*x+e)+3*sec(
f*x+e)*tan(f*x+e))-6*c^2*d/f*(a*(sec(f*x+e)+1))^(1/2)*(cot(f*x+e)-csc(f*x+
e))+2*c*d^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(2*sin(f*x+e)+tan(f*
x+e))
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.91

$$\int\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$$

$$=\left[\frac{15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+}{\cos(fx+e)+1}\right.}{15\left(f\cos(fx+\right.}\right.$$

$$-\frac{2\left(15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(3\,d^3+\left(45\,c^2d+30\,cd\right.\right.\right.}{15\left(f\cos(fx+e)^3+f\cos(f\right.}$$

input
```
integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(
f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*d^3 + (45
*c^2*d + 30*c*d^2 + 8*d^3)*cos(f*x + e)^2 + (15*c*d^2 + 4*d^3)*cos(f*x + e
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3
 + f*cos(f*x + e)^2), -2/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*
sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(
a)*sin(f*x + e))) - (3*d^3 + (45*c^2*d + 30*c*d^2 + 8*d^3)*cos(f*x + e)^2
+ (15*c*d^2 + 4*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

3.148.6 Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx = \int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}(c+d\sec\left(e+fx\right))^3\,dx$$

input
```
integrate((c+d*sec(f*x+e))**3*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**3, x)
```

3.148.7 Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx = \int \sqrt{a\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)^3\,dx$$

input
```
integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/30*(15*((c^3*cos(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*
f*x + 2*e) + c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c
^3*cos(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*f*x + 2*e) +
c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^3*f*cos(2*
f*x + 2*e)^2 + c^3*f*sin(2*f*x + 2*e)^2 + 2*c^3*f*cos(2*f*x + 2*e) + c^3*f
)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(
2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 +
sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) +
3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e)
+ 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e
) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e
) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)...
```

3.148.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx=\int \sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)^3\,dx$$

input
```
integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c+\frac{d}{\cos(e+f\,x)}\right)^3\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3, x)
```

3.149 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$

3.149.1 Optimal result

Integrand size = 27, antiderivative size = 144

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx = \frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^{3/2}c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a*d*(2*c+d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*d^2*(a-a*sec(f*x+e))
*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2)*c^2*arctanh((a-a*sec(f*x+e)
)^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2
)
```

3.149.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.98 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.08

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx = \frac{\csc^3\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}(c+d\sec(e+fx))^2\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2\left(\cdots\right.}}{\cdots}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2,x]
```

output
```
(Csc[(e + f*x)/2]^3*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec
[e + f*x])^2*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(e + f*x
)/2]^2]*(256*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[(e + f*x)/2]
^2]*Sin[(e + f*x)/2]^6*(c + d - 2*c*Sin[(e + f*x)/2]^2)^2 + 1024*Hypergeom
etric2F1[3/2, 7/2, 9/2, 2*Sin[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^6*(d^2 + c*
d*(2 - 3*Sin[(e + f*x)/2]^2) + c^2*(1 - 3*Sin[(e + f*x)/2]^2 + 2*Sin[(e +
f*x)/2]^4)) - (7*Sqrt[2]*(-3*ArcSin[Sqrt[2]*Sqrt[Sin[(e + f*x)/2]^2]] + Sq
rt[2]*Sqrt[Sin[(e + f*x)/2]^2]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*(3 + 4*Sin[(
e + f*x)/2]^2))*(15*d^2 + 10*c*d*(3 - 2*Sin[(e + f*x)/2]^2) + c^2*(15 - 20
*Sin[(e + f*x)/2]^2 + 12*Sin[(e + f*x)/2]^4)))/Sqrt[Sin[(e + f*x)/2]^2]))/
(672*f*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^(5/2))
```

3.149.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3042, 4428, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^2\tan(e+fx)\int \left(\frac{\cos(e+fx)c^2}{\sqrt{a-a\sec(e+fx)}}-\frac{d^2\sqrt{a-a\sec(e+fx)}}{a}+\frac{d(2c+d)}{\sqrt{a-a\sec(e+fx)}}\right)d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{2d^2(a-a\sec(e+fx))^{3/2}}{3a^2}-\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d(2c+d)\sqrt{a-a\sec(e+fx)}}{a}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^2*((-2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*d*
(2*c + d)*Sqrt[a - a*Sec[e + f*x]])/a + (2*d^2*(a - a*Sec[e + f*x])^(3/2))
/(3*a^2))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x
]]))
```

3.149.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.149.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10

method	result
parts	$\frac{2c^2\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}+\frac{2d^2\sqrt{a(\sec(fx+e)+1)}\,(2\sin(fx+e)+\tan(fx-$ ... ${3f(\cos(fx+e)+1)}$
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(3\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)c^2\cos(fx+e)+3\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{}{(\cos(\ldots}\right.\right.}{3f(\cos(fx+e)+1)}$

input
```
int((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*c^2/f*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctan
h(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+2/3*d^2/f*
(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(2*sin(f*x+e)+tan(f*x+e))-4*c*d/f*
(a*(sec(f*x+e)+1))^(1/2)*(cot(f*x+e)-csc(f*x+e))
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.22

$$\int\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$$

$$=\left[\frac{3\left(c^2\cos(fx+e)^2+c^2\cos(fx+e)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-\ldots}{\cos(fx+e)+1}\right.}{3\left(f\cos(fx+e)^2+f\cos(fx+e)\right)}\right.$$

$$-\frac{2\left(3\left(c^2\cos(fx+e)^2+c^2\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(d^2+2\left(3\,cd+d^2\right)\cos(f\ldots\right.\right.}{3\left(f\cos(fx+e)^2+f\cos(fx+e)\right)}$$

input
```
integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x
+ e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(d^2 + 2*(3*c*d
+ d^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))
/(f*cos(f*x + e)^2 + f*cos(f*x + e)), -2/3*(3*(c^2*cos(f*x + e)^2 + c^2*co
s(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)/(sqrt(a)*sin(f*x + e))) - (d^2 + 2*(3*c*d + d^2)*cos(f*x + e))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos
(f*x + e))]
```

3.149.6 Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx = \int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^2\,dx$$

input
```
integrate((c+d*sec(f*x+e))**2*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**2, x)
```

3.149.7 Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx = \int \sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)^2\,dx$$

input
```
integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(8*(3*c*d*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*si
n(2*f*x + 2*e) - (3*c*d*cos(2*f*x + 2*e) + 3*c*d + d^2)*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) + 3*((c^2*cos(2*f*x + 2*e)^
2 + c^2*sin(2*f*x + 2*e)^2 + 2*c^2*cos(2*f*x + 2*e) + c^2)*arctan2((cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c^2*cos(2*f*x + 2*e)^2 + c^2*sin(2
*f*x + 2*e)^2 + 2*c^2*cos(2*f*x + 2*e) + c^2)*arctan2((cos(2*f*x + 2*e)^2
+ sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1)) - 1) - 2*(c^2*f*cos(2*f*x + 2*e)^2 + c^2*f*sin(2*f*x +
2*e)^2 + 2*c^2*f*cos(2*f*x + 2*e) + c^2*f)*integrate((((cos(6*f*x + 6*e)*c
os(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2
 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*
e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x +...
```

3.149.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx=\int \sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)^2\,dx$$

input
```
integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c+\frac{d}{\cos(e+f\,x)}\right)^2 dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2, x)
```

3.150 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$

3.150.1 Optimal result

Integrand size = 25, antiderivative size = 66

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx = \frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{2ad\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f+2*a*d*tan(
f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

3.150.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx = \frac{\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}\Big(\sqrt{2}c\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{\cos(e+fx)}+2d\sin\big(\frac{1}{2}(e+fx}{f}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

output
```
(Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt[2]*c*ArcSin[Sqrt[2]*Sin
[(e + f*x)/2]]*Sqrt[Cos[e + f*x]] + 2*d*Sin[(e + f*x)/2]))/f
```

3.150.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4403, 3042, 4261, 216, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4403}$$

$$c\int \sqrt{\sec(e+fx)a+a}dx+d\int \sec(e+fx)\sqrt{\sec(e+fx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+d\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4261}$$

$$d\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-\frac{2ac\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$d\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+\frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

$$\Big\downarrow \text{4279}$$

$$\frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2ad\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

output
```
(2*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f +
(2*a*d*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

3.150.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4403
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Si
mp[d   Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.150.4 Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.45

method	result
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)c-d\cot(fx+e)+d\csc(fx+e)\right)}{f}$
parts	$\frac{2c\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}-\frac{2d\sqrt{a(\sec(fx+e)+1)}\,(\cot(fx+e)-\csc(fx+e))}{f}$

input
```
int((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*(a*(sec(f*x+e)+1))^(1/2)*((-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(s
in(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*c-d*cot(f*x+e
)+d*csc(f*x+e))
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.56

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$$

$$= \left[\frac{(c\cos(fx+e)+c)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+2\,d\sqrt{\frac{a\cos(f}{\cos(f}}}{f\cos(fx+e)+f} \right.$$

$$\left. -\frac{2\left((c\cos(fx+e)+c)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-d\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)\right)}{f\cos(fx+e)+f}\right]$$

input
```
integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[((c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x +
 e) - a)/(cos(f*x + e) + 1)) + 2*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e))/(f*cos(f*x + e) + f), -2*((c*cos(f*x + e) + c)*sqrt(a)*arct
an(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
 e))) - d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
 + e) + f)]
```

3.150.6 Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx = \int \sqrt{a\,(\sec(e+fx)+1)}(c+d\sec(e+fx))\,dx$$

input
```
integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x)), x)
```

3.150.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(58) = 116.

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.23

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$$
$$= \frac{\sqrt{a}c\arctan\left(\left(\cos\left(2\,fx+2\,e\right)^2+\sin\left(2\,fx+2\,e\right)^2+2\,\cos\left(2\,fx+2\,e\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin\left(2\,fx+2\,e\right.\right.\right.}{}$$

input
```
integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*c*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
+ sin(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
cos(f*x + e))/f
```

3.150.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx = \int \sqrt{a\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)\,dx$$

input
```
integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx = \int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)), x)
```

3.151 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

3.151.1 Optimal result

Integrand size = 27, antiderivative size = 105

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} - \frac{2\sqrt{a}\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{c\sqrt{c+d}f}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c/f-2*arctan(a
^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)*d^(1
/2)/c/f/(c+d)^(1/2)
```

3.151.2 Mathematica [A] (verified)

Time = 2.89 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2\left(\sqrt{c+d}\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)-\sqrt{d}\arctan\left(\frac{\sqrt{d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{a(1+\sec(e+fx))}}{c\sqrt{c+d}f}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

output
```
(2*(Sqrt[c + d]*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x
])]] - Sqrt[d]*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e +
 f*x]/(1 + Cos[e + f*x])])])*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[a*
(1 + Sec[e + f*x])])/(c*Sqrt[c + d]*f)
```

3.151.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3042, 4413, 3042, 4261, 216, 4455, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{c+d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4413}$$

$$\frac{\int \sqrt{\sec(e+fx)a+a}dx}{c}-\frac{d\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{c+d\sec(e+fx)}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}$$

$$\Big\downarrow \text{4261}$$

$$-\frac{2a\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}$$

$$\Big\downarrow \text{4455}$$

$$\frac{2ad\int \frac{1}{\frac{a^2 d\tan^2(e+fx)}{\sec(e+fx)a+a}+a(c+d)}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}+\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{2\sqrt{a}\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{cf\sqrt{c+d}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

output
```
(2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f)
- (2*Sqrt[a]*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sq
rt[a + a*Sec[e + f*x]])])/(c*Sqrt[c + d]*f)
```

3.151.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4413
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[1/c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[d/c   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 -
 b^2, 0] || EqQ[c^2 - d^2, 0])
```

3.151. $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

rule 4455
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[-2*(b/f)   Subst[In
t[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0]
```

3.151.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(85) = 170$.

Time = 15.15 (sec) , antiderivative size = 499, normalized size of antiderivative = 4.75

method	result
default	$\sqrt{2}\,\Big(2\sqrt{(c+d)(c-d)}\,\sqrt{\frac{d}{c-d}}\,\operatorname{arctanh}\Big(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^{2}\csc(fx+e)^{2}-1}}\Big)-d\ln\Big(-\frac{2\big(\sqrt{(1-\cos(fx+e))^{2}\csc(fx+e)^{2}-1}\,\sqrt{2}\,\sqrt{\frac{d}{c-d}}\,c-\sqrt{2}\,\sqrt{\frac{c}{c-}} \cdots}{-c(-\cot(fx+e)+\csc(\cdots}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*2^(1/2)/(d/(c-d))^(1/2)/((c+d)*(c-d))^(1/2)/c*(2*((c+d)*(c-d))^(1/2)
*(d/(c-d))^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(
-cot(f*x+e)+csc(f*x+e)))-d*ln(-2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*
2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*
x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-
cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))+d*
ln(2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^
(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d-((c+d)*(c-
d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e)+csc(f*x+e))-(-cot(
f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2))))*((1-cos(f*x+e))^2*csc(f*x+e)^2
-1)^(1/2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 669, normalized size of antiderivative = 6.37

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{-\frac{ad}{c+d}}\log\left(\frac{2\,(c+d)\sqrt{-\frac{ad}{c+d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac+2\,ad)\cos(fx+e)^2-ad+(ac+ad)\cos(fx+e)}{c\cos(fx+e)^2+(c+d)\cos(fx+e)+d}\right)+\sqrt{-a}\log\Bigg(}{cf}\right.$$
$$-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\sqrt{-\frac{ad}{c+d}}\log\left(\frac{2\,(c+d)\sqrt{-\frac{ad}{c+d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac+2\,ad)\,\mathrm{co}}{c\cos(fx+e)^2+(c+d)\cos(fx+e)+}\right.}{cf}$$
$$\left.-\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\sqrt{\frac{ad}{c+d}}\arctan\left(\frac{(c+d)\sqrt{\frac{ad}{c+d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{ad\sin(fx+e)}\right)\right)}{cf}\right]$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[(sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e
)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x
+ e) + d)) + sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(
cos(f*x + e) + 1)))/(c*f), -(2*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(-a*d/(c + d))*log(
(2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*c
os(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/(c*f), (2*sqr
t(a*d/(c + d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x + e))) + sqrt(-a)*log((2*a*cos(f*x
 + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*
sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -2*(sqrt(a)
*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(
f*x + e))) - sqrt(a*d/(c + d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x + e))))/(c*f)]
```

3.151.6 Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c + d\sec(e + fx)}\,dx = \int \frac{\sqrt{a\left(\sec\left(e + fx\right) + 1\right)}}{c + d\sec\left(e + fx\right)}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x)), x)
```

3.151.7 Maxima [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c + d\sec(e + fx)}\,dx = \int \frac{\sqrt{a\sec\left(fx + e\right) + a}}{d\sec\left(fx + e\right) + c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

3.151.8 Giac [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c + d\sec(e + fx)}\,dx = \int \frac{\sqrt{a\sec\left(fx + e\right) + a}}{d\sec\left(fx + e\right) + c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
sage0*x
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c + d\sec(e + fx)}\,dx = \int \frac{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}{c + \frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)), x)
```

3.152 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx$

3.152.1 Optimal result

Integrand size = 27, antiderivative size = 219

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx = \frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^{3/2}\sqrt{d}(3c+2d)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
-a*d*tan(f*x+e)/c/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2
)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/(a-a*sec(f*x+e)
)^(1/2)/(a+a*sec(f*x+e))^(1/2)-a^(3/2)*(3*c+2*d)*arctanh(d^(1/2)*(a-a*sec(
f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*d^(1/2)*tan(f*x+e)/c^2/(c+d)^(3/2)/f/(a
-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.152.2 Mathematica [A] (warning: unable to verify)

Time = 5.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{(d+c\cos(e+fx))^2\sec^{\frac{3}{2}}(e+fx)\sqrt{a(1+\sec(e+fx))}\left(\frac{2\left(2(c+d)^{3/2}\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)-\sqrt{d}(3c+2d)\arctan\left(\frac{\sqrt{d}\ldots}{\sqrt{c+\ldots}}\right)\ldots\right)}{(c+d)^{3/2}}\ldots\right)}{2c^2f(c+d\sec(e+fx))^2}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^2,x]
```

output
```
((d + c*Cos[e + f*x])^2*Sec[e + f*x]^(3/2)*Sqrt[a*(1 + Sec[e + f*x])]*((2*
(2*(c + d)^(3/2)*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*
x])]] - Sqrt[d]*(3*c + 2*d)*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]
*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])])*Sqrt[Cos[e + f*x]/(1 + Cos[e + f
*x])^2]*Sqrt[1 + Sec[e + f*x]])/(c + d)^(3/2) - (2*c*d*Tan[(e + f*x)/2])/(
(c + d)*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]])))/(2*c^2*f*(c + d*Sec[e +
 f*x])^2)
```

3.152.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3042, 4428, 114, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c+d\sec(e+fx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4428$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 114$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a\cos(e+fx)(2(c+d)-d\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{ac(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{\cos(e+fx)(2(c+d)-d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{2c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 174$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{2(c+d)\int \frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{c}-\frac{d(3c+2d)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{c}}{2c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{2d(3c+2d)\int \frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{4(c+d)\int \frac{1}{1-\frac{a-a\sec(e+fx)}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}}{2c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 219$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{2d(3c+2d)\int \frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{4(c+d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 221$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{2\sqrt{d}(3c+2d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c\sqrt{c+d}}-\frac{4(c+d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

3.152. $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^2*(((-4*(c + d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c
) + (2*Sqrt[d]*(3*c + 2*d)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqr
t[a]*Sqrt[c + d])])/(Sqrt[a]*c*Sqrt[c + d]))/(2*c*(c + d)) + (d*Sqrt[a - a
*Sec[e + f*x]])/(a*c*(c + d)*(c + d*Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[
a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.152.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.152.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 38051 vs. $2(189) = 378$.

Time = 15.52 (sec) , antiderivative size = 38052, normalized size of antiderivative = 173.75

method	result	size
default	Expression too large to display	38052

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.152.5 Fricas [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 1413, normalized size of antiderivative = 6.45

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*c*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) - ((3*c^2 + 2*c*d)*cos(f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2 + 5*c*d +
2*d^2)*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c +
 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2
 + (c + d)*cos(f*x + e) + d)) - 2*((c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2
+ (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*s
qrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
+ a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d)*f*cos(f*x + e)^2
 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), -1/2*(
2*c*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) +
4*((c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x +
e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) - ((3*c^2 + 2*c*d)*cos(f*x + e)^2 + 3*c*d + 2*d^2 +
(3*c^2 + 5*c*d + 2*d^2)*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sq
rt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/
(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/((c^4 + c^3*d)*f*cos(f*x +
 e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), -
(c*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) ...
```

3.152.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx=\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(c+d\sec\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**2, x)
```

3.152.7 Maxima [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx=\int \frac{\sqrt{a\sec(fx+e)+a}}{(d\sec(fx+e)+c)^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^2, x)
```

3.152.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx=\int \frac{\sqrt{a\sec(fx+e)+a}}{(d\sec(fx+e)+c)^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^2, x)
```

3.153 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$

3.153.1 Optimal result

Integrand size = 27, antiderivative size = 287

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx &= \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &\quad - \frac{a^{3/2}\sqrt{d}(15c^2+20cd+8d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4c^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &\quad - \frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} \\ &\quad - \frac{ad(7c+4d)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\end{aligned}$$

output
```
-1/2*a*d*tan(f*x+e)/c/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/
4*a*d*(7*c+4*d)*tan(f*x+e)/c^2/(c+d)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))
^(1/2)+2*a^(3/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3/f/
(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*a^(3/2)*(15*c^2+20*c*d+8
*d^2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*d^(1/2)*
tan(f*x+e)/c^3/(c+d)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.153.2 Mathematica [A] (warning: unable to verify)

Time = 7.57 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{(d+c\cos(e+fx))^3\sec\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{5}{2}}(e+fx)\sqrt{a(1+\sec(e+fx))}\left(\frac{\left(8(c+d)^{5/2}\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)-\sqrt{d}(\cdots\right.}{\cdots}\right.}{8c^3\cdots}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^3,x]
```

output
```
((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]*Sec[e + f*x]^(5/2)*Sqrt[a*(1 + Se
c[e + f*x])]*(((8*(c + d)^(5/2)*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/
(1 + Cos[e + f*x])]] - Sqrt[d]*(15*c^2 + 20*c*d + 8*d^2)*ArcTan[(Sqrt[d]*T
an[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])])*Sqr
t[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sec[(e + f*x)/2]*Sqrt[1 + Sec[e + f*x]]
)/((c + d)^(5/2)*Sqrt[(1 + Cos[e + f*x])^(-1)]) - (2*c*d*(d*(7*c + 4*d) +
3*c*(3*c + 2*d)*Cos[e + f*x])*Sec[e + f*x]^(3/2)*Sin[(e + f*x)/2])/((c + d
)^2*(c + d*Sec[e + f*x])^2)))/(8*c^3*f*(c + d*Sec[e + f*x])^3)
```

3.153.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {3042, 4428, 114, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 114$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a\cos(e+fx)(4(c+d)-3d\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2ac(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{\cos(e+fx)(4(c+d)-3d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{4c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 168$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{a\cos(e+fx)\left(8(c+d)^2-d(7c+4d)\sec(e+fx)\right)}{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{ac(c+d)}+\frac{d(7c+4d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}}{4c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{\cos(e+fx)\left(8(c+d)^2-d(7c+4d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{2c(c+d)}+\frac{d(7c+4d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}}{4c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 174$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{8(c+d)^2\int \frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{c}-\frac{d\left(15c^2+20cd+8d^2\right)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{c}}{2c(c+d)}+\frac{d(7c+4d)\sqrt{a-a\sec(e\cdots}}{ac(c+d)(c+d\sec(e+\cdots}}{4c(c+d)}\cdots\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 73$$

3.153. $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{2d\left(15c^2+20cd+8d^2\right)\int \frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{16(c+d)^2\int \frac{1}{1-\frac{a-a\sec(e+fx)}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}}{2c(c+d)}+\frac{d(7c+4d)\sqrt{a-\cdots}}{ac(c+d)(c+d\cdots}}{4c(c+d)}\cdots\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 219

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{2d\left(15c^2+20cd+8d^2\right)\int \frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{16(c+d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}+\frac{d(7c+4d)\sqrt{a-a\sec(e\cdots}}{ac(c+d)(c+d\sec(e+\cdots}}{4c(c+d)}\cdots\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 221

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{2\sqrt{d}\left(15c^2+20cd+8d^2\right)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c\sqrt{c+d}}-\frac{16(c+d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}+\frac{d(7c+4d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}}{4c(c+d)}+\cdots\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^2*((d*Sqrt[a - a*Sec[e + f*x]])/(2*a*c*(c + d)*(c + d*Sec[e + f*x])^2
) + (((-16*(c + d)^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c
) + (2*Sqrt[d]*(15*c^2 + 20*c*d + 8*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e
 + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c*Sqrt[c + d]))/(2*c*(c + d)) +
 (d*(7*c + 4*d)*Sqrt[a - a*Sec[e + f*x]])/(a*c*(c + d)*(c + d*Sec[e + f*x]
)))/(4*c*(c + d)))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Se
c[e + f*x]]))
```

3.153. $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$

3.153.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.153. $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.153.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 76268 vs. $2(249) = 498$.

Time = 17.66 (sec) , antiderivative size = 76269, normalized size of antiderivative = 265.75

method	result	size
default	Expression too large to display	76269

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.153.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(249) = 498$.

Time = 4.36 (sec) , antiderivative size = 2368, normalized size of antiderivative = 8.25

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/8*((15*c^2*d^2 + 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos
(f*x + e)^3 + (15*c^4 + 50*c^3*d + 48*c^2*d^2 + 16*c*d^3)*cos(f*x + e)^2 +
 (30*c^3*d + 55*c^2*d^2 + 36*c*d^3 + 8*d^4)*cos(f*x + e))*sqrt(-a*d/(c + d
))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c +
 a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + 8*(c^
2*d^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f*x + e)^3 + (c^4 +
4*c^3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d + 5*c^2*d^2 + 4*c
*d^3 + d^4)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*
x + e) - a)/(cos(f*x + e) + 1)) - 2*(3*(3*c^3*d + 2*c^2*d^2)*cos(f*x + e)^
2 + (7*c^2*d^2 + 4*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4
*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 +
4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f),
-1/8*(16*(c^2*d^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f*x + e)
^3 + (c^4 + 4*c^3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d + 5*c
^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*c^2*d^2 + 20
*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*c...
```

3.153.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx=\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(c+d\sec\left(e+fx\right)\right)^3}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**3, x)
```

3.153.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.153.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(d\sec\left(fx+e\right)+c\right)^3}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^3, x)
```

3.154 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx$

3.154.1 Optimal result

Integrand size = 27, antiderivative size = 241

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx = \frac{2a^{5/2}c^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(2(36c^3+243c^2d+189cd^2+52d^3)+d(24c^2+111cd+52d^2)\sec(e+fx))\tan(e+fx)}{105f\sqrt{a+a\sec(e+fx)}}$$

output
```
2/35*a^2*(6*c+13*d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
+2/7*a^2*(c+d*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/105*a^2*
(72*c^3+486*c^2*d+378*c*d^2+104*d^3+d*(24*c^2+111*c*d+52*d^2)*sec(f*x+e))*
tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(5/2)*c^3*arctanh((a-a*sec(f*x+e))
^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.154.2 Mathematica [A] (verified)

Time = 3.63 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.91

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx = \frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\left(420\sqrt{2}c^3\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+f\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3,x]
```

output
```
(a*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*(420*Sqrt[2]
*c^3*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(7/2) + 2*(210*c^2*d +
378*c*d^2 + 164*d^3 + 9*(35*c^3 + 175*c^2*d + 154*c*d^2 + 52*d^3)*Cos[e +
f*x] + 2*d*(105*c^2 + 189*c*d + 52*d^2)*Cos[2*(e + f*x)] + 105*c^3*Cos[3*(
e + f*x)] + 525*c^2*d*Cos[3*(e + f*x)] + 378*c*d^2*Cos[3*(e + f*x)] + 104*
d^3*Cos[3*(e + f*x)])*Sin[(e + f*x)/2]))/(420*f)
```

3.154.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3042, 4428, 27, 170, 27, 170, 27, 164, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{a\cos(e+fx)(\sec(e+fx)+1)(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{170}$$

$$-\frac{a^3\tan(e+fx)\left(-\frac{2\int -\frac{a\cos(e+fx)(c+d\sec(e+fx))^2(7c+(6c+13d)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{7a}-\frac{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{7a}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{7}\int\frac{\cos(e+fx)(c+d\sec(e+fx))^2(7c+(6c+13d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{7a}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{170}$$

$$-\frac{a^3\tan(e+fx)\Bigg(\frac{1}{7}\Bigg(-\frac{2\int-\frac{a\cos(e+fx)(c+d\sec(e+fx))\left(35c^2+\left(24c^2+111dc+52d^2\right)\sec(e+fx)\right)}{2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{5a}-\frac{2(6c+13d)\sqrt{a-a\sec(e+fx)}(c\cdots}{5a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{7}\left(\frac{1}{5}\int\frac{\cos(e+fx)(c+d\sec(e+fx))(35c^2+(24c^2+111dc+52d^2)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{2(6c+13d)\sqrt{a-a\sec(e+\cdots}}{5a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{164}$$

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{7}\left(\frac{1}{5}\left(35c^3\int\frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{2\sqrt{a-a\sec(e+fx)}(d(24c^2+111cd+52d^2)\sec(e+fx)+2(36c^3+243\cdots}{3a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e\cdots}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^3\tan(e+fx)\Bigg(\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(-\frac{70c^3\int\frac{1}{1-\frac{a-a\sec(e+fx)}{a}}d\sqrt{a-a\sec(e+fx)}}{a}-\frac{2\sqrt{a-a\sec(e+fx)}(d(24c^2+111cd+52d^2)\sec(e+fx)+2(36c^3+\cdots}{3a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(\cdots}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^3\tan(e+fx)\Bigg(\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(-\frac{70c^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{a-a\sec(e+fx)}(d(24c^2+111cd+52d^2)\sec(e+fx)+2(36c^3+243c^2d+\cdots}{3a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e-\cdots}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3,x]
```

3.154. $\int(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx$

output
```
-((a^3*((-2*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3)/(7*a) + ((-2*
(6*c + 13*d)*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2)/(5*a) + ((-7
0*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*Sqrt[a - a*S
ec[e + f*x]]*(2*(36*c^3 + 243*c^2*d + 189*c*d^2 + 52*d^3) + d*(24*c^2 + 11
1*c*d + 52*d^2)*Sec[e + f*x]))/(3*a))/5)/7)*Tan[e + f*x])/(f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.154.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.154.4 Maple [A] (verified)

Time = 6.62 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.23

method	result
default	$\frac{2a\sqrt{a(\sec(fx+e)+1)}\left(105\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,c^3\cos(fx+e)+105\,\text{arctanh}\left(\frac{\sin(f}{(\cos(fx+e)+1)\sqrt{}}\right.\right.}{}$
parts	$\frac{2c^3a\sqrt{a(\sec(fx+e)+1)}\left(\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos(fx+e)+\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(f}{\cos(fx+}}}\right.\right.}{f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
2/105*a/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(105*arctanh(sin(f*x+e)/
(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*c^3*cos(f*x+e)+105*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x
+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c^3+105*sin(
f*x+e)*c^3+525*sin(f*x+e)*c^2*d+378*sin(f*x+e)*c*d^2+104*sin(f*x+e)*d^3+10
5*c^2*d*tan(f*x+e)+189*c*d^2*tan(f*x+e)+52*d^3*tan(f*x+e)+63*c*d^2*tan(f*x
+e)*sec(f*x+e)+39*d^3*tan(f*x+e)*sec(f*x+e)+15*d^3*tan(f*x+e)*sec(f*x+e)^2
)
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.00

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx = \left[\frac{105\left(ac^3\cos(fx+e)^4+ac^3\cos(fx+e)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\text{co}}{\cos(fx+e}\right.}{}\right.$$

$$-\frac{2\left(105\left(ac^3\cos(fx+e)^4+ac^3\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(15\,ad^3+(105\,ac^3+5\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f*x + e)^3)*sqrt(-a)*log((2*
a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*a*
d^3 + (105*a*c^3 + 525*a*c^2*d + 378*a*c*d^2 + 104*a*d^3)*cos(f*x + e)^3 +
 (105*a*c^2*d + 189*a*c*d^2 + 52*a*d^3)*cos(f*x + e)^2 + 3*(21*a*c*d^2 + 1
3*a*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e
))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a*c^3*cos(f*x + e)^
4 + a*c^3*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*a*d^3 + (105*a*c^3 + 525
*a*c^2*d + 378*a*c*d^2 + 104*a*d^3)*cos(f*x + e)^3 + (105*a*c^2*d + 189*a*
c*d^2 + 52*a*d^3)*cos(f*x + e)^2 + 3*(21*a*c*d^2 + 13*a*d^3)*cos(f*x + e))
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 +
 f*cos(f*x + e)^3)]
```

3.154.6 Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx = \int (a(\sec(e+fx)+1))^{\frac{3}{2}}\,(c+d\sec(e+fx))^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**3, x)
```

3.154.7 Maxima [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/210*(4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)^(1/4)*(7*(15*(a*c^3 + 3*a*c^2*d)*sin(6*f*x + 6*e) + 5*(9*a*c^3 + 33*a*c
^2*d + 18*a*c*d^2 + 4*a*d^3)*sin(4*f*x + 4*e) + (45*a*c^3 + 195*a*c^2*d +
144*a*c*d^2 + 52*a*d^3)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) - (105*a*c^3 + 525*a*c^2*d + 378*a*c*d^2 + 104*a*
d^3 + 105*(a*c^3 + 3*a*c^2*d)*cos(6*f*x + 6*e) + 35*(9*a*c^3 + 33*a*c^2*d
+ 18*a*c*d^2 + 4*a*d^3)*cos(4*f*x + 4*e) + 7*(45*a*c^3 + 195*a*c^2*d + 144
*a*c*d^2 + 52*a*d^3)*cos(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1)))*sqrt(a) + 105*((a*c^3*cos(2*f*x + 2*e)^4 + a*c^3*si
n(2*f*x + 2*e)^4 + 4*a*c^3*cos(2*f*x + 2*e)^3 + 6*a*c^3*cos(2*f*x + 2*e)^2
 + 4*a*c^3*cos(2*f*x + 2*e) + a*c^3 + 2*(a*c^3*cos(2*f*x + 2*e)^2 + 2*a*c^
3*cos(2*f*x + 2*e) + a*c^3)*sin(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)) + 1) - (a*c^3*cos(2*f*x + 2*e)^4 + a*c^3*sin(2*f*x +
2*e)^4 + 4*a*c^3*cos(2*f*x + 2*e)^3 + 6*a*c^3*cos(2*f*x + 2*e)^2 + 4*a*c^3
*cos(2*f*x + 2*e) + a*c^3 + 2*(a*c^3*cos(2*f*x + 2*e)^2 + 2*a*c^3*cos(2*f*
x + 2*e) + a*c^3)*sin(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x ...
```

3.154.8 Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx=\int(a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx=\int\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^3dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3, x)
```

3.155 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx$

3.155.1 Optimal result

Integrand size = 27, antiderivative size = 176

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \frac{2a^{5/2}c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(2(6c^2+25cd+9d^2)+d(4c+9d)\sec(e+fx))\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}}$$

output
```
2/5*a^2*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/15*a^2*(1
2*c^2+50*c*d+18*d^2+d*(4*c+9*d)*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^
(1/2)+2*a^(5/2)*c^2*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(
a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.155.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.82

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\left(30\sqrt{2}c^2\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2,x]
```

output
```
(a*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*(30*Sqrt[2]*
c^2*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(5/2) + 2*(15*c^2 + 50*c
*d + 24*d^2 + 2*d*(10*c + 9*d)*Cos[e + f*x] + (15*c^2 + 50*c*d + 18*d^2)*C
os[2*(e + f*x)])*Sin[(e + f*x)/2]))/(30*f)
```

3.155.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3042, 4428, 27, 170, 27, 164, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \ 4428$$

$$-\frac{a^2\tan(e+fx)\int \frac{a\cos(e+fx)(\sec(e+fx)+1)(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^3\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{170}$$

$$-\frac{a^3\tan(e+fx)\left(-\frac{2\int -\frac{a\cos(e+fx)(c+d\sec(e+fx))(5c+(4c+9d)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{5a}-\frac{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{5a}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{5}\int \frac{\cos(e+fx)(c+d\sec(e+fx))(5c+(4c+9d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{5a}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{164}$$

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{5}\left(5c^2\int \frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{2\sqrt{a-a\sec(e+fx)}(2(6c^2+25cd+9d^2)+d(4c+9d)\sec(e+fx))}{3a}\right)-2\sqrt{\cdots}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{5}\left(-\frac{10c^2\int \frac{1}{1-\frac{a-a\sec(e+fx)}{a}}d\sqrt{a-a\sec(e+fx)}}{a}-\frac{2\sqrt{a-a\sec(e+fx)}(2(6c^2+25cd+9d^2)+d(4c+9d)\sec(e+fx))}{3a}\right)-\cdots\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{5}\left(-\frac{10c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{a-a\sec(e+fx)}(2(6c^2+25cd+9d^2)+d(4c+9d)\sec(e+fx))}{3a}\right)-\frac{2\sqrt{a-a\cdots}}{\cdots}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^3*((-2*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2)/(5*a) + ((-10
*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*Sqrt[a - a*Se
c[e + f*x]]*(2*(6*c^2 + 25*c*d + 9*d^2) + d*(4*c + 9*d)*Sec[e + f*x]))/(3*
a))/5)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
)
```

3.155. $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx$

3.155.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.155. $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx$

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.155.4 Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.32

method	result
default	$\frac{2a\sqrt{a(\sec(fx+e)+1)}\left(15\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)c^2\cos(fx+e)+15\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\cdots\right.\right.}{15\cdots}$
parts	$\frac{2c^2a\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(f\cdots}{\cos(fx+\cdots}}}\cdots\right.\right.}{f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/15*a/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(15*(-cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1
))^(1/2))*c^2*cos(f*x+e)+15*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(sin
(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*c^2+15*sin(f*x+
e)*c^2+50*sin(f*x+e)*c*d+18*sin(f*x+e)*d^2+10*c*d*tan(f*x+e)+9*d^2*tan(f*x
+e)+3*d^2*tan(f*x+e)*sec(f*x+e))
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.26

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \left[\frac{15\left(ac^2\cos(fx+e)^3+ac^2\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(\cdots}{\cos(fx+e)+\cdots}\right.}{\cdots}\right.$$

$$-\frac{2\left(15\left(ac^2\cos(fx+e)^3+ac^2\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(3\,ad^2+(15\,ac^2+50\,a\cdots\right.\right.}{15\left(f\cos(fx+e)^3+f\cos(\cdots\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a)*log((2*a*
cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*a*d^2
 + (15*a*c^2 + 50*a*c*d + 18*a*d^2)*cos(f*x + e)^2 + (10*a*c*d + 9*a*d^2)*
cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos
(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*c
os(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)/(sqrt(a)*sin(f*x + e))) - (3*a*d^2 + (15*a*c^2 + 50*a*c*d + 18*a*
d^2)*cos(f*x + e)^2 + (10*a*c*d + 9*a*d^2)*cos(f*x + e))*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)
]
```

3.155.6 Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \int (a(\sec(e+fx)+1))^{\frac{3}{2}}\,(c+d\sec(e+fx))^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**2, x)
```

3.155.7 Maxima [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/30*(15*((a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*
cos(2*f*x + 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
 1) - (a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*cos(2
*f*x + 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) -
 2*(a*c^2*f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2*a*c^2*f*co
s(2*f*x + 2*e) + a*c^2*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) +
2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e
)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e
)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2
*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6
*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*...
```

3.155.8 Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^2dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2, x)
```

3.156 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$

3.156.1 Optimal result

Integrand size = 25, antiderivative size = 105

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \frac{2a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{2a^2(3c+4d)\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2ad\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{3f}$$

output
```
2*a^(3/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f+2/3*a^2*(3
*c+4*d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a*d*(a+a*sec(f*x+e))^(1/2)
*tan(f*x+e)/f
```

3.156.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\left(3\sqrt{2}c\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)c\right.}{3f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

output
```
(a*Sec[(e + f*x)/2]*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*(3*Sqrt[2]*c*A
rcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(3/2) + 2*(d + (3*c + 5*d)*Co
s[e + f*x])*Sin[(e + f*x)/2]))/(3*f)
```

3.156.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4405, 27, 3042, 4403, 3042, 4261, 216, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4405}$$

$$\frac{2}{3}\int \frac{1}{2}\sqrt{\sec(e+fx)a+a}(3ac+a(3c+4d)\sec(e+fx))dx+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \sqrt{\sec(e+fx)a+a}(3ac+a(3c+4d)\sec(e+fx))dx+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(3ac+a(3c+4d)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{4403}$$

$$\frac{1}{3}\left(a(3c+4d)\int \sec(e+fx)\sqrt{\sec(e+fx)a+a}dx+3ac\int \sqrt{\sec(e+fx)a+a}dx\right)+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a(3c+4d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+3ac\int\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx\right)+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$\Big\downarrow$ 4261

$$\frac{1}{3}\left(a(3c+4d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-\frac{6a^2c\int\frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}\right)+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$\Big\downarrow$ 216

$$\frac{1}{3}\left(a(3c+4d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+\frac{6a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}\right)+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$\Big\downarrow$ 4279

$$\frac{1}{3}\left(\frac{6a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2a^2(3c+4d)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}\right)+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

output
```
(2*a*d*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(3*f) + ((6*a^(3/2)*c*ArcTan
[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (2*a^2*(3*c + 4*d)*
Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]))/3
```

3.156. $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$

3.156.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4403
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Si
mp[d   Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4405
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Simp[1/m   Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
 (b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2
*m]
```

3.156.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(91) = 182$.

Time = 1.46 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.96

method	result
default	$\frac{2ca\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx}{\cos(fx+}}}\right.\right.}{f(\cos(fx+e)+1)}$
parts	$\frac{2ca\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx}{\cos(fx+}}}\right.\right.}{f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2*c/f*a*(a*(sec(f*x+e)+1))^(1/2)*(arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(
f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e
)+arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+sin(f*x+e))/(cos(f*x+e)+1)+2/3*d/f*a*(a*(
sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(5*sin(f*x+e)+tan(f*x+e))
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.01

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \left[\frac{3\left(ac\cos(fx+e)^2+ac\cos(fx+e)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\cos(fx+e)+1}\right.}{3\left(f\cos(fx+e)^2\right.}\right.$$

$$-\frac{2\left(3\left(ac\cos(fx+e)^2+ac\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-(ad+(3\,ac+5\,ad)\cos(fx\right.}{3\left(f\cos(fx+e)^2+f\cos(fx+e)\right)}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x
+ e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(a*d + (3*a*c +
5*a*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))
/(f*cos(f*x + e)^2 + f*cos(f*x + e)), -2/3*(3*(a*c*cos(f*x + e)^2 + a*c*co
s(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)/(sqrt(a)*sin(f*x + e))) - (a*d + (3*a*c + 5*a*d)*cos(f*x + e))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos
(f*x + e))]
```

3.156.6 Sympy [F]

$$\int (a + a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int \left(a(\sec{(e+fx)}+1)\right)^{\frac{3}{2}}(c+d\sec{(e+fx)})\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e)),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x)), x)
```

3.156.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(91) = 182$.

Time = 0.39 (sec) , antiderivative size = 998, normalized size of antiderivative = 9.50

$$\int (a + a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*((a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))) + 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) - 1) - a*arctan2((cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e),...
```

3.156.8 Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
sage0*x
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx=\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)), x)
```

3.157 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$

3.157.1 Optimal result

Integrand size = 27, antiderivative size = 110

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx=\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{2a^{3/2}(c-d)\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{c\sqrt{d}\sqrt{c+d}f}$$

output
```
2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f+2*a^(3/2)*
(c-d)*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(1/2)
)/c/f/d^(1/2)/(c+d)^(1/2)
```

3.157.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.23

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx=\frac{\sqrt{2}a\left(\sqrt{d}\sqrt{c+d}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)+(c-d)\arctan\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+f\right.}{\sqrt{c+d}\sqrt{\cos(e+f}}\right.\right.}{c\sqrt{d}\sqrt{c+d}f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

output
```
(Sqrt[2]*a*(Sqrt[d]*Sqrt[c + d]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (c - d)
*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]]
)])*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(c*Sqr
t[d]*Sqrt[c + d]*f)
```

3.157.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3042, 4415, 3042, 4261, 216, 4455, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{c+d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4415}$$

$$\frac{a(c-d)\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{c+d\sec(e+fx)}dx}{c}+\frac{a\int \sqrt{\sec(e+fx)a+a}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}+\frac{a\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c}$$

$$\Big\downarrow \text{4261}$$

$$\frac{a(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}-\frac{2a^2\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}$$

$$\Big\downarrow \text{216}$$

$$\frac{a(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}+\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

$$\Big\downarrow \text{4455}$$

$$\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{2a^2(c-d)\int\frac{1}{\frac{a^2d\tan^2(e+fx)}{\sec(e+fx)a+a}+a(c+d)}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2a^{3/2}(c-d)\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{c\sqrt{d}f\sqrt{c+d}}+\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

output
```
(2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f)
+ (2*a^(3/2)*(c - d)*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sq
rt[a + a*Sec[e + f*x]])])/(c*Sqrt[d]*Sqrt[c + d]*f)
```

3.157.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4415
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Simp[a/c   Int[Sqrt[a + b*Csc[e + f*x]], x], x]
+ Simp[(b*c - a*d)/c   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Cs
c[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

rule 4455
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[-2*(b/f)   Subst[In
t[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0]
```

3.157.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 839 vs. $2(90) = 180$.

Time = 14.28 (sec) , antiderivative size = 840, normalized size of antiderivative = 7.64

method	result
default	$\sqrt{2}\,a\left(2\sqrt{(c+d)(c-d)}\,\sqrt{\frac{d}{c-d}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\ln\left(-\frac{2\left(\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\sqrt{2}\,\sqrt{\frac{d}{c-d}}\,c-\sqrt{2}\,\sqrt{\frac{d}{c-}}\right.}{-c(-\cot(fx+e)+\csc(f}\right.\right.$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*2^(1/2)*a/((c+d)*(c-d))^(1/2)/c/(d/(c-d))^(1/2)*(2*((c+d)*(c-d))^(1/
2)*(d/(c-d))^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)
*(-cot(f*x+e)+csc(f*x+e)))+ln(-2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*
2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*
x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-
cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))*c-
d*ln(-2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c
-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*
(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-
cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))-ln(2*(((1-cos(f*x+e))^2*csc
(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-c
os(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d-((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(
f*x+e))-c+d)/(c*(-cot(f*x+e)+csc(f*x+e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+d)
*(c-d))^(1/2)))*c+d*ln(2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*
(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1
)^(1/2)*d-((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e
)+csc(f*x+e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2))))*((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/
2)
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 731, normalized size of antiderivative = 6.65

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \left[-\frac{(ac-ad)\sqrt{-\frac{a}{cd+d^2}}\log\left(\frac{2\,(cd+d^2)\sqrt{-\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac+2}{c\cos(fx+e)^2+(c+d)\cos(fx+}\right.}{}\right.$$

$$-\frac{2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+(ac-ad)\sqrt{-\frac{a}{cd+d^2}}\log\left(\frac{2\,(cd+d^2)\sqrt{-\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e}{c\cos(fx+e)^2+(c+d)}\right.}{cf}$$

$$-\frac{2\,(ac-ad)\sqrt{\frac{a}{cd+d^2}}\arctan\left(\frac{(c+d)\sqrt{\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{a\sin(fx+e)}\right)-\sqrt{-a}a\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{\cos(fx+}\right.}{cf}$$

$$\left.-\frac{2\left((ac-ad)\sqrt{\frac{a}{cd+d^2}}\arctan\left(\frac{(c+d)\sqrt{\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{a\sin(fx+e)}\right)+a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)\right)}{cf}\right]$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[-((a*c - a*d)*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2)
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c
 + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)
^2 + (c + d)*cos(f*x + e) + d)) - sqrt(-a)*a*log((2*a*cos(f*x + e)^2 - 2*s
qrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
+ a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -(2*a^(3/2)*arctan(sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) +
(a*c - a*d)*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c +
2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2
+ (c + d)*cos(f*x + e) + d)))/(c*f), -(2*(a*c - a*d)*sqrt(a/(c*d + d^2))*a
rctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)/(a*sin(f*x + e))) - sqrt(-a)*a*log((2*a*cos(f*x + e)^2 - 2*sq
rt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) +
 a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -2*((a*c - a*d)*sqrt(a/(c
*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) + a^(3/2)*arctan(sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*f)]
```

3.157.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx=\int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{c+d\sec(e+fx)}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x)), x)
```

3.157.7 Maxima [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{d\sec(fx+e)+c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

3.157.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{d\sec(fx+e)+c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
sage0*x
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)), x)
```

3.158 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx$

3.158.1 Optimal result

Integrand size = 27, antiderivative size = 229

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^{5/2}(c^2-3cd-2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2\sqrt{d}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^2(c-d)\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
a^2*(c-d)*tan(f*x+e)/c/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*a
^(5/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/(a-a*sec(f
*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+a^(5/2)*(c^2-3*c*d-2*d^2)*arctanh(d^(1
/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/(c+d)^(3/2)
/f/d^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.158.2 Mathematica [A] (warning: unable to verify)

Time = 4.74 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.28

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{(d+c\cos(e+fx))^2\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}(a(1+\sec(e+fx)))^{3/2}}{}$$

input

```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^2,x]
```

output

```
((d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]^3*Sqrt[Sec[e + f*x]]*(a*(1 + Sec[
e + f*x]))^(3/2)*(((2*Sqrt[d]*(c + d)^(3/2)*ArcTan[Tan[(e + f*x)/2]/Sqrt[C
os[e + f*x]/(1 + Cos[e + f*x])]] + (c^2 - 3*c*d - 2*d^2)*ArcTan[(Sqrt[d]*T
an[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])*Sqr
t[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sec[(e + f*x)/2]*Sqrt[1 + Sec[e + f*x]]
)/(Sqrt[d]*(c + d)^(3/2)*Sqrt[(1 + Cos[e + f*x])^(-1)]) + (2*c*(c - d)*Sqr
t[Sec[e + f*x]]*Sin[(e + f*x)/2])/((c + d)*(c + d*Sec[e + f*x]))))/(4*c^2*
f*(c + d*Sec[e + f*x])^2)
```

3.158.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4428, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{a\cos(e+fx)(\sec(e+fx)+1)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^3\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 168$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\int \frac{a\cos(e+fx)(2(c+d)+(c-d)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{ac(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\int \frac{\cos(e+fx)(2(c+d)+(c-d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{2c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 174$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\frac{\left(c^2-3cd-2d^2\right)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{c}+\frac{2(c+d)\int \frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{c}}{2c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^3\tan(e+fx)\left(\frac{-\frac{2\left(c^2-3cd-2d^2\right)\int \frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{4(c+d)\int \frac{1}{1-\frac{a-a\sec(e+fx)}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}}{2c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 219$$

$$-\frac{a^3\tan(e+fx)\left(\frac{-\frac{2\left(c^2-3cd-2d^2\right)\int \frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{4(c+d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 221$$

3.158. $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx$

$$-\frac{a^3\tan(e+fx)\left(\frac{-\frac{2(c^2-3cd-2d^2)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c\sqrt{d}\sqrt{c+d}}-\frac{4(c+d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^3*(((-4*(c + d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c
) - (2*(c^2 - 3*c*d - 2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(S
qrt[a]*Sqrt[c + d])])/(Sqrt[a]*c*Sqrt[d]*Sqrt[c + d]))/(2*c*(c + d)) - ((c
 - d)*Sqrt[a - a*Sec[e + f*x]])/(a*c*(c + d)*(c + d*Sec[e + f*x])))*Tan[e
+ f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.158.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.158.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 25447 vs. $2(199) = 398$.

Time = 15.93 (sec) , antiderivative size = 25448, normalized size of antiderivative = 111.13

method	result	size
default	Expression too large to display	25448

input
```
int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.158.5 Fricas [A] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 1640, normalized size of antiderivative = 7.16

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2*d - 2*
a*c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*cos(f*
x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*
d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c
 + d)*cos(f*x + e) + d)) + 2*(a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)
^2 + (a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e
)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d)*f*cos(f
*x + e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f
), 1/2*(2*(a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) - 4*(a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a
*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (a*c^2*d - 3*a*c
*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2*d - 2*a*c*d^2)*cos(f*x + e)^2 + (a*c^3 -
 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((
2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a
*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/((c^4...
```

3.158.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{(c+d\sec(e+fx))^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x))**2, x)
```

3.158.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.158.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^2, x)
```

3.159 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx$

3.159.1 Optimal result

Integrand size = 27, antiderivative size = 310

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx = \frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^{5/2}(3c^3-15c^2d-20cd^2-8d^3)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4c^3\sqrt{d}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^2(3c^2-7cd-4d^2)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
1/2*a^2*(c-d)*tan(f*x+e)/c/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/
2)+1/4*a^2*(3*c^2-7*c*d-4*d^2)*tan(f*x+e)/c^2/(c+d)^2/f/(c+d*sec(f*x+e))/(
a+a*sec(f*x+e))^(1/2)+2*a^(5/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*ta
n(f*x+e)/c^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+1/4*a^(5/2)*(
3*c^3-15*c^2*d-20*c*d^2-8*d^3)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1
/2)/(c+d)^(1/2))*tan(f*x+e)/c^3/(c+d)^(5/2)/f/d^(1/2)/(a-a*sec(f*x+e))^(1/
2)/(a+a*sec(f*x+e))^(1/2)
```

3.159.2 Mathematica [A] (warning: unable to verify)

Time = 5.86 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.16

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx = \frac{(d+c\cos(e+fx))^3\sec^3\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{3}{2}}(e+fx)(a(1+\sec(e+fx)))^{3/2}\Big(}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^3,x]
```

output
```
((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]^3*Sec[e + f*x]^(3/2)*(a*(1 + Sec[
e + f*x]))^(3/2)*((Sqrt[2]*(8*Sqrt[d]*(c + d)^(5/2)*ArcTan[Tan[(e + f*x)/2
]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] + (3*c^3 - 15*c^2*d - 20*c*d^2 -
8*d^3)*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1
 + Cos[e + f*x])]])]*Sec[(e + f*x)/2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2
]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]])/(Sqrt[d]*(c + d)^(5/2)*Sqrt[Sec[(
e + f*x)/2]^2]) + (2*c*Sqrt[Sec[e + f*x]]*(c*(5*c^2 - 7*c*d - 6*d^2) + d*(
3*c^2 - 7*c*d - 4*d^2)*Sec[e + f*x])*Sin[(e + f*x)/2])/((c + d)^2*(c + d*S
ec[e + f*x])^2)))/(16*c^3*f*(c + d*Sec[e + f*x])^3)
```

3.159.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {3042, 4428, 27, 168, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{a\cos(e+fx)(\sec(e+fx)+1)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^3\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 168$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\int \frac{a\cos(e+fx)(4(c+d)+3(c-d)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2ac(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\int \frac{\cos(e+fx)(4(c+d)+3(c-d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{4c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 168$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\frac{\int \frac{a\cos(e+fx)\left(8(c+d)^2+\left(3c^2-7dc-4d^2\right)\sec(e+fx)\right)}{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{ac(c+d)}-\frac{\left(3c^2-7cd-4d^2\right)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}}{4c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx}}{2ac(c+d)(c+d\sec(e+fx)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\frac{\int \frac{\cos(e+fx)\left(8(c+d)^2+\left(3c^2-7dc-4d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{2c(c+d)}-\frac{\left(3c^2-7cd-4d^2\right)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}}{4c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 174$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\frac{\frac{\left(3c^3-15c^2d-20cd^2-8d^3\right)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{c}+\frac{8(c+d)^2\int \frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{c}}{2c(c+d)}-\frac{\left(3c^2-7cd-4d^2\right.}{ac(c+d)(c}}{4c(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 73$$

3.159. $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx$

$$-\frac{a^3\tan(e+fx)\left(\frac{-\frac{2\left(3c^3-15c^2d-20cd^2-8d^3\right)\int\frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{16(c+d)^2\int\frac{1}{1-\frac{a-a\sec(e+fx)}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}}{2c(c+d)}-\frac{\left(3c^2-\right.}{\;}}{4c(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 219

$$-\frac{a^3\tan(e+fx)\left(\frac{-\frac{2\left(3c^3-15c^2d-20cd^2-8d^3\right)\int\frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{16(c+d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}-\frac{\left(3c^2-7cd-4\right.}{ac(c+d)}}{4c(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 221

$$-\frac{a^3\tan(e+fx)\left(\frac{-\frac{2\left(3c^3-15c^2d-20cd^2-8d^3\right)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c\sqrt{d}\sqrt{c+d}}-\frac{16(c+d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}-\frac{\left(3c^2-7cd-4d^2\right)\sqrt{a-a}}{ac(c+d)(c+d\sec(e}}{4c(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^3*(-1/2*((c - d)*Sqrt[a - a*Sec[e + f*x]])/(a*c*(c + d)*(c + d*Sec[e
+ f*x])^2) + (((-16*(c + d)^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(
Sqrt[a]*c) - (2*(3*c^3 - 15*c^2*d - 20*c*d^2 - 8*d^3)*ArcTanh[(Sqrt[d]*Sqr
t[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c*Sqrt[d]*Sqrt[c +
 d]))/(2*c*(c + d)) - ((3*c^2 - 7*c*d - 4*d^2)*Sqrt[a - a*Sec[e + f*x]])/(
a*c*(c + d)*(c + d*Sec[e + f*x])))/(4*c*(c + d)))*Tan[e + f*x])/(f*Sqrt[a
- a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.159. $\int\frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx$

3.159.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.159. $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx$

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.159.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 55535 vs. $2(272) = 544$.

Time = 16.91 (sec) , antiderivative size = 55536, normalized size of antiderivative = 179.15

method	result	size
default	Expression too large to display	55536

input
```
int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.159.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(272) = 544$.

Time = 11.67 (sec) , antiderivative size = 2729, normalized size of antiderivative = 8.80

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c + d\sec(e + fx))^3} \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[-1/8*((3*a*c^3*d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*
a*c^4*d - 20*a*c^3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*
d - 50*a*c^3*d^2 - 48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d
- 27*a*c^3*d^2 - 55*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(-
a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e
)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x
+ e) + d)) - 8*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d + a*c^2
*d^2)*cos(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)*cos(f
*x + e)^2 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e))*sq
rt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) +
1)) - 2*((5*a*c^4 - 7*a*c^3*d - 6*a*c^2*d^2)*cos(f*x + e)^2 + (3*a*c^3*d -
 7*a*c^2*d^2 - 4*a*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4
*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 +
4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f),
-1/8*(16*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*
cos(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)*cos(f*x + e
)^2 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e))*sqrt(...
```

3.159.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx = \int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{(c+d\sec(e+fx))^3}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x))**3, x)
```

3.159.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c + d\sec(e + fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.159.8 Giac [F]

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c + d\sec(e + fx))^3}\,dx = \int \frac{(a\sec(fx + e) + a)^{\frac{3}{2}}}{(d\sec(fx + e) + c)^3}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c + d\sec(e + fx))^3}\,dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c + \frac{d}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^3, x)
```

3.160 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx$

3.160.1 Optimal result

Integrand size = 27, antiderivative size = 336

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = \frac{2a^3(3c^3+12c^2d+12cd^2+4d^3)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}c^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2ad(3c^2+15cd+13d^2)(a-a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} - \frac{6d^2(c+2d)(a-a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3(a-a\sec(e+fx))^4\tan(e+fx)}{9af\sqrt{a+a\sec(e+fx)}} - \frac{2(c^3+12c^2d+24cd^2+12d^3)(a^3-a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^3*(3*c^3+12*c^2*d+12*c*d^2+4*d^3)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+
2/5*a*d*(3*c^2+15*c*d+13*d^2)*(a-a*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x
+e))^(1/2)-6/7*d^2*(c+2*d)*(a-a*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e)
)^(1/2)+2/9*d^3*(a-a*sec(f*x+e))^4*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)-2
/3*(c^3+12*c^2*d+24*c*d^2+12*d^3)*(a^3-a^3*sec(f*x+e))*tan(f*x+e)/f/(a+a*s
ec(f*x+e))^(1/2)+2*a^(7/2)*c^3*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan
(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.160.2 Mathematica [A] (verified)

Time = 4.73 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.85

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = \frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{a(1+\sec(e+fx))}\left(2520\sqrt{2}c^3\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3,x]
```

output
```
(a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*(2520*Sqrt
[2]*c^3*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(9/2) + 2*(2520*c^3
+ 8883*c^2*d + 8370*c*d^2 + 2908*d^3 + (630*c^3 + 5292*c^2*d + 7290*c*d^2
+ 2792*d^3)*Cos[e + f*x] + 4*(840*c^3 + 2898*c^2*d + 2610*c*d^2 + 803*d^3)
*Cos[2*(e + f*x)] + 210*c^3*Cos[3*(e + f*x)] + 1764*c^2*d*Cos[3*(e + f*x)]
 + 2070*c*d^2*Cos[3*(e + f*x)] + 584*d^3*Cos[3*(e + f*x)] + 840*c^3*Cos[4*
(e + f*x)] + 2709*c^2*d*Cos[4*(e + f*x)] + 2070*c*d^2*Cos[4*(e + f*x)] + 5
84*d^3*Cos[4*(e + f*x)])*Sin[(e + f*x)/2]))/(2520*f)
```

3.160.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{a^2\cos(e+fx)(\sec(e+fx)+1)^2(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^4 \tan(e+fx) \int \frac{\cos(e+fx)(\sec(e+fx)+1)^2(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 198$$

$$-\frac{a^4 \tan(e+fx) \int \left(\frac{d^3(a-a\sec(e+fx))^{7/2}}{a^4} - \frac{3d^2(c+2d)(a-a\sec(e+fx))^{5/2}}{a^3} + \frac{d(3c^2+15dc+13d^2)(a-a\sec(e+fx))^{3/2}}{a^2} + \frac{(-c^3-12dc^2}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^4 \tan(e+fx) \left(-\frac{2d^3(a-a\sec(e+fx))^{9/2}}{9a^5} + \frac{6d^2(c+2d)(a-a\sec(e+fx))^{7/2}}{7a^4} - \frac{2d(3c^2+15cd+13d^2)(a-a\sec(e+fx))^{5/2}}{5a^3} + \frac{2(c^3+12c^2}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\,\mathrm{se}}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^4*((-2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*(3
*c^3 + 12*c^2*d + 12*c*d^2 + 4*d^3)*Sqrt[a - a*Sec[e + f*x]])/a + (2*(c^3
+ 12*c^2*d + 24*c*d^2 + 12*d^3)*(a - a*Sec[e + f*x])^(3/2))/(3*a^2) - (2*d
*(3*c^2 + 15*c*d + 13*d^2)*(a - a*Sec[e + f*x])^(5/2))/(5*a^3) + (6*d^2*(c
 + 2*d)*(a - a*Sec[e + f*x])^(7/2))/(7*a^4) - (2*d^3*(a - a*Sec[e + f*x])^
(9/2))/(9*a^5))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e
 + f*x]]))
```

3.160.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.160.4 Maple [A] (verified)

Time = 90.86 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.09

method	result
default	$\frac{2a^2\sqrt{a(\sec(fx+e)+1)}\left(315\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,c^3\cos(fx+e)+315\,\text{arctanh}\left(\frac{\sin(\ldots}{(\cos(fx+e)+1)\ldots}\right.\right.}{\ldots}$
parts	$\frac{2c^3a^2\sqrt{a(\sec(fx+e)+1)}\left(3\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos(fx+e)+3\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\ldots}}\right.\right.}{3f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
2/315*a^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(315*arctanh(sin(f*x+e
)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*c^3*cos(f*x+e)+315*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f
*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c^3+840*si
n(f*x+e)*c^3+2709*sin(f*x+e)*c^2*d+2070*sin(f*x+e)*c*d^2+584*sin(f*x+e)*d^
3+105*c^3*tan(f*x+e)+882*c^2*d*tan(f*x+e)+1035*c*d^2*tan(f*x+e)+292*d^3*ta
n(f*x+e)+189*c^2*d*tan(f*x+e)*sec(f*x+e)+540*c*d^2*tan(f*x+e)*sec(f*x+e)+2
19*d^3*tan(f*x+e)*sec(f*x+e)+135*c*d^2*tan(f*x+e)*sec(f*x+e)^2+130*d^3*tan
(f*x+e)*sec(f*x+e)^2+35*d^3*tan(f*x+e)*sec(f*x+e)^3)
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.85

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = \left[\frac{315\left(a^2c^3\cos(fx+e)^5+a^2c^3\cos(fx+e)^4\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{\cos(fx}\right.}{}\right.$$

$$-\frac{2\left(315\left(a^2c^3\cos(fx+e)^5+a^2c^3\cos(fx+e)^4\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-(35a^2d^3+(840a^2c^3}{}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(-a)*log
((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3
5*a^2*d^3 + (840*a^2*c^3 + 2709*a^2*c^2*d + 2070*a^2*c*d^2 + 584*a^2*d^3)*
cos(f*x + e)^4 + (105*a^2*c^3 + 882*a^2*c^2*d + 1035*a^2*c*d^2 + 292*a^2*d
^3)*cos(f*x + e)^3 + 3*(63*a^2*c^2*d + 180*a^2*c*d^2 + 73*a^2*d^3)*cos(f*x
 + e)^2 + 5*(27*a^2*c*d^2 + 26*a^2*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4), -
2/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(a)*arcta
n(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
e))) - (35*a^2*d^3 + (840*a^2*c^3 + 2709*a^2*c^2*d + 2070*a^2*c*d^2 + 584*
a^2*d^3)*cos(f*x + e)^4 + (105*a^2*c^3 + 882*a^2*c^2*d + 1035*a^2*c*d^2 +
292*a^2*d^3)*cos(f*x + e)^3 + 3*(63*a^2*c^2*d + 180*a^2*c*d^2 + 73*a^2*d^3
)*cos(f*x + e)^2 + 5*(27*a^2*c*d^2 + 26*a^2*d^3)*cos(f*x + e))*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x +
 e)^4)]
```

3.160.6 Sympy [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx=\int \left(a(\sec(e+fx)+1)\right)^{\frac{5}{2}}\left(c+d\sec(e+fx)\right)^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**3, x)
```

3.160.7 Maxima [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx=\text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.160.8 Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^3\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3, x)
```

3.161 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx$

3.161.1 Optimal result

Integrand size = 27, antiderivative size = 258

$$\begin{aligned}\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx &= \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}\\ &+\frac{2a^{7/2}c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &+\frac{2ad(2c+5d)(a-a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}}\\ &-\frac{2d^2(a-a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}}\\ &-\frac{2(c^2+8cd+8d^2)(a^3-a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

output
```
2*a^3*(c+2*d)*(3*c+2*d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/5*a*d*(2*c+5
*d)*(a-a*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/7*d^2*(a-a*se
c(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*(c^2+8*c*d+8*d^2)*(a^3
-a^3*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(7/2)*c^2*arctanh
((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*
sec(f*x+e))^(1/2)
```

3.161.2 Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.74

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx = \frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\left(420\sqrt{2}c^2\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+f\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2,x]
```

output
```
(a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*(420*Sqrt[
2]*c^2*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(7/2) + 4*(35*c^2 + 1
96*c*d + 145*d^2 + (420*c^2 + 987*c*d + 465*d^2)*Cos[e + f*x] + (35*c^2 +
196*c*d + 115*d^2)*Cos[2*(e + f*x)] + 140*c^2*Cos[3*(e + f*x)] + 301*c*d*C
os[3*(e + f*x)] + 115*d^2*Cos[3*(e + f*x)])*Sin[(e + f*x)/2]))/(420*f)
```

3.161.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{a^2\cos(e+fx)(\sec(e+fx)+1)^2(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)^2(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{a^4 \tan(e+fx) \int \left(-\frac{d^2(a-a\sec(e+fx))^{5/2}}{a^3} + \frac{d(2c+5d)(a-a\sec(e+fx))^{3/2}}{a^2} + \frac{(-c^2-8dc-8d^2)\sqrt{a-a\sec(e+fx)}}{a} + \frac{3c^2+8dc+4d^2}{\sqrt{a-a\sec(e+fx)}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4 \tan(e+fx) \left(\frac{2d^2(a-a\sec(e+fx))^{7/2}}{7a^4} - \frac{2d(2c+5d)(a-a\sec(e+fx))^{5/2}}{5a^3} + \frac{2(c^2+8cd+8d^2)(a-a\sec(e+fx))^{3/2}}{3a^2} - \frac{2c^2\text{arctanh}\left(\sqrt{\right.}}{\sqrt{}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^4*((-2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*(c
 + 2*d)*(3*c + 2*d)*Sqrt[a - a*Sec[e + f*x]])/a + (2*(c^2 + 8*c*d + 8*d^2)
*(a - a*Sec[e + f*x])^(3/2))/(3*a^2) - (2*d*(2*c + 5*d)*(a - a*Sec[e + f*x
])^(5/2))/(5*a^3) + (2*d^2*(a - a*Sec[e + f*x])^(7/2))/(7*a^4))*Tan[e + f*
x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.161.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.161.4 Maple [A] (verified)

Time = 21.68 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.09

method	result
default	$\frac{2a^2\sqrt{a(\sec(fx+e)+1)}\left(105\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)c^2\cos(fx+e)+105\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\right.}{}$
parts	$\frac{2c^2a^2\sqrt{a(\sec(fx+e)+1)}\left(3\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+3\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{c}{c}}}\right.\right.}{3f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/105*a^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(105*(-cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+
e)+1))^(1/2))*c^2*cos(f*x+e)+105*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctan
h(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*c^2+280*si
n(f*x+e)*c^2+602*sin(f*x+e)*c*d+230*sin(f*x+e)*d^2+35*c^2*tan(f*x+e)+196*c
*d*tan(f*x+e)+115*d^2*tan(f*x+e)+42*c*d*tan(f*x+e)*sec(f*x+e)+60*d^2*tan(f
*x+e)*sec(f*x+e)+15*d^2*tan(f*x+e)*sec(f*x+e)^2)
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.94

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx = \left[\frac{105\left(a^2c^2\cos(fx+e)^4+a^2c^2\cos(fx+e)^3\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cdots}{\cos(fx\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{2\left(105\left(a^2c^2\cos(fx+e)^4+a^2c^2\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(15a^2d^2+2(140a^2\cdots\right.}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(-a)*log
((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(1
5*a^2*d^2 + 2*(140*a^2*c^2 + 301*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^3 + (
35*a^2*c^2 + 196*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^2 + 6*(7*a^2*c*d + 10
*a^2*d^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x +
e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a^2*c^2*cos(f*x +
e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*a^2*d^2 + 2*(140*a^
2*c^2 + 301*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^3 + (35*a^2*c^2 + 196*a^2*
c*d + 115*a^2*d^2)*cos(f*x + e)^2 + 6*(7*a^2*c*d + 10*a^2*d^2)*cos(f*x + e
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4
 + f*cos(f*x + e)^3)]
```

3.161.6 Sympy [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx = \int \left(a(\sec(e+fx)+1)\right)^{\frac{5}{2}}(c+d\sec(e+fx))^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**2, x)
```

3.161.7 Maxima [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/210*(105*((a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 + 2*
a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) + 1) - (a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 +
 2*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1)) - 1) - 2*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + a^2*c^2*f*sin(2*f*x +
 2*e)^2 + 2*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*integrate((((cos(6*f*x
 + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x
 + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f
*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4
*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2
*f*x + 2*e))*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin
(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*...
```

3.161.8 Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^2\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2, x)
```

3.162 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

3.162.1 Optimal result

Integrand size = 25, antiderivative size = 142

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \frac{2a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{2a^3(35c+32d)\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(5c+8d)\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{15f} + \frac{2ad(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{5f}$$

output
```
2*a^(5/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f+2/5*a*d*(a
+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f+2/15*a^3*(35*c+32*d)*tan(f*x+e)/f/(a+a*s
ec(f*x+e))^(1/2)+2/15*a^2*(5*c+8*d)*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

3.162.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\left(30\sqrt{2}c\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

output
```
(a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*(30*Sqrt[2
]*c*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(5/2) + 2*(40*c + 49*d +
 2*(5*c + 14*d)*Cos[e + f*x] + (40*c + 43*d)*Cos[2*(e + f*x)])*Sin[(e + f*
x)/2]))/(30*f)
```

3.162.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4405, 27, 3042, 4405, 27, 3042, 4403, 3042, 4261, 216, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4405}$$

$$\frac{2}{5}\int \frac{1}{2}(\sec(e+fx)a+a)^{3/2}(5ac+a(5c+8d)\sec(e+fx))dx+\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int (\sec(e+fx)a+a)^{3/2}(5ac+a(5c+8d)\sec(e+fx))dx+\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(5ac+a(5c+8d)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{4405}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{1}{2}\sqrt{\sec(e+fx)a+a}(15ca^2+(35c+32d)\sec(e+fx)a^2)\,dx+\frac{2a^2(5c+8d)\tan(e+fx)\sqrt{a\sec(e+fx)}\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{3}\int \sqrt{\sec(e+fx)a+a}(15ca^2+(35c+32d)\sec(e+fx)a^2)\,dx+\frac{2a^2(5c+8d)\tan(e+fx)\sqrt{a\sec(e+fx)+}}{3f}\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(15ca^2+(35c+32d)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx+\frac{2a^2(5c+8d)\tan(e+fx)\sqrt{a}}{3f}\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$\downarrow$ 4403

$$\frac{1}{5}\left(\frac{1}{3}\left(a^2(35c+32d)\int \sec(e+fx)\sqrt{\sec(e+fx)a+a}dx+15a^2c\int \sqrt{\sec(e+fx)a+a}dx\right)+\frac{2a^2(5c+8d)\tan(}{}\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(a^2(35c+32d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+15a^2c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx\right)\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$\downarrow$ 4261

$$\frac{1}{5}\left(\frac{1}{3}\left(a^2(35c+32d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-\frac{30a^3c\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}\right.\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$\downarrow$ 216

3.162. $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

$$\frac{1}{5}\left(\frac{1}{3}\left(a^2(35c+32d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+\frac{30a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}\right)+\frac{2a\right.$$

$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{4279}$$

$$\frac{1}{5}\left(\frac{2a^2(5c+8d)\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}+\frac{1}{3}\left(\frac{30a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2a^3(35c+32d)\tan(e+}{f\sqrt{a\sec(e+fx)+}}\right.\right.$$

$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

output
```
(2*a*d*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f) + ((2*a^2*(5*c + 8*d
)*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(3*f) + ((30*a^(5/2)*c*ArcTan[(Sq
rt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (2*a^3*(35*c + 32*d)*Ta
n[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]))/3)/5
```

3.162.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4403
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Si
mp[d   Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4405
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Simp[1/m   Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
 (b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2
*m]
```

3.162.4 Maple [A] (verified)

Time = 7.70 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.49

method	result
default	$\frac{2a^2\sqrt{a(\sec(fx+e)+1)}\left(15\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)c\cos(fx+e)+15\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\right.\right.}{15f(\cos(fx+}$
parts	$\frac{2c\,a^2\sqrt{a(\sec(fx+e)+1)}\left(3\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+3\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{c}{\cos}}}\right.\right.}{3f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/15*a^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(15*(-cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)
+1))^(1/2))*c*cos(f*x+e)+15*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(sin
(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*c+40*sin(f*x+e)
*c+43*sin(f*x+e)*d+5*c*tan(f*x+e)+14*d*tan(f*x+e)+3*d*tan(f*x+e)*sec(f*x+e
))
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.75

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \left[\frac{15\left(a^2c\cos(fx+e)^3+a^2c\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(f}{\cos(fx+e)+}\right.}{}\right.$$

$$-\frac{2\left(15\left(a^2c\cos(fx+e)^3+a^2c\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(3a^2d+(40a^2c+43a^2\right.}{15\left(f\cos(fx+e)^3+f\cos(fx+\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/15*(15*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a)*log((2*a*
cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*a^2*d
 + (40*a^2*c + 43*a^2*d)*cos(f*x + e)^2 + (5*a^2*c + 14*a^2*d)*cos(f*x + e
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3
 + f*cos(f*x + e)^2), -2/15*(15*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)
^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) - (3*a^2*d + (40*a^2*c + 43*a^2*d)*cos(f*x + e)^2 +
(5*a^2*c + 14*a^2*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

3.162.6 Sympy [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int (a(\sec(e+fx)+1))^{\frac{5}{2}}(c+d\sec(e+fx))\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e)),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x)), x)
```

3.162.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1396 vs. $2(124) = 248$.

Time = 0.43 (sec) , antiderivative size = 1396, normalized size of antiderivative = 9.83

$$\int (a + a\sec(e + fx))^{5/2}(c + d\sec(e + fx))\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
1/6*(30*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)
*((12*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x +
 2*e) - 3*a^2*sin(2*f*x + 2*e) - 4*(3*a^2*cos(2*f*x + 2*e) + 4*a^2)*sin(3/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1)) + (12*a^2*sin(2*f*x + 2*e)*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*a^2*cos(2*f*x + 2*e) - a^2 + 4*(3
*a^2*cos(2*f*x + 2*e) + 4*a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(
a) + 3*((a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*f*x
 + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))) + 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*...
```

3.162.8 Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
sage0*x
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{5/2}(c + d\sec(e + fx))\,dx = \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^{5/2} \left(c + \frac{d}{\cos(e + f\,x)}\right)\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)), x)
```

3.163 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx$

3.163.1 Optimal result

Integrand size = 27, antiderivative size = 203

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \frac{2a^3\tan(e+fx)}{df\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2a^{7/2}(c-d)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{cd^{3/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

output

```
2*a^3*tan(f*x+e)/d/f/(a+a*sec(f*x+e))^(1/2)+2*a^(7/2)*arctanh((a-a*sec(f*x
+e))^(1/2)/a^(1/2))*tan(f*x+e)/c/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))
^(1/2)-2*a^(7/2)*(c-d)^2*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c
+d)^(1/2))*tan(f*x+e)/c/d^(3/2)/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*
sec(f*x+e))^(1/2)
```

3.163.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.71 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.69

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \frac{\cos^{\frac{3}{2}}(e+fx)(d+c\cos(e+fx))\sec^5\left(\frac{1}{2}(e+fx)\right)(a(1+\sec(e+fx)))^{5/2}\left(\cdots\right.}{\cdots}$$

input

```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x]),x]
```

output

```
(Cos[e + f*x]^(3/2)*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^5*(a*(1 + Sec[e
+ f*x]))^(5/2)*((10*(c - d)^2*(c + 3*d + 2*c*Cos[e + f*x])*Csc[(e + f*x)/2
]*(-ArcTanh[Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]] + Sqrt[-((d*(-1 + Se
c[e + f*x]))/(c + d))]))/(d*(c + d)*Sqrt[Cos[e + f*x]]*Sqrt[-((d*(-1 + Sec
[e + f*x]))/(c + d))]) + (20*(3*c - d)*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]
] - (16*(c - d)^2*d*(d + c*Cos[e + f*x])*Hypergeometric2F1[2, 5/2, 7/2, (-
2*d*Sec[e + f*x]*Sin[(e + f*x)/2]^2)/(c + d)]*Sin[(e + f*x)/2]^3)/((c + d)
^3*Cos[e + f*x]^(5/2)) + 10*c*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] -
(2*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]])))/(40*c^2*f*(c + d*Sec[e + f*x]))
```

3.163.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{c+d\sec(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4428$$

$$-\frac{a^2\tan(e+fx)\int\frac{a^2\cos(e+fx)(\sec(e+fx)+1)^2}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\tan(e+fx)\int\frac{\cos(e+fx)(\sec(e+fx)+1)^2}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{a^4\tan(e+fx)\int\left(-\frac{(c-d)^2}{cd\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{\cos(e+fx)}{c\sqrt{a-a\sec(e+fx)}}+\frac{1}{d\sqrt{a-a\sec(e+fx)}}\right)d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4\tan(e+fx)\left(\frac{2(c-d)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}cd^{3/2}\sqrt{c+d}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}-\frac{2\sqrt{a-a\sec(e+fx)}}{ad}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x]),x]
```

output
```
-((a^4*((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c) + (2*(c
 - d)^2*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])
/(Sqrt[a]*c*d^(3/2)*Sqrt[c + d]) - (2*Sqrt[a - a*Sec[e + f*x]])/(a*d))*Tan
[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.163.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.163. $\int\frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.163.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1401 vs. $2(173) = 346$.

Time = 23.45 (sec) , antiderivative size = 1402, normalized size of antiderivative = 6.91

method	result	size
default	Expression too large to display	1402

input
```
int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*a^2/c/((c+d)*(c-d))^(1/2)/d/(d/(c-d))^(1/2)*(2*((c+d)*(c-d))^(1/2)*2
^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*arctanh(2^(1/2)/((1-cos(f*x
+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(d/(c-d))^(1/2)*d-2
^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*ln(-2*(((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-
cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc
(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+e))*d+((c+
d)*(c-d))^(1/2)))*c^2+2*2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*ln
(-2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(
1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d
))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(
f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))*c*d-2^(1/2)*((1-cos(f*x+e))^2*c
sc(f*x+e)^2-1)^(1/2)*ln(-2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2
)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2
-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*
x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))*d^2+2^(1
/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*ln(-2*(-((1-cos(f*x+e))^2*csc(
f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c+2^(1/2)*(d/(c-d))^(1/2)*((1-co
s(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f
*x+e))+c-d)/(c*(-cot(f*x+e)+csc(f*x+e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+...
```

3.163.5 Fricas [A] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 1140, normalized size of antiderivative = 5.62

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[(2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + (a^2*c^2
- 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x + e))*sqrt
(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x +
 e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*
x + e) + d)) + (a^2*d*cos(f*x + e) + a^2*d)*sqrt(-a)*log((2*a*cos(f*x + e)
^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f
*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*d*f*cos(f*x + e) + c
*d*f), (2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 2*(
a^2*d*cos(f*x + e) + a^2*d)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c^2 - 2*a^2*c*d + a^2
*d^2 + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*
log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*
c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/(c*
d*f*cos(f*x + e) + c*d*f), (2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sin(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2*a^2*c*d +
 a^2*d^2)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d
^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))
) + (a^2*d*cos(f*x + e) + a^2*d)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*s...
```

3.163.6 Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{c + d\sec(e + fx)}\,dx = \int \frac{(a(\sec{(e + fx)} + 1))^{\frac{5}{2}}}{c + d\sec{(e + fx)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(5/2)/(c + d*sec(e + f*x)), x)
```

3.163.7 Maxima [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{d\sec(fx+e)+c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c), x)
```

3.163.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{d\sec(fx+e)+c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
sage0*x
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x)), x)
```

3.164 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx$

3.164.1 Optimal result

Integrand size = 27, antiderivative size = 329

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{2a^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^{7/2}(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{cd^{3/2}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}(c-d)\sqrt{c+d}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2d^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^3(c-d)^2\tan(e+fx)}{cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
-a^3*(c-d)^2*tan(f*x+e)/c/d/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2
)+2*a^(7/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/(a-a*
sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-a^(7/2)*(c-d)^2*arctanh(d^(1/2)*(
a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/d^(3/2)/(c+d)^(3/2
)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*a^(7/2)*(c-d)*arctanh(
d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*(c+d)^(1/2)*tan(f*x+e)
/c^2/d^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.164.2 Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.85

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{\sqrt{\cos(e+fx)}(d+c\cos(e+fx))^2\sec^5\left(\frac{1}{2}(e+fx)\right)(a(1+\sec(e+fx)))^{5/2}}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^2,x]
```

output
```
(Sqrt[Cos[e + f*x]]*(d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]^5*(a*(1 + Sec[
e + f*x]))^(5/2)*(2*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (4*Sqrt[2]*
(c - d)*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e
+ f*x]])])/(Sqrt[d]*Sqrt[c + d]) - ((c - d)^2*(2*c*Cos[e + f*x] - (2*(c +
2*d)*ArcTanh[Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]]*(d + c*Cos[e + f*x]
))/((c + d)*Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]))*Sin[(e + f*x)/2])/(
d*(c + d)*Sqrt[Cos[e + f*x]]*(d + c*Cos[e + f*x]))))/(8*c^2*f*(c + d*Sec[e
 + f*x])^2)
```

3.164.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{a^2\cos(e+fx)(\sec(e+fx)+1)^2}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 27

$$-\frac{a^4\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)^2}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 198

$$-\frac{a^4\tan(e+fx)\int \left(-\frac{(c-d)^2}{cd\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}+\frac{\cos(e+fx)}{c^2\sqrt{a-a\sec(e+fx)}}+\frac{c^2-d^2}{c^2d\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}\right)d\sec(e+}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 2009

$$-\frac{a^4\tan(e+fx)\left(-\frac{2\sqrt{c+d}(c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2d^{3/2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^2}+\frac{(c-d)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+}}{\sqrt{a}\sqrt{c+d}}\right.}{\sqrt{a}cd^{3/2}(c+d)^{3/2}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^4*((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^2) + ((c
 - d)^2*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])
/(Sqrt[a]*c*d^(3/2)*(c + d)^(3/2)) - (2*(c - d)*Sqrt[c + d]*ArcTanh[(Sqrt[
d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^2*d^(3/2))
 + ((c - d)^2*Sqrt[a - a*Sec[e + f*x]])/(a*c*d*(c + d)*(c + d*Sec[e + f*x]
)))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.164.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.164.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 20137 vs. 2(281) = 562.

Time = 53.69 (sec) , antiderivative size = 20138, normalized size of antiderivative = 61.21

method	result	size
default	Expression too large to display	20138

input
```
int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.164.5 Fricas [A] (verification not implemented)

Time = 13.17 (sec) , antiderivative size = 2031, normalized size of antiderivative = 6.17

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a^2*c^3*d + 4*a^2*c^2*d^2 - 3*a^2*
c*d^3 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*d^2 - 2*a^2*c*d^3)*
cos(f*x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 - 5*a^2*c*d^3 - 2*a^
2*d^4)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d
+ d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
+ (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*
x + e)^2 + (c + d)*cos(f*x + e) + d)) - 2*(a^2*c*d^2 + a^2*d^3 + (a^2*c^2*
d + a^2*c*d^2)*cos(f*x + e)^2 + (a^2*c^2*d + 2*a^2*c*d^2 + a^2*d^3)*cos(f*
x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f
*x + e) + 1)))/((c^4*d + c^3*d^2)*f*cos(f*x + e)^2 + (c^4*d + 2*c^3*d^2 +
c^2*d^3)*f*cos(f*x + e) + (c^3*d^2 + c^2*d^3)*f), -1/2*(2*(a^2*c^3 - 2*a^2
*c^2*d + a^2*c*d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) + 4*(a^2*c*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*c*d^2)*cos(f*x + e
)^2 + (a^2*c^2*d + 2*a^2*c*d^2 + a^2*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))
+ (a^2*c^3*d + 4*a^2*c^2*d^2 - 3*a^2*c*d^3 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*
c^3*d - 3*a^2*c^2*d^2 - 2*a^2*c*d^3)*cos(f*x + e)^2 + (a^2*c^4 + 5*a^2*c^3
*d + a^2*c^2*d^2 - 5*a^2*c*d^3 - 2*a^2*d^4)*cos(f*x + e))*sqrt(-a/(c*d ...
```

3.164.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx=\int \frac{(a(\sec(e+fx)+1))^{\frac{5}{2}}}{(c+d\sec(e+fx))^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(5/2)/(c + d*sec(e + f*x))**2, x)
```

3.164.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.164.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^2, x)
```

3.165 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx$

3.165.1 Optimal result

Integrand size = 27, antiderivative size = 536

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx &= \frac{2a^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&- \frac{3a^{7/2}(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4cd^{3/2}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&+ \frac{a^{7/2}(c-d)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2d^{3/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&- \frac{2a^{7/2}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&- \frac{a^3(c-d)^2\tan(e+fx)}{2cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\\
&+ \frac{a^3(c-d)\tan(e+fx)}{c^2df\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\\
&- \frac{3a^3(c-d)^2\tan(e+fx)}{4cd(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\end{aligned}$$

output
```
-1/2*a^3*(c-d)^2*tan(f*x+e)/c/d/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)
)^(1/2)+a^3*(c-d)*tan(f*x+e)/c^2/d/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/
2)-3/4*a^3*(c-d)^2*tan(f*x+e)/c/d/(c+d)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+
e))^(1/2)+2*a^(7/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3
/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-3/4*a^(7/2)*(c-d)^2*arcta
nh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/d^(3/2
)/(c+d)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+a^(7/2)*(c-d
)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c
^2/d^(3/2)/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2*a
^(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*d^(1/2)
*tan(f*x+e)/c^3/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2
)
```

3.165.2 Mathematica [A] (warning: unable to verify)

Time = 9.97 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.03

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = \frac{\left(8d^{3/2}(c+d)^2\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right) - \frac{(c^4+10c^3d-15c^2d^2-20cd^3-8d^4)\operatorname{arctanh}\left(\cdots\right)}{\sqrt{-c-d}} \cdots\right.}{\cdots}$$

$$+\frac{(d+c\cos(e+fx))^3\sec^5\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(a(1+\sec(e+fx)))^{5/2}\left(-\frac{(c^3-12c^2d+5cd^2+6d^3)\sin\left(\frac{1}{2}(e+fx)\right)}{16c^3d(c+d)^2}+\cdots\right.}{f(c+d\sec\cdots}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^3,x]
```

output
```
((8*d^(3/2)*(c + d)^2*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e
 + f*x])]] - ((c^4 + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*ArcTanh[(Sq
rt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])
])])/Sqrt[-c - d])*(d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]^6*Sqrt[Cos[e +
f*x]*Sec[(e + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e
+ f*x]]*(a*(1 + Sec[e + f*x]))^(5/2))/(16*Sqrt[2]*c^3*d^(3/2)*(c + d)^2*f*
Sqrt[Sec[(e + f*x)/2]^2]*(c + d*Sec[e + f*x])^3) + ((d + c*Cos[e + f*x])^3
*Sec[(e + f*x)/2]^5*Sec[e + f*x]*(a*(1 + Sec[e + f*x]))^(5/2)*(-1/16*((c^3
 - 12*c^2*d + 5*c*d^2 + 6*d^3)*Sin[(e + f*x)/2])/(c^3*d*(c + d)^2) + (-(c^
2*d*Sin[(e + f*x)/2]) + 2*c*d^2*Sin[(e + f*x)/2] - d^3*Sin[(e + f*x)/2])/(
8*c^3*(c + d)*(d + c*Cos[e + f*x])^2) + (3*c^3*Sin[(e + f*x)/2] - 14*c^2*d
*Sin[(e + f*x)/2] + 3*c*d^2*Sin[(e + f*x)/2] + 8*d^3*Sin[(e + f*x)/2])/(16
*c^3*(c + d)^2*(d + c*Cos[e + f*x])))))/(f*(c + d*Sec[e + f*x])^3)
```

3.165.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{a^2\cos(e+fx)(\sec(e+fx)+1)^2}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)^2}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{a^4\tan(e+fx)\int\left(-\frac{(c-d)^2}{cd\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}+\frac{\cos(e+fx)}{c^3\sqrt{a-a\sec(e+fx)}}-\frac{d}{c^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{}{c^2d\sqrt{a-a}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\big\downarrow$ 2009

$$-\frac{a^4\tan(e+fx)\left(\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^3\sqrt{c+d}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^3}-\frac{(c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2d^{3/2}\sqrt{c+d}}+\frac{3(c}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^4*((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^3) + (3*
(c - d)^2*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])
])/(4*Sqrt[a]*c*d^(3/2)*(c + d)^(5/2)) - ((c - d)*ArcTanh[(Sqrt[d]*Sqrt[a
- a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^2*d^(3/2)*Sqrt[c + d
]) + (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c
 + d])])/(Sqrt[a]*c^3*Sqrt[c + d]) + ((c - d)^2*Sqrt[a - a*Sec[e + f*x]])/
(2*a*c*d*(c + d)*(c + d*Sec[e + f*x])^2) - ((c - d)*Sqrt[a - a*Sec[e + f*x
]])/(a*c^2*d*(c + d*Sec[e + f*x])) + (3*(c - d)^2*Sqrt[a - a*Sec[e + f*x]]
)/(4*a*c*d*(c + d)^2*(c + d*Sec[e + f*x]))))*Tan[e + f*x])/(f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.165.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.165. $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.165.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 42562 vs. $2(460) = 920$.

Time = 235.82 (sec) , antiderivative size = 42563, normalized size of antiderivative = 79.41

method	result	size
default	Expression too large to display	42563

input
```
int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.165.5 Fricas [A] (verification not implemented)

Time = 22.58 (sec) , antiderivative size = 3351, normalized size of antiderivative = 6.25

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c + d\sec(e + fx))^3}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[-1/8*((a^2*c^4*d^2 + 10*a^2*c^3*d^3 - 15*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a
^2*d^6 + (a^2*c^6 + 10*a^2*c^5*d - 15*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2
*c^2*d^4)*cos(f*x + e)^3 + (a^2*c^6 + 12*a^2*c^5*d + 5*a^2*c^4*d^2 - 50*a^
2*c^3*d^3 - 48*a^2*c^2*d^4 - 16*a^2*c*d^5)*cos(f*x + e)^2 + (2*a^2*c^5*d +
 21*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 55*a^2*c^2*d^4 - 36*a^2*c*d^5 - 8*a^2*d
^6)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d
^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (
a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x +
 e)^2 + (c + d)*cos(f*x + e) + d)) - 8*(a^2*c^2*d^3 + 2*a^2*c*d^4 + a^2*d^
5 + (a^2*c^4*d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*c^4*d
+ 4*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^2*c^3
*d^2 + 5*a^2*c^2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(-a)*log((
2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*((a^
2*c^5 - 12*a^2*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*cos(f*x + e)^2 - (a^
2*c^4*d + 10*a^2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*cos(f*x + e))*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d + 2*c^6*d^2 + c^
5*d^3)*f*cos(f*x + e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 + 2*c^4*d^4)*f*co
s(f*x + e)^2 + (2*c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5)*f*cos(f*x + e
) + (c^5*d^3 + 2*c^4*d^4 + c^3*d^5)*f), -1/8*(16*(a^2*c^2*d^3 + 2*a^2*c...
```

3.165.6 Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c + d\sec(e + fx))^3}\, dx = \int \frac{(a(\sec(e + fx) + 1))^{\frac{5}{2}}}{(c + d\sec(e + fx))^3}\, dx$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(5/2)/(c + d*sec(e + f*x))**3, x)
```

3.165.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.165.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^3}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^3, x)
```

3.166 $\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.166.1 Optimal result

Integrand size = 27, antiderivative size = 258

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2(3c-d)d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^3(1-\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}(c-d)^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

output
```
2*(3*c-d)*d^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*d^3*tan(f*x+e)/f/(a+a*
sec(f*x+e))^(1/2)-2/3*d^3*(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/
2)+2*c^3*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*a^(1/2)*tan(f*x+e)/f/(a-a
*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(c-d)^3*arctanh(1/2*(a-a*sec(f*x
+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)*tan(f*x+e)/f/(a-a*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.166.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.07 (sec) , antiderivative size = 787, normalized size of antiderivative = 3.05

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)(c+d\sec(e+fx))^3\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}\left(\frac{2c(c^2+3d^2)\sin\left(\frac{1}{2}(e+fx)\right)}{3\left(1-2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)^{3/2}}-\cdots\right.}{}$$

input
```
Integrate[(c + d*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Cos[(e + f*x)/2]*(c + d*Sec[e + f*x])^3*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)
^(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*((2*c*(c^2 + 3*d^2)*Sin[(e + f*x)/2]
)/(3*(1 - 2*Sin[(e + f*x)/2]^2)^(3/2)) - (4*c^2*(c + 3*d)*Sin[(e + f*x)/2]
^3)/(3*(1 - 2*Sin[(e + f*x)/2]^2)^(3/2)) + (4*c*(c^2 + 3*d^2)*Sin[(e + f*x
)/2])/(3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]) + (c^3*Csc[(e + f*x)/2]*Sqrt[1 -
2*Sin[(e + f*x)/2]^2]*((4*Sin[(e + f*x)/2]^4)/(1 - 2*Sin[(e + f*x)/2]^2)^2
 - (6*Sin[(e + f*x)/2]^2)/(1 - 2*Sin[(e + f*x)/2]^2) + (3*Sqrt[2]*ArcSin[S
qrt[2]*Sin[(e + f*x)/2]]*Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2])
)/3 - ((c - d)^3*Csc[(e + f*x)/2]^5*(-12*Cos[(e + f*x)/2]^4*Hypergeometric
PFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2)
)]*Sin[(e + f*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin[(e + f*x)/
2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*Sin[(e + f*x)/2]^8*(4 - 7*Sin[(e + f*x)/
2]^2 + 3*Sin[(e + f*x)/2]^4) + 7*Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e +
 f*x)/2]^2))]*(1 - 2*Sin[(e + f*x)/2]^2)^3*(15 - 20*Sin[(e + f*x)/2]^2 + 8
*Sin[(e + f*x)/2]^4)*((3 - 7*Sin[(e + f*x)/2]^2)*Sqrt[-(Sin[(e + f*x)/2]^2
/(1 - 2*Sin[(e + f*x)/2]^2))] - 3*ArcTanh[Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2
*Sin[(e + f*x)/2]^2))]]*(1 - 2*Sin[(e + f*x)/2]^2))))/(63*(1 - 2*Sin[(e +
f*x)/2]^2)^(7/2))))/(f*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^(5/2)*Sqrt[a*(1
 + Sec[e + f*x])])
```

3.166.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^3}{a(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^3}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{a\tan(e+fx)\int \left(\frac{\cos(e+fx)c^3}{\sqrt{a-a\sec(e+fx)}}+\frac{(3c-d)d^2}{\sqrt{a-a\sec(e+fx)}}+\frac{d^3\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^3}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}\right)d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\tan(e+fx)\left(\frac{2d^3(a-a\sec(e+fx))^{3/2}}{3a^2}-\frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{2}(c-d)^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-2d^2(3c-d)\sqrt{\ldots}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(c + d*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((a*((-2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] + (Sqrt[2
]*(c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a] -
 (2*(3*c - d)*d^2*Sqrt[a - a*Sec[e + f*x]])/a - (2*d^3*Sqrt[a - a*Sec[e +
f*x]])/a + (2*d^3*(a - a*Sec[e + f*x])^(3/2))/(3*a^2))*Tan[e + f*x])/(f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.166.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.166. $\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.166.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(227) = 454$.

Time = 5.60 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.97

method	result
parts	$-\frac{c^3\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\right.}{fa}$
default	$\frac{\left(3\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)c^3-3\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}\ln\left(c\right.\right.}{}$

input
```
int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-c^3/f/a*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2^(1
/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x
+e)^2-1)^(1/2))-2*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)))+1/3*d^3/f/a*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*
(3*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-c
os(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)-4*(1-cos(f*x+e))^3*csc(f*x+e)^3)/((1-co
s(f*x+e))^2*csc(f*x+e)^2-1)+3*c^2*d/f/a*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*(a*(sec(f*x+e)+1))^(1/2)*2^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2
*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))-3*c*d^2/f/a*(a*(sec(f*x+e)+1
))^(1/2)*(2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(csc(f*x+e)-cot(f*x
+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))+2*cot(f*x
+e)-2*csc(f*x+e))
```

3.166.5 Fricas [A] (verification not implemented)

Time = 7.80 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.40

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \left[-\frac{3\sqrt{2}\left((ac^3-3\,ac^2d+3\,acd^2-ad^3)\cos\left(fx+e\right)^2+(ac^3-3\,ac^2d+3\,acd^2-ad^3)\cos\left(fx+e\right)\right)\sqrt{-\frac{1}{a}}\,\mathrm{l}\cdots}{\cdots}\right.$$

$$-\frac{6\left(c^3\cos\left(fx+e\right)^2+c^3\cos\left(fx+e\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-2\left(d^3+(9\,cd^2-d^3)\cos\left(fx+\cdots\right.\right.}{3\left(af\,\mathrm{c}\cdots\right.}$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(2)*((a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e)^2 +
 (a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e))*sqrt(-1/a)*log(-(2*
sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*si
n(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*co
s(f*x + e) + 1)) + 6*(c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(d^
3 + (9*c*d^2 - d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sin(f*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e)), -1/3*(6*(c^3*cos(f*
x + e)^2 + c^3*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(d^3 + (9*c*d^2 - d^3)*
cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 3*sqr
t(2)*((a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e)^2 + (a*c^3 - 3*
a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f
*cos(f*x + e)^2 + a*f*cos(f*x + e))]
```

3.166.6 Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(c+d\sec(e+fx))^3}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**3/sqrt(a*(sec(e + f*x) + 1)), x)
```

3.166.7 Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e)+c)^3}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)^3/sqrt(a*sec(f*x + e) + a), x)
```

3.166.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2), x)
```

3.167 $\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.167.1 Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

output
```
2*d^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*c^2*arctanh((a-a*sec(f*x+e))^(
1/2)/a^(1/2))*a^(1/2)*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))
^(1/2)-(c-d)^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)
*a^(1/2)*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.167.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.73 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.61

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$=\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\cos^{\frac{3}{2}}(e+fx)(c+d\sec(e+fx))^2\left(-\frac{(c-d)^2\sqrt{-1+\cos(e+fx)}(2+\cos(e+fx))\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-2\text{arctan}\right.}{2\sqrt{2}}\right.}{}$$

input
```
Integrate[(c + d*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Cos[(e + f*x)/2]*Cos[e + f*x]^(3/2)*(c + d*Sec[e + f*x])^2*(-1/2*((c -
d)^2*Sqrt[-1 + Cos[e + f*x]]*(2 + Cos[e + f*x])*Csc[(e + f*x)/2]^3*(-2*Arc
Tanh[Sqrt[-(Sec[e + f*x]*Sin[(e + f*x)/2]^2)]] + Sqrt[2 - 2*Sec[e + f*x]])
)/Sqrt[2] + (4*c*d*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]] + c^2*(Sqrt[2]*Arc
Sin[Sqrt[2]*Sin[(e + f*x)/2]] - (2*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]]) -
 ((c - d)^2*Hypergeometric2F1[2, 5/2, 7/2, -(Sec[e + f*x]*Sin[(e + f*x)/2]
^2)]*Sin[(e + f*x)/2]*Sin[e + f*x]^2)/(10*Cos[e + f*x]^(5/2))))/(f*(d + c*
Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

3.167.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a\sec(e+fx)+a}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}\,dx$$
$$\Big\downarrow \text{4428}$$
$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^2}{a(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a \tan(e+fx) \int \frac{\cos(e+fx)(c+d\sec(e+fx))^2}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 198$$

$$-\frac{a \tan(e+fx) \int \left(\frac{\cos(e+fx)c^2}{\sqrt{a-a\sec(e+fx)}}+\frac{d^2}{\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^2}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}\right)d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{a \tan(e+fx)\left(-\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{2}(c-d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d^2\sqrt{a-a\sec(e+fx)}}{a}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(c + d*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((a*((-2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] + (Sqrt[2
]*(c - d)^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a] -
 (2*d^2*Sqrt[a - a*Sec[e + f*x]])/a)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f
*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.167.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.167. $\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.167.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(158) = 316$.

Time = 5.11 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.90

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left(\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\sqrt{2}\,c^2-\sqrt{(1-\cos(fx\cdots}\right.}{\cdots}$
parts	$-\frac{c^2\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\cdots\right.}{fa}$

input
```
int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/a*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2
)*(-cot(f*x+e)+csc(f*x+e)))*2^(1/2)*c^2-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^
(1/2)*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*c^
2+2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+((1-c
os(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*c*d-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^
(1/2)*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*d^
2+2*d^2*(-cot(f*x+e)+csc(f*x+e)))
```

3.167.5 Fricas [A] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.63

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \left[\frac{4\,d^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)+\sqrt{2}(ac^2-2\,acd+ad^2+(ac^2-2\,acd+ad^2)\cos(fx+e))\sqrt{-\frac{1}{a}}\log\left(\frac{2}{}\right.}{}\right.$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(4*d^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + sqrt(2)
*(a*c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-
1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos
(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x +
 e)^2 + 2*cos(f*x + e) + 1)) - 2*(c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*
a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*f*cos(
f*x + e) + a*f), (2*d^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x +
e) - 2*(c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + sqrt(2)*(a*c^2 - 2*a*c
*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e))*arctan(sqrt(2)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqr
t(a))/(a*f*cos(f*x + e) + a*f)]
```

3.167.6 Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(c+d\sec(e+fx))^2}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**2/sqrt(a*(sec(e + f*x) + 1)), x)
```

3.167.7 Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e)+c)^2}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)^2/sqrt(a*sec(f*x + e) + a), x)
```

3.167.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2), x)
```

3.168 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.168.1 Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-(c-d)*arct
an(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2
)
```

3.168.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$=\frac{2\left(\sqrt{2}c\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)+(-c+d)\arctan\left(\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(c + d*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*(Sqrt[2]*c*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (-c + d)*ArcTan[Sin[(e +
f*x)/2]/Sqrt[Cos[e + f*x]]])*Cos[(e + f*x)/2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[
a*(1 + Sec[e + f*x])])
```

3.168.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4408}$$

$$\frac{c\int\sqrt{\sec(e+fx)a+a}dx}{a}-(c-d)\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\int\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{a}-(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4261}$$

$$-\frac{2c\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}-\left((c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4282}$$

$$\frac{2(c-d)\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+2a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}+\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}$$

$$\Big\downarrow \text{216}$$

$$\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}$$

input
```
Int[(c + d*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
- (Sqrt[2]*(c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
 + f*x]])])/(Sqrt[a]*f)
```

3.168.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.168.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(76) = 152$.

Time = 2.40 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.23

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(c\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2}\right.\right.}{fa}$
parts	$\frac{c\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(2\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)-\sqrt{2}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2}\right.\right.}{fa}$

input
```
int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)^(1/2)*(c*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+
e)^2-1)^(1/2))-d*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1
)^(1/2))-c*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)
*(-cot(f*x+e)+csc(f*x+e))))
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.45

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$=\left[-\frac{\sqrt{2}(ac-ad)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)-3\cos(fx+e)^2-2\cos(fx+e)+1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+2\sqrt{-ac}\log}{2af}\right.$$
$$\left.-\frac{2\sqrt{ac}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\frac{\sqrt{2}(ac-ad)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)}{\sqrt{a}}}{af}\right]$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(2)*(a*c - a*d)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^
2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-a
)*c*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1))
)/(a*f), -(2*sqrt(a)*c*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(2)*(a*c - a*d)*arctan(sqrt(2)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/s
qrt(a))/(a*f)]
```

3.168.6 Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{c+d\sec{(e+fx)}}{\sqrt{a\,(\sec{(e+fx)}+1)}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*sec(e + f*x))/sqrt(a*(sec(e + f*x) + 1)), x)
```

3.168.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 699, normalized size of antiderivative = 7.68

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx=$$

$$-\frac{\left(\sqrt{2}\sqrt{a}\arctan\left(\frac{\left(\left|2\,e^{(i\,fx+i\,e)}+2\right|^{4}+16\,\cos(fx+e)^{4}+16\,\sin(fx+e)^{4}+8\left(\cos(fx+e)^{2}-\sin(fx+e)^{2}-2\,\cos(fx+e)+1\right)\left|2\,e^{(i\,fx+i\,e)}+2\right|\cdots}{\cdots}\right.\right.}{\cdots}$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*arctan2(((abs(2*e^(I*f*x + I*e) + 2)^4 + 16*cos(f*x + e)
^4 + 16*sin(f*x + e)^4 + 8*(cos(f*x + e)^2 - sin(f*x + e)^2 - 2*cos(f*x +
e) + 1)*abs(2*e^(I*f*x + I*e) + 2)^2 - 64*cos(f*x + e)^3 + 32*(cos(f*x + e
)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)^2 + 96*cos(f*x + e)^2 - 64*cos(f*x
+ e) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(f*x + e) - 1)*sin(f*x + e)/abs(2*e
^(I*f*x + I*e) + 2)^2, (abs(2*e^(I*f*x + I*e) + 2)^2 + 4*cos(f*x + e)^2 -
4*sin(f*x + e)^2 - 8*cos(f*x + e) + 4)/abs(2*e^(I*f*x + I*e) + 2)^2)) + 2*
sin(f*x + e))/abs(2*e^(I*f*x + I*e) + 2), ((abs(2*e^(I*f*x + I*e) + 2)^4 +
 16*cos(f*x + e)^4 + 16*sin(f*x + e)^4 + 8*(cos(f*x + e)^2 - sin(f*x + e)^
2 - 2*cos(f*x + e) + 1)*abs(2*e^(I*f*x + I*e) + 2)^2 - 64*cos(f*x + e)^3 +
 32*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)^2 + 96*cos(f*x + e)
^2 - 64*cos(f*x + e) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(f*x + e) - 1)*sin(
f*x + e)/abs(2*e^(I*f*x + I*e) + 2)^2, (abs(2*e^(I*f*x + I*e) + 2)^2 + 4*c
os(f*x + e)^2 - 4*sin(f*x + e)^2 - 8*cos(f*x + e) + 4)/abs(2*e^(I*f*x + I*
e) + 2)^2)) + 2*cos(f*x + e) - 2)/abs(2*e^(I*f*x + I*e) + 2)) - sqrt(a)*ar
ctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(f*x +
e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(f*x + e)
))*c/(a*f)
```

3.168.8 Giac [F(-2)]

Exception generated.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{c+\frac{d}{\cos(e+f\,x)}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2), x)
```

3.169 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

3.169.1 Optimal result

Integrand size = 27, antiderivative size = 166

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2d^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c(c-d)\sqrt{c+d}f}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f/a^(1/2)-arctan(1/2
*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d)/f/a^(1/2
)+2*d^(3/2)*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))
^(1/2))/c/(c-d)/f/a^(1/2)/(c+d)^(1/2)
```

3.169.2 Mathematica [A] (verified)

Time = 3.44 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$$
$$= \frac{2\left(-c\sqrt{c+d}\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{2}\left((c-d)\sqrt{c+d}\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)+d^{3/2}\arctan\left(\frac{\sqrt{d}\tan(}{\sqrt{c+d}\sqrt{}}\right.\right.\right.}{c(c-d)\sqrt{c+d}f\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{a(1+\sec(e+}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output
```
(2*(-(c*Sqrt[c + d]*ArcSin[Tan[(e + f*x)/2]]) + Sqrt[2]*((c - d)*Sqrt[c +
d]*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] + d^(3/2
)*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1 + Co
s[e + f*x])])]))*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*(d + c*Cos[e + f*x]
)*Sec[e + f*x]^(3/2)*Sqrt[1 + Sec[e + f*x]])/(c*(c - d)*Sqrt[c + d]*f*Sqrt
[Sec[(e + f*x)/2]^2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x]))
```

3.169.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {3042, 4417, 3042, 4408, 3042, 4261, 216, 4282, 216, 4455, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \ 4417$$
$$\frac{d^2\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{c+d\sec(e+fx)}dx}{ac(c-d)}+\frac{\int \frac{a(c-d)-ad\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{ac(c-d)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}+\frac{\int \frac{a(c-d)-ad\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{ac(c-d)}$$

$$\Big\downarrow \text{ 4408}$$

$$\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}+\frac{(c-d)\int \sqrt{\sec(e+fx)a+a}dx-ac\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{ac(c-d)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}+\frac{(c-d)\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-ac\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{ac(c-d)}$$

$$\Big\downarrow \text{ 4261}$$

$$\frac{-\frac{2a(c-d)\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}-ac\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{ac(c-d)}+\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{2\sqrt{a}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-ac\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{ac(c-d)}+\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}$$

$$\Big\downarrow \text{ 4282}$$

$$\frac{\frac{2ac\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+2a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}+\frac{2\sqrt{a}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}}{ac(c-d)}+\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}$$

$$\Big\downarrow \text{ 216}$$

3.169. $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}dx$

$$\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}+\frac{\frac{2\sqrt{a}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{\sqrt{2}\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{f}}{ac(c-d)}$$

$$\Big\downarrow \; 4455$$

$$\frac{\frac{2\sqrt{a}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{\sqrt{2}\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{f}}{ac(c-d)}-\frac{2d^2\int \frac{1}{\frac{a^2d\tan^2(e+fx)}{\sec(e+fx)a+a}+a(c+d)}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf(c-d)}$$

$$\Big\downarrow \; 218$$

$$\frac{2d^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}cf(c-d)\sqrt{c+d}}+\frac{\frac{2\sqrt{a}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{\sqrt{2}\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{f}}{ac(c-d)}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output
```
((2*Sqrt[a]*(c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]
])/f - (Sqrt[2]*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a +
a*Sec[e + f*x]])])/f)/(a*c*(c - d)) + (2*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*T
an[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*c*(c - d)*S
qrt[c + d]*f)
```

3.169.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.169. $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4417
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Simp[1/(c*(b*c - a*d))   Int[(b*c - a*d - b*d*
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[d^2/(c*(b*c - a*d))
 Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] |
| EqQ[c^2 - d^2, 0])
```

rule 4455
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[-2*(b/f)   Subst[In
t[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0]
```

3.169.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(137) = 274$.

Time = 15.29 (sec) , antiderivative size = 659, normalized size of antiderivative = 3.97

method	result
default	$\Big(2\sqrt{\frac{d}{c-d}}\sqrt{2}\sqrt{(c+d)(c-d)}\,\operatorname{arctanh}\Big(\frac{\sqrt{2}(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\Big)c-2\sqrt{\frac{d}{c-d}}\sqrt{2}\sqrt{(c+d)(c-d)}\,\operatorname{arctanh}\Big(\frac{\sqrt{2}(-\cot(fx+e)+\csc(}{\sqrt{(1-\cos(fx+e))^2\csc(f}}$

input
```
int(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(d/(c-d))^(1/2)/(c-d)/c/((c+d)*(c-d))^(1/2)/a*(2*(d/(c-d))^(1/2)*2^(
1/2)*((c+d)*(c-d))^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*c-2*(d/(c-d))^(1/2)*2^(1/2)*((c+d)*(c-d))
^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e
)+csc(f*x+e)))*d-2*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*ln(csc(f*x+e)-cot(f
*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*c+2^(1/2)*ln(-2*(((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^
(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(
f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+
e))*d+((c+d)*(c-d))^(1/2)))*d^2-2^(1/2)*ln(2*(((1-cos(f*x+e))^2*csc(f*x+e)
^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^(1/2)*d-((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))
-c+d)/(c*(-cot(f*x+e)+csc(f*x+e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))
^(1/2)))*d^2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-2*a/((1-cos(f*x+e)
)^2*csc(f*x+e)^2-1))^(1/2)
```

3.169.5 Fricas [A] (verification not implemented)

Time = 10.60 (sec) , antiderivative size = 1050, normalized size of antiderivative = 6.33

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(2)*a*c*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*co
s(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*a*d*sqrt(-d/(a*
c + a*d))*log((2*(c + d)*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) + (c + 2*d)*cos(f*x + e)^2 + (c + d)
*cos(f*x + e) - d)/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + 2*sqrt
(-a)*(c - d)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x +
 e) + 1)))/((a*c^2 - a*c*d)*f), -1/2*(sqrt(2)*a*c*sqrt(-1/a)*log(-(2*sqrt(
2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x
 + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x
 + e) + 1)) + 4*a*d*sqrt(d/(a*c + a*d))*arctan((c + d)*sqrt(d/(a*c + a*d))
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(d*sin(f*x + e))) +
2*sqrt(-a)*(c - d)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos
(f*x + e) + 1)))/((a*c^2 - a*c*d)*f), -(a*d*sqrt(-d/(a*c + a*d))*log((2*(c
 + d)*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e) + (c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)/
(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) - sqrt(2)*sqrt(a)*c*arctan(
sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*s...
```

3.169.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))), x)
```

3.169.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{1}{\sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

3.169.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.170 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx$

3.170.1 Optimal result

Integrand size = 27, antiderivative size = 416

$$\begin{aligned}
&\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx \\
&= \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\sqrt{2}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\sqrt{a}d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c(c-d)(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2\sqrt{a}(2c-d)d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c-d)^2\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{d^2\tan(e+fx)}{c(c^2-d^2)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}
\end{aligned}$$

output
```
d^2*tan(f*x+e)/c/(c^2-d^2)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arc
tanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*a^(1/2)*tan(f*x+e)/c^2/f/(a-a*sec(f*x
+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+d^(3/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))
^(1/2)/a^(1/2)/(c+d)^(1/2))*a^(1/2)*tan(f*x+e)/c/(c-d)/(c+d)^(3/2)/f/(a-a*
sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-arctanh(1/2*(a-a*sec(f*x+e))^(1/2
)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)*tan(f*x+e)/(c-d)^2/f/(a-a*sec(f*x+e))^(
1/2)/(a+a*sec(f*x+e))^(1/2)+2*(2*c-d)*d^(3/2)*arctanh(d^(1/2)*(a-a*sec(f*x
+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*a^(1/2)*tan(f*x+e)/c^2/(c-d)^2/f/(c+d)^(1/
2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.170.2 Mathematica [A] (warning: unable to verify)

Time = 11.36 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{(d+c\cos(e+fx))^2\sec^{\frac{5}{2}}(e+fx)\left(\frac{c(c-d)d^2\sin(e+fx)}{(c+d)(d+c\cos(e+fx))\sqrt{\sec(e+fx)}}+\frac{\left(\sqrt{2}\left(2(c-d)^2(c+d)^{3/2}\text{arctanh}\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{-\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\right.\right.}{}\right.}{c^2(c-d)^2f\sqrt{a(1}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2),x]
```

output
```
((d + c*Cos[e + f*x])^2*Sec[e + f*x]^(5/2)*((c*(c - d)*d^2*Sin[e + f*x])/(
(c + d)*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) + ((Sqrt[2]*(2*(c - d)^2*
(c + d)^(3/2)*ArcTanh[Tan[(e + f*x)/2]/Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*
x]))]] + d^(3/2)*(5*c^2 + c*d - 2*d^2)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/2])/
(Sqrt[c + d]*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))])]) - 2*c^2*(c + d)^(
3/2)*ArcTanh[Tan[(e + f*x)/2]/Sqrt[-1 + Tan[(e + f*x)/2]^2]])*Sqrt[Cos[(e
+ f*x)/2]^2*Sec[e + f*x]]*Sqrt[-1 + Tan[(e + f*x)/2]^2])/((c + d)^(3/2)*Sq
rt[Sec[(e + f*x)/2]^2])))/(c^2*(c - d)^2*f*Sqrt[a*(1 + Sec[e + f*x])]*(c +
 d*Sec[e + f*x])^2)
```

3.170.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{a\tan(e+fx)\int \left(\frac{(2c-d)d^2}{c^2(c-d)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{d^2}{c(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}+\frac{\cos(e+fx)}{c^2\sqrt{a-a\sec(e+fx)}}-\frac{}{(c\cdots}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\tan(e+fx)\left(-\frac{2d^{3/2}(2c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2(c-d)^2\sqrt{c+d}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^2}-\frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c(c-d)(c+d)^{3/2}}\cdots\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2),x]
```

output
```
-((a*((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^2) + (Sqrt
[2]*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*(c - d)^
2) - (d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c +
 d])])/(Sqrt[a]*c*(c - d)*(c + d)^(3/2)) - (2*(2*c - d)*d^(3/2)*ArcTanh[(S
qrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^2*(c -
 d)^2*Sqrt[c + d]) - (d^2*Sqrt[a - a*Sec[e + f*x]])/(a*c*(c^2 - d^2)*(c +
d*Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec
[e + f*x]]))
```

3.170.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.170.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 54029 vs. $2(355) = 710$.

Time = 16.58 (sec) , antiderivative size = 54030, normalized size of antiderivative = 129.88

method	result	size
default	Expression too large to display	54030

input
```
int(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.170.5 Fricas [A] (verification not implemented)

Time = 108.96 (sec) , antiderivative size = 2508, normalized size of antiderivative = 6.03

$$\int \frac{1}{\sqrt{a + a\sec(e + fx)}(c + d\sec(e + fx))^2}\, dx = \text{Too large to display}$$

input
```
integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
[1/2*(2*(c^2*d^2 - c*d^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + sqrt(2)*(a*c^3*d + a*c^2*d^2 + (a*c^4 + a*c^3*d)*cos(f
*x + e)^2 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e))*sqrt(-1/a)*log((
2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*
sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*
cos(f*x + e) + 1)) - (5*a*c^2*d^2 + a*c*d^3 - 2*a*d^4 + (5*a*c^3*d + a*c^2
*d^2 - 2*a*c*d^3)*cos(f*x + e)^2 + (5*a*c^3*d + 6*a*c^2*d^2 - a*c*d^3 - 2*
a*d^4)*cos(f*x + e))*sqrt(-d/(a*c + a*d))*log((2*(c + d)*sqrt(-d/(a*c + a*
d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (c
 + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)/(c*cos(f*x + e)^2 + (c
+ d)*cos(f*x + e) + d)) - 2*(c^3*d - c^2*d^2 - c*d^3 + d^4 + (c^4 - c^3*d
- c^2*d^2 + c*d^3)*cos(f*x + e)^2 + (c^4 - 2*c^2*d^2 + d^4)*cos(f*x + e))*
sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e)
+ 1)))/((a*c^6 - a*c^5*d - a*c^4*d^2 + a*c^3*d^3)*f*cos(f*x + e)^2 + (a*c^
6 - 2*a*c^4*d^2 + a*c^2*d^4)*f*cos(f*x + e) + (a*c^5*d - a*c^4*d^2 - a*c^3
*d^3 + a*c^2*d^4)*f), 1/2*(2*(c^2*d^2 - c*d^3)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e) + sqrt(2)*(a*c^3*d + a*c^2*d^2 + (a
*c^4 + a*c^3*d)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x +
 e))*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*...
```

3.170.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**2), x)
```

3.170.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)^2}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^2), x)
```

3.170.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+fx)}}\left(c+\frac{d}{\cos(e+fx)}\right)^2}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2), x)
```

3.171 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx$

3.171.1 Optimal result

Integrand size = 27, antiderivative size = 653

$$\begin{aligned}
\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx
&= \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\sqrt{2}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{(c-d)^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{3\sqrt{a}d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4c(c-d)(c+d)^{5/2} f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\sqrt{a}(2c-d)d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c-d)^2(c+d)^{3/2} f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2\sqrt{a}d^{3/2}(3c^2-3cd+d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^3(c-d)^3\sqrt{c+d} f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{d^2\tan(e+fx)}{2c\left(c^2-d^2\right) f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} \\
&\quad + \frac{3d^2\tan(e+fx)}{4c(c-d)(c+d)^2 f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad + \frac{(2c-d)d^2\tan(e+fx)}{c^2(c-d)^2(c+d) f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}
\end{aligned}$$

output
```
1/2*d^2*tan(f*x+e)/c/(c^2-d^2)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)
+3/4*d^2*tan(f*x+e)/c/(c-d)/(c+d)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1
/2)+(2*c-d)*d^2*tan(f*x+e)/c^2/(c-d)^2/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f
*x+e))^(1/2)+2*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*a^(1/2)*tan(f*x+e)/
c^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+3/4*d^(3/2)*arctanh(d^
(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*a^(1/2)*tan(f*x+e)/c/(c-
d)/(c+d)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+(2*c-d)*d^(
3/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*a^(1/2)*t
an(f*x+e)/c^2/(c-d)^2/(c+d)^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)
)^(1/2)-arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2
)*tan(f*x+e)/(c-d)^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*d^(
3/2)*(3*c^2-3*c*d+d^2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d
)^(1/2))*a^(1/2)*tan(f*x+e)/c^3/(c-d)^3/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/
2)/(a+a*sec(f*x+e))^(1/2)
```

3.171.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2940 vs. $2(653) = 1306$.

Time = 22.45 (sec) , antiderivative size = 2940, normalized size of antiderivative = 4.50

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3),x]
```

output
```
(Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^4*(-1/2*(d^2*(-13*c^
2 - c*d + 6*d^2)*Sin[(e + f*x)/2])/(c^3*(-c + d)^2*(c + d)^2) - (d^4*Sin[(
e + f*x)/2])/(c^3*(-c + d)*(c + d)*(d + c*Cos[e + f*x])^2) + (-15*c^2*d^3*
Sin[(e + f*x)/2] - c*d^4*Sin[(e + f*x)/2] + 8*d^5*Sin[(e + f*x)/2])/(2*c^3
*(-c + d)^2*(c + d)^2*(d + c*Cos[e + f*x]))))/(f*Sqrt[a*(1 + Sec[e + f*x])
]*(c + d*Sec[e + f*x])^3) - (Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])^3*((Sqr
t[2]*d^(3/2)*(35*c^4 + 14*c^3*d - 21*c^2*d^2 - 4*c*d^3 + 8*d^4)*ArcTan[(Sq
rt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x
]))])])/(Sqrt[-c - d]*(c - d)) - 2*Sqrt[2]*(c^2 - d^2)^2*Log[Sec[(e + f*x)
/2]^2*(-1 + 2*Cos[e + f*x] - 2*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))]*Si
n[e + f*x])] + 2*Sqrt[2]*(c^2 - d^2)^2*Log[Sec[(e + f*x)/2]^2*(-1 + 2*Cos[
e + f*x] + 2*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))]*Sin[e + f*x])] + (8*
c^3*(c + d)^2*Log[Tan[(e + f*x)/2] + Sqrt[-1 + Tan[(e + f*x)/2]^2]])/(c -
d))*((-2*c*d*Sec[(e + f*x)/2])/((-c + d)^2*(c + d)^2*(d + c*Cos[e + f*x])*
Sqrt[Sec[e + f*x]]) - (13*d^2*Sec[(e + f*x)/2])/(8*(-c + d)^2*(c + d)^2*(d
 + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) + (d^3*Sec[(e + f*x)/2])/(8*c*(-c +
 d)^2*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) + (d^4*Sec[(e + f
*x)/2])/(2*c^2*(-c + d)^2*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]
]) + (c^2*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(2*(-c + d)^2*(c + d)^2*(d
+ c*Cos[e + f*x])) + (3*d^2*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(8*(-c...
```

3.171.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 27

$$-\frac{a\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 198

$$-\frac{a\tan(e+fx)\int \left(\frac{(3c^2-3dc+d^2)d^2}{c^3(c-d)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{(2c-d)d^2}{c^2(c-d)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}+\frac{d^2}{c(c-d)\sqrt{a-a\sec(e+fx)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+f}}$$

$\Big\downarrow$ 2009

$$-\frac{a\tan(e+fx)\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^3}-\frac{d^{3/2}(2c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2(c-d)^2(c+d)^{3/2}}-\frac{2d^{3/2}(3c^2-3cd+d^2)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{}}{}\right.}{\sqrt{a}c^3(c-d)^3\sqrt{c+d}}\right.}{}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3),x]
```

output
```
-((a*((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^3) + (Sqrt
[2]*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*(c - d)^
3) - (3*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c
 + d])])/(4*Sqrt[a]*c*(c - d)*(c + d)^(5/2)) - ((2*c - d)*d^(3/2)*ArcTanh[
(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^2*(c
 - d)^2*(c + d)^(3/2)) - (2*d^(3/2)*(3*c^2 - 3*c*d + d^2)*ArcTanh[(Sqrt[d]
*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^3*(c - d)^3*
Sqrt[c + d]) - (d^2*Sqrt[a - a*Sec[e + f*x]])/(2*a*c*(c^2 - d^2)*(c + d*Se
c[e + f*x])^2) - (3*d^2*Sqrt[a - a*Sec[e + f*x]])/(4*a*c*(c - d)*(c + d)^2
*(c + d*Sec[e + f*x])) - ((2*c - d)*d^2*Sqrt[a - a*Sec[e + f*x]])/(a*c^2*(
c - d)^2*(c + d)*(c + d*Sec[e + f*x]))))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e
 + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.171. $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx$

3.171.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.171.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 97276 vs. $2(564) = 1128$.

Time = 18.67 (sec) , antiderivative size = 97277, normalized size of antiderivative = 148.97

method	result	size
default	Expression too large to display	97277

input
```
int(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.171.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.171.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^3}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**3), x)
```

3.171.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \int \frac{1}{\sqrt{a\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)^3}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^3), x)
```

3.171.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a + a\sec(e + fx)}(c + d\sec(e + fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + a\sec(e + fx)}(c + d\sec(e + fx))^3}\,dx = \int \frac{1}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}\,\left(c + \frac{d}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3), x)
```

3.172 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.172.1 Optimal result

Integrand size = 27, antiderivative size = 324

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c-d)^2(c+2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

output
```
2*d^3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)-1/2*(c-d)^3*tan(f*x+e)/a/f/(1+
sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^3*arctanh((a-a*sec(f*x+e))^(1/2)/a^
(1/2))*tan(f*x+e)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-
1/4*(c-d)^3*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)
/f*2^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(c-d)^2*(
c+2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*tan(f*x
+e)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.172.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.09 (sec) , antiderivative size = 856, normalized size of antiderivative = 2.64

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2\cos^3\left(\frac{1}{2}(e+fx)\right)(c+d\sec(e+fx))^3\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(e+\right.}}{}$$

input
```
Integrate[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*Cos[(e + f*x)/2]^3*(c + d*Sec[e + f*x])^3*Sqrt[(1 - 2*Sin[(e + f*x)/2]^
2)^(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*((-3*(c - d)^3*ArcTan[(1 - 2*Sin[(
e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]])/2 + (3*(c - d)^3*ArcTan[(1 +
 2*Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]])/2 - (4*c^2*(c - 3*d)
*Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2] + ((c - d)^3*(1 - 2*Sin[
(e + f*x)/2]))/(4*(1 + Sin[(e + f*x)/2])*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]) -
 ((c - d)^3*(1 + 2*Sin[(e + f*x)/2]))/(4*(1 - Sin[(e + f*x)/2])*Sqrt[1 - 2
*Sin[(e + f*x)/2]^2]) - ((c - d)^3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2])/(1 - Si
n[(e + f*x)/2]) + ((c - d)^3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2])/(1 + Sin[(e +
 f*x)/2]) - (2*c^3*(-(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]) + 2*Sqrt[2
]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Sin[(e + f*x)/2]^2 + 2*Sin[(e + f*x)/2]
*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]))/(1 - 2*Sin[(e + f*x)/2]^2) - ((c - d)^2*
(11*c + d)*Sin[(e + f*x)/2]*((2*Cos[(e + f*x)/2]^2*Hypergeometric2F1[2, 5/
2, 7/2, -(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*Sin[(e + f*x)/2]
^2)/(1 - 2*Sin[(e + f*x)/2]^2) + 5*Csc[(e + f*x)/2]^4*Sqrt[-(Sin[(e + f*x)
/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*(1 - 2*Sin[(e + f*x)/2]^2)^2*(3 - 2*Sin
[(e + f*x)/2]^2)*(-ArcTanh[Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/
2]^2))]] + Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))])))/(10*(
1 - 2*Sin[(e + f*x)/2]^2)^(3/2))))/(f*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^
(3/2)*(a*(1 + Sec[e + f*x]))^(3/2))
```

3.172.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^3}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^3}{a^2(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^3}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{\tan(e+fx)\int \left(\frac{\cos(e+fx)c^3}{\sqrt{a-a\sec(e+fx)}}+\frac{d^3}{\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^2(c+2d)}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^3}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}\right)d\sec}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(-\frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{(c-d)^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}}+\frac{\sqrt{2}(c-d)^2(c+2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right.}{\sqrt{a}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((((-2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] + ((c - d)^
3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*Sqrt[a])
 + (Sqrt[2]*(c - d)^2*(c + 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*
Sqrt[a])])/Sqrt[a] - (2*d^3*Sqrt[a - a*Sec[e + f*x]])/a + ((c - d)^3*Sqrt[
a - a*Sec[e + f*x]])/(2*a*(1 + Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[a - a
*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.172.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.172.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(280) = 560$.

Time = 6.06 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.81

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left(4\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\sqrt{2}\,c^3+c^3(1-\cos(f\right.}{}$
parts	$\frac{c^3\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(4\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{(1-\cos(f\right.}{4f\,a^2}$

input
```
int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^2/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(4*arctanh(2^(1/2
)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-co
s(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*c^3+c^3*(1-cos(f*x+e))^3*csc(f*x
+e)^3-3*c^2*d*(1-cos(f*x+e))^3*csc(f*x+e)^3+3*c*d^2*(1-cos(f*x+e))^3*csc(f
*x+e)^3-d^3*(1-cos(f*x+e))^3*csc(f*x+e)^3-5*ln(csc(f*x+e)-cot(f*x+e)+((1-c
os(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2
)*c^3+3*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*
((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*c^2*d+9*ln(csc(f*x+e)-cot(f*x+e)+(
(1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^
(1/2)*c*d^2-7*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(
1/2))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d^3-c^3*(-cot(f*x+e)+csc(f*x
+e))+3*c^2*d*(-cot(f*x+e)+csc(f*x+e))-3*c*d^2*(-cot(f*x+e)+csc(f*x+e))+9*d
^3*(-cot(f*x+e)+csc(f*x+e)))
```

3.172.5 Fricas [A] (verification not implemented)

Time = 14.30 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.16

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{\sqrt{2}\left(5\,c^3-3\,c^2d-9\,cd^2+7\,d^3+\left(5\,c^3-3\,c^2d-9\,cd^2+7\,d^3\right)\cos\left(fx+e\right)\right.}{}\right.$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(2)*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3 + (5*c^3 - 3*c^2*d - 9*c
*d^2 + 7*d^3)*cos(f*x + e)^2 + 2*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f
*x + e))*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e)
 + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c^3*cos(f*x + e)^2 + 2*c
^3*cos(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x +
 e) - a)/(cos(f*x + e) + 1)) - 4*(4*d^3 - (c^3 - 3*c^2*d + 3*c*d^2 - 5*d^3
)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^2
*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), 1/4*(sqrt(2)*(5*c^3 - 3
*c^2*d - 9*c*d^2 + 7*d^3 + (5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e
)^2 + 2*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e))*sqrt(a)*arctan(s
qrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f
*x + e))) - 8*(c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sqrt(a)*arct
an(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
 e))) + 2*(4*d^3 - (c^3 - 3*c^2*d + 3*c*d^2 - 5*d^3)*cos(f*x + e))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a
^2*f*cos(f*x + e) + a^2*f)]
```

3.172.6 Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(c+d\sec(e+fx))^3}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**3/(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.172.7 Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(d\sec(fx+e)+c)^3}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)^3/(a*sec(f*x + e) + a)^(3/2), x)
```

3.172.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2), x)
```

3.173 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.173.1 Optimal result

Integrand size = 27, antiderivative size = 290

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{(c-d)^2\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c^2-d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

output
```
-1/2*(c-d)^2*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^2*ar
ctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/a^(1/2)/(a-a*sec(f*x+e)
)^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*(c-d)^2*arctanh(1/2*(a-a*sec(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/f*2^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(
a+a*sec(f*x+e))^(1/2)-(c^2-d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*2^(1/2)*tan(f*x+e)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x
+e))^(1/2)
```

3.173.2 Mathematica [A] (verified)

Time = 5.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.61

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{-\sqrt{2}(5c^2-2cd-3d^2)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sec(e+fx}{af(1}$$

input
```
Integrate[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(-(Sqrt[2]*(5*c^2 - 2*c*d - 3*d^2)*ArcSin[Tan[(e + f*x)/2]]*Cos[(e + f*x)/
2]^4*Sec[e + f*x]*Sqrt[(1 + Sec[e + f*x])^(-1)]) + 8*c^2*ArcTan[Tan[(e + f
*x)/2]/Sqrt[(1 + Sec[e + f*x])^(-1)]]*Cos[(e + f*x)/2]^4*Sec[e + f*x]*Sqrt
[(1 + Sec[e + f*x])^(-1)] - ((c - d)^2*Sin[e + f*x])/2)/(a*f*(1 + Cos[e +
f*x])*Sqrt[a*(1 + Sec[e + f*x])])
```

3.173.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^2}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^2}{a^2(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^2}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{\tan(e+fx)\int\left(\frac{\cos(e+fx)c^2}{\sqrt{a-a\sec(e+fx)}}+\frac{d^2-c^2}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^2}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}\right)d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(\frac{\sqrt{2}(c^2-d^2)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{(c-d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}}+\frac{(c\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((((-2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] + ((c - d)^
2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*Sqrt[a])
 + (Sqrt[2]*(c^2 - d^2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])
])/Sqrt[a] + ((c - d)^2*Sqrt[a - a*Sec[e + f*x]])/(2*a*(1 + Sec[e + f*x]))
)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.173.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.173. $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.173.4 Maple [A] (warning: unable to verify)

Time = 3.46 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.34

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-4c^2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)-\sqrt{(1-c\cdots}{\cdots}$
parts	$\frac{c^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{(1-\cos(f\cdots}{4f\,a^2}$

input
```
int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2)*(-4*c^2*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*
csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-((1-cos(f*x+e))^2*csc(f*x+
e)^2-1)^(1/2)*c^2*(-cot(f*x+e)+csc(f*x+e))+2*((1-cos(f*x+e))^2*csc(f*x+e)^
2-1)^(1/2)*c*d*(-cot(f*x+e)+csc(f*x+e))-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^
(1/2)*d^2*(-cot(f*x+e)+csc(f*x+e))+5*c^2*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(
f*x+e))^2*csc(f*x+e)^2-1)^(1/2))-2*c*d*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(1/2))-3*d^2*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^(1/2)))
```

3.173.5 Fricas [A] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 620, normalized size of antiderivative = 2.14

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{4\left(c^2-2\,cd+d^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\sqrt{2}\left(\left(5\,c^2-2\,c\right.\right.}{}\right.$$

$$-\frac{2\left(c^2-2\,cd+d^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\sqrt{2}\left(\left(5\,c^2-2\,cd-3\,d^2\right)\cos\left(fx+e\right)^2+5\,c^2-\right.}{}$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(4*(c^2 - 2*c*d + d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f
*x + e)*sin(f*x + e) - sqrt(2)*((5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e)^2 + 5
*c^2 - 2*c*d - 3*d^2 + 2*(5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e))*sqrt(-a)*lo
g((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2
 + 2*cos(f*x + e) + 1)) + 8*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2
)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e
) + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/4*(2*(c
^2 - 2*c*d + d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) - sqrt(2)*((5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e)^2 + 5*c^2 - 2*c*
d - 3*d^2 + 2*(5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e))*sqrt(a)*arctan(sqrt(2)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e
))) + 8*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))
/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

3.173.6 Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(c+d\sec(e+fx))^2}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**2/(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.173.7 Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e)+c)^2}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)^2/(a*sec(f*x + e) + a)^(3/2), x)
```

3.173.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2), x)
```

3.174 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.174.1 Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}-\frac{(5c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f-1/4*(5*c-d
)*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f*
2^(1/2)-1/2*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

3.174.2 Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.81

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(-\left((5c-d)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\cos\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

input
```
Integrate[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(Cos[(e + f*x)/2]*Sec[e + f*x]*(-((5*c - d)*ArcSin[Tan[(e + f*x)/2]]*Cos[(
e + f*x)/2]*Sqrt[Sec[e + f*x]]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[1 + Sec[
e + f*x]]) + Sqrt[2]*(4*c*ArcTan[Tan[(e + f*x)/2]/Sqrt[(1 + Sec[e + f*x])^
(-1)]]*Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]]*Sqrt[(1 + Sec[e + f*x])^(-1)]*S
qrt[1 + Sec[e + f*x]] - (c - d)*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sin[(e + f*x
)/2]))/(f*Sqrt[Sec[(e + f*x)/2]^2]*(a*(1 + Sec[e + f*x]))^(3/2))
```

3.174.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4410, 27, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+d\sec(e+fx)}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4410}$$

$$-\frac{\int -\frac{4ac-a(c-d)\sec(e+fx)}{2\sqrt{\sec(e+fx)a+a}}dx}{2a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4ac-a(c-d)\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4ac-a(c-d)\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{4408}$$

$$\frac{4c\int \sqrt{\sec(e+fx)a+a}dx-a(5c-d)\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4c\int\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-a(5c-d)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 4261

$$\frac{-\frac{8ac\int\frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}-a(5c-d)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 216

$$\frac{\frac{8\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-a(5c-d)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 4282

$$\frac{\frac{2a(5c-d)\int\frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+2a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}+\frac{8\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 216

$$\frac{\frac{8\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{\sqrt{2}\sqrt{a}(5c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{f}}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

input
```
Int[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
((8*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (Sqrt[2]*Sqrt[a]*(5*c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[e + f*x]])])/f)/(4*a^2) - ((c - d)*Tan[e + f*x])/(2*f*(a + a*Sec[e
 + f*x])^(3/2))
```

3.174. $\int\frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}dx$

3.174.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4410
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&
EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

3.174.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(106) = 212$.

Time = 3.02 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.28

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-4c\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)-c\sqrt{(1-c\cdots}}{\cdots}$
parts	$\frac{c\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{(1-\cos(fx\cdots}}{4f\,a^2}$

input
```
int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2)*(-4*c*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-c*((1-cos(f*x+e))^2*csc(f*x+
e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e))+d*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
^(1/2)*(-cot(f*x+e)+csc(f*x+e))+5*c*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e
))^2*csc(f*x+e)^2-1)^(1/2))-d*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*c
sc(f*x+e)^2-1)^(1/2)))
```

3.174.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(106) = 212$.

Time = 1.88 (sec) , antiderivative size = 548, normalized size of antiderivative = 4.31

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{4\,(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-\sqrt{2}\big((5\,c-d)\cos(fx+\cdots}{\cdots}\right.$$

$$-\frac{2\,(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-\sqrt{2}\big((5\,c-d)\cos(fx+e)^2+2\,(5\,c-d)\cos(fx+e)+\cdots}{4\left(a^2f\cos(fx+\cdots\right.}$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(4*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) - sqrt(2)*((5*c - d)*cos(f*x + e)^2 + 2*(5*c - d)*cos(f*x + e) +
5*c - d)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e)
- a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c*cos(f*x + e)^2 + 2*c*co
s(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a
)/(cos(f*x + e) + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*
f), -1/4*(2*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) - sqrt(2)*((5*c - d)*cos(f*x + e)^2 + 2*(5*c - d)*cos(f*x + e)
 + 5*c - d)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 8*(c*cos(f*x + e)^2 + 2*c*cos(f*x
+ e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
 e)/(sqrt(a)*sin(f*x + e))))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e)
+ a^2*f)]
```

3.174.6 Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{c+d\sec{(e+fx)}}{(a(\sec{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*sec(e + f*x))/(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.174.7 Maxima [F]

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{d\sec{(fx+e)}+c}{(a\sec{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)/(a*sec(f*x + e) + a)^(3/2), x)
```

3.174.8 Giac [F(-2)]

Exception generated.

$$\int \frac{c + d\sec(e + fx)}{(a + a\sec(e + fx))^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c + d\sec(e + fx)}{(a + a\sec(e + fx))^{3/2}}\, dx = \int \frac{c + \frac{d}{\cos(e+f\,x)}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\, dx$$

input
```
int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2), x)
```

3.175 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx$

3.175.1 Optimal result

Integrand size = 27, antiderivative size = 394

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx &= \\ &-\frac{\tan(e+fx)}{2a(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ &+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &-\frac{\sqrt{2}(c-2d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &-\frac{2d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c(c-d)^2\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

output
```
-1/2*tan(f*x+e)/a/(c-d)/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh(
(a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c/f/a^(1/2)/(a-a*sec(f*x+e))^(1
/2)/(a+a*sec(f*x+e))^(1/2)-1/4*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*tan(f*x+e)/(c-d)/f*2^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*se
c(f*x+e))^(1/2)-(c-2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*2^(1/2)*tan(f*x+e)/(c-d)^2/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x
+e))^(1/2)-2*d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^
(1/2))*tan(f*x+e)/c/(c-d)^2/f/a^(1/2)/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(
a+a*sec(f*x+e))^(1/2)
```

3.175.2 Mathematica [A] (warning: unable to verify)

Time = 6.10 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \frac{\cos^2\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\sec^{\frac{5}{2}}(e+fx)\left(\left(\frac{-c(5}{}\right.\right.}{}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])),x]
```

output
```
(Cos[(e + f*x)/2]^2*(d + c*Cos[e + f*x])*Sec[e + f*x]^(5/2)*(((-(c*(5*c -
9*d)*Sqrt[c + d]*ArcSin[Tan[(e + f*x)/2]]) + 4*Sqrt[2]*((c - d)^2*Sqrt[c +
 d]*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] - d^(5/
2)*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1 + C
os[e + f*x])])]))*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[1 + Sec[e + f
*x]])/(c*Sqrt[c + d]*Sqrt[Sec[(e + f*x)/2]^2]) + (c - d)*Sqrt[Sec[e + f*x]
]*(-Sin[e + f*x] + Tan[(e + f*x)/2])))/((c - d)^2*f*(a*(1 + Sec[e + f*x]))
^(3/2)*(c + d*Sec[e + f*x]))
```

3.175.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.69, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a^2(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 198}$$

$$-\frac{\tan(e+fx)\int \left(-\frac{d^3}{c(c-d)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{\cos(e+fx)}{c\sqrt{a-a\sec(e+fx)}}+\frac{2d-c}{(c-d)^2(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}-\frac{}{(c-d)(}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\tan(e+fx)\left(\frac{2d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c(c-d)^2\sqrt{c+d}}+\frac{\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}(c-d)}+\frac{\sqrt{2}(c-2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}(c-d)^2}-\frac{2\text{arc}}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])),x]
```

output
```
-((((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c) + (Sqrt[2]*
(c - 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*(c
 - d)^2) + ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*
Sqrt[a]*(c - d)) + (2*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(
Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c*(c - d)^2*Sqrt[c + d]) + Sqrt[a - a*Sec[
e + f*x]]/(2*a*(c - d)*(1 + Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.175.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.175.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1418 vs. $2(334) = 668$.

Time = 16.33 (sec) , antiderivative size = 1419, normalized size of antiderivative = 3.60

method	result	size
default	Expression too large to display	1419

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/(d/(c-d))^(1/2)/(c-d)^2/c/((c+d)*(c-d))^(1/2)/a^2*(((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(3/2)*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c^2*(-cot(f*x+
e)+csc(f*x+e))-2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*((c+d)*(c-d))^(1/
2)*(d/(c-d))^(1/2)*c*d*(-cot(f*x+e)+csc(f*x+e))+((1-cos(f*x+e))^2*csc(f*x+
e)^2-1)^(3/2)*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*d^2*(-cot(f*x+e)+csc(f*x
+e))-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*((c+d)*(c-d))^(1/2)*(d/(c-d))
^(1/2)*c^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*((1-cos(f*x+e))^2*csc(f*x+e)^2-
1)^(1/2)*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c*d*(1-cos(f*x+e))^3*csc(f*x+
e)^3-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*((c+d)*(c-d))^(1/2)*(d/(c-d))
^(1/2)*d^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+5*((1-cos(f*x+e))^2*csc(f*x+e)^2-
1)^(1/2)*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c^2*(-cot(f*x+e)+csc(f*x+e))-
6*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1
/2)*c*d*(-cot(f*x+e)+csc(f*x+e))+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(
(c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*d^2*(-cot(f*x+e)+csc(f*x+e))+16*((c+d)*
(c-d))^(1/2)*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/
2)*(-cot(f*x+e)+csc(f*x+e)))*(d/(c-d))^(1/2)*c^2-32*((c+d)*(c-d))^(1/2)*2^
(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)
+csc(f*x+e)))*(d/(c-d))^(1/2)*c*d+16*((c+d)*(c-d))^(1/2)*2^(1/2)*arctanh(2
^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(
d/(c-d))^(1/2)*d^2-20*((c+d)*(c-d))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+((1-...
```

3.175.5 Fricas [A] (verification not implemented)

Time = 215.92 (sec) , antiderivative size = 2033, normalized size of antiderivative = 5.16

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[-1/8*(4*(c^2 - c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*
sin(f*x + e) - sqrt(2)*((5*c^2 - 9*c*d)*cos(f*x + e)^2 + 5*c^2 - 9*c*d + 2
*(5*c^2 - 9*c*d)*cos(f*x + e))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)
^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 8*(a*d
^2*cos(f*x + e)^2 + 2*a*d^2*cos(f*x + e) + a*d^2)*sqrt(-d/(a*c + a*d))*log
((2*(c + d)*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) + (c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e)
 - d)/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + 8*((c^2 - 2*c*d + d
^2)*cos(f*x + e)^2 + c^2 - 2*c*d + d^2 + 2*(c^2 - 2*c*d + d^2)*cos(f*x + e
))*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x +
e) + 1)))/((a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f*cos(f*x + e)^2 + 2*(a^2*c
^3 - 2*a^2*c^2*d + a^2*c*d^2)*f*cos(f*x + e) + (a^2*c^3 - 2*a^2*c^2*d + a^
2*c*d^2)*f), -1/8*(4*(c^2 - c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) - sqrt(2)*((5*c^2 - 9*c*d)*cos(f*x + e)^2 + 5*c^2
 - 9*c*d + 2*(5*c^2 - 9*c*d)*cos(f*x + e))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*
cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) +
1)) - 16*(a*d^2*cos(f*x + e)^2 + 2*a*d^2*cos(f*x + e) + a*d^2)*sqrt(d/(...
```

3.175.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx=\int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))), x)
```

3.175.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c)), x)
```

3.175.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))), x)
```

3.176 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx$

3.176.1 Optimal result

Integrand size = 27, antiderivative size = 560

$$\begin{aligned}
\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx =& -\frac{\tan(e+fx)}{2a(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&-\frac{\sqrt{2}(c-3d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&-\frac{d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c(c-d)^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&-\frac{2(3c-d)d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^2(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&-\frac{d^3\tan(e+fx)}{ac(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}
\end{aligned}$$

output
```
-1/2*tan(f*x+e)/a/(c-d)^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-d^3*tan(
f*x+e)/a/c/(c-d)^2/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arcta
nh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/a^(1/2)/(a-a*sec(f*x+e
))^(1/2)/(a+a*sec(f*x+e))^(1/2)-d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(
1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/(c-d)^2/(c+d)^(3/2)/f/a^(1/2)/(a-a*
sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*arctanh(1/2*(a-a*sec(f*x+e))^
(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/(c-d)^2/f*2^(1/2)/a^(1/2)/(a-a*sec(f*x+e
))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(c-3*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)
*2^(1/2)/a^(1/2))*2^(1/2)*tan(f*x+e)/(c-d)^3/f/a^(1/2)/(a-a*sec(f*x+e))^(1
/2)/(a+a*sec(f*x+e))^(1/2)-2*(3*c-d)*d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+
e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/(c-d)^3/f/a^(1/2)/(c+d)^(1/2
)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.176.2 Mathematica [A] (warning: unable to verify)

Time = 11.85 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \frac{\left((-c-d)^{3/2}\left(-c^2(5c-13d)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+\cdots\right.\right.}{\cdots} + \frac{\cos^3\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))^2\sec^4(e+fx)\left(-\frac{2(c^3+c^2d+2d^3)\sin\left(\frac{1}{2}(e+fx)\right)}{c^2(-c+d)^2(c+d)}+\frac{4d^4\sin\left(\frac{1}{2}(e+fx)\right)}{c^2(-c+d)^2(c+d)(d+c\cos(e+fx))}\cdots\right)}{f(a(1+\sec(e+fx)))^{3/2}(c+d\sec(e+fx))^2}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2),x]
```

output
```
(((-c - d)^(3/2)*(-(c^2*(5*c - 13*d)*ArcSin[Tan[(e + f*x)/2]]) + 4*Sqrt[2]
*(c - d)^3*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])
 + 2*Sqrt[2]*d^(5/2)*(-7*c^2 - 3*c*d + 2*d^2)*ArcTanh[(Sqrt[d]*Tan[(e + f*
x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])])*(d + c*Cos[e
 + f*x])^2*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sec[e + f*x]^(7/2)*Sqrt[C
os[(e + f*x)/2]^2*Sec[e + f*x]])/(c^2*(-c - d)^(3/2)*(c - d)^3*f*(Sec[(e +
 f*x)/2]^2)^(3/2)*(a*(1 + Sec[e + f*x]))^(3/2)*(c + d*Sec[e + f*x])^2) + (
Cos[(e + f*x)/2]^3*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^4*((-2*(c^3 + c^2*d
 + 2*d^3)*Sin[(e + f*x)/2])/(c^2*(-c + d)^2*(c + d)) + (4*d^4*Sin[(e + f*x
)/2])/(c^2*(-c + d)^2*(c + d)*(d + c*Cos[e + f*x])) + (Sec[(e + f*x)/2]*Ta
n[(e + f*x)/2])/(-c + d)^2))/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(c + d*Sec[e
+ f*x])^2)
```

3.176.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a^2(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 198}$$

$$-\frac{\tan(e+fx)\int \left(-\frac{(3c-d)d^3}{c^2(c-d)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}-\frac{d^3}{c(c-d)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}+\frac{\cos(e+fx)}{c^2\sqrt{a-a\sec(e+fx)}}+ \cdots}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx \cdots}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\tan(e+fx)\left(\frac{2d^{5/2}(3c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2(c-d)^3\sqrt{c+d}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^2}+\frac{d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c(c-d)^2(c+d)^{3/2}}+ \cdots}{f\sqrt{a-a\sec(e+fx) \cdots}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2),x]
```

output
```
-((((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^2) + (Sqrt[2
]*(c - 3*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*
(c - d)^3) + ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2
]*Sqrt[a]*(c - d)^2) + (d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])
/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c*(c - d)^2*(c + d)^(3/2)) + (2*(3*c - d
)*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])
])/(Sqrt[a]*c^2*(c - d)^3*Sqrt[c + d]) + Sqrt[a - a*Sec[e + f*x]]/(2*a*(c
- d)^2*(1 + Sec[e + f*x])) + (d^3*Sqrt[a - a*Sec[e + f*x]])/(a*c*(c - d)^2
*(c + d)*(c + d*Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]]))
```

3.176.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.176.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 69594 vs. 2(480) = 960.

Time = 17.96 (sec) , antiderivative size = 69595, normalized size of antiderivative = 124.28

method	result	size
default	Expression too large to display	69595

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.176.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas
")
```

output
```
Timed out
```

3.176.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{3}{2}}(c+d\sec(e+fx))^2}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**2), x)
```

3.176.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)^2}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima
")
```

output
```
integrate(1/((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c)^2), x)
```

3.176.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2), x)
```

3.177 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx$

3.177.1 Optimal result

Integrand size = 27, antiderivative size = 802

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx =
-\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}
+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
-\frac{\sqrt{2}(c-4d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
-\frac{3d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4\sqrt{a}c(c-d)^2(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
-\frac{(3c-d)d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^2(c-d)^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
-\frac{2d^{5/2}(6c^2-4cd+d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^3(c-d)^4\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
-\frac{d^3\tan(e+fx)}{2ac(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}
-\frac{(3c-d)d^3\tan(e+fx)}{ac^2(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}
-\frac{3d^3\tan(e+fx)}{4ac(c^2-d^2)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
-1/2*tan(f*x+e)/a/(c-d)^3/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-1/2*d^3*
tan(f*x+e)/a/c/(c-d)^2/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-(
3*c-d)*d^3*tan(f*x+e)/a/c^2/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+
e))^(1/2)-3/4*d^3*tan(f*x+e)/a/c/(c^2-d^2)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f
*x+e))^(1/2)+2*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3/f/a^
(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-3/4*d^(5/2)*arctanh(d^
(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/(c-d)^2/(c+
d)^(5/2)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(3*c-d)*d
^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x
+e)/c^2/(c-d)^3/(c+d)^(3/2)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+
e))^(1/2)-1/4*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+
e)/(c-d)^3/f*2^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
-(c-4*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*tan(f
*x+e)/(c-d)^4/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2*d^
(5/2)*(6*c^2-4*c*d+d^2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+
d)^(1/2))*tan(f*x+e)/c^3/(c-d)^4/f/a^(1/2)/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1
/2)/(a+a*sec(f*x+e))^(1/2)
```

3.177.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2632 vs. $2(802) = 1604$.

Time = 18.76 (sec) , antiderivative size = 2632, normalized size of antiderivative = 3.28

$$\int \frac{1}{(a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3),x]
```

output
```
(Cos[(e + f*x)/2]^3*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^5*(-(((-2*c^5 - 4*
c^4*d - 2*c^3*d^2 - 17*c^2*d^3 - 5*c*d^4 + 6*d^5)*Sin[(e + f*x)/2])/(c^3*(
-c + d)^3*(c + d)^2)) - (2*d^5*Sin[(e + f*x)/2])/(c^3*(-c + d)^2*(c + d)*(
d + c*Cos[e + f*x])^2) + (-19*c^2*d^4*Sin[(e + f*x)/2] - 5*c*d^5*Sin[(e +
f*x)/2] + 8*d^6*Sin[(e + f*x)/2])/(c^3*(-c + d)^3*(c + d)^2*(d + c*Cos[e +
 f*x])) - (Sec[(e + f*x)/2]*Tan[(e + f*x)/2])/(-c + d)^3))/(f*(a*(1 + Sec[
e + f*x]))^(3/2)*(c + d*Sec[e + f*x])^3) - ((2*c^3*(5*c - 17*d)*(c + d)^2*
ArcSin[Tan[(e + f*x)/2]] - 8*Sqrt[2]*(c - d)^4*(c + d)^2*ArcTan[Tan[(e + f
*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] - (Sqrt[2]*d^(5/2)*(63*c^4 +
 54*c^3*d - 17*c^2*d^2 - 12*c*d^3 + 8*d^4)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/
2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])])/Sqrt[-c - d])*C
os[(e + f*x)/2]^3*(d + c*Cos[e + f*x])^3*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2
]^2]*((c^3*Sec[(e + f*x)/2])/(2*(-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x])*
Sqrt[Sec[e + f*x]]) - (c^2*d*Sec[(e + f*x)/2])/((-c + d)^3*(c + d)^2*(d +
c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (19*c*d^2*Sec[(e + f*x)/2])/(2*(-c +
 d)^3*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (33*d^3*Sec[(e
+ f*x)/2])/(4*(-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]
) - (3*d^4*Sec[(e + f*x)/2])/(4*c*(-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x]
)*Sqrt[Sec[e + f*x]]) + (d^5*Sec[(e + f*x)/2])/(c^2*(-c + d)^3*(c + d)^2*(
d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (c^3*Sec[(e + f*x)/2]*Sqrt[Se...
```

3.177.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 579, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a^2(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 198$$

$$-\frac{\tan(e+fx)\int \left(-\frac{(6c^2-4dc+d^2)d^3}{c^3(c-d)^4\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}-\frac{(3c-d)d^3}{c^2(c-d)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}-\frac{d^3}{c(c-d)^2\sqrt{a-a\sec(e+f}\right.}{f\sqrt{a-a\sec(e+}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\tan(e+fx)\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^3}+\frac{d^{5/2}(3c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2(c-d)^3(c+d)^{3/2}}+\frac{2d^{5/2}(6c^2-4cd+d^2)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-}}{\sqrt{}}\right.}{\sqrt{a}c^3(c-d)^4\sqrt{c+d}}\right.}{}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3),x]
```

output
```
-((((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^3) + (Sqrt[2
]*(c - 4*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*
(c - d)^4) + ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2
]*Sqrt[a]*(c - d)^3) + (3*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]
])/(Sqrt[a]*Sqrt[c + d])])/(4*Sqrt[a]*c*(c - d)^2*(c + d)^(5/2)) + ((3*c -
 d)*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d
])])/(Sqrt[a]*c^2*(c - d)^3*(c + d)^(3/2)) + (2*d^(5/2)*(6*c^2 - 4*c*d + d
^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sq
rt[a]*c^3*(c - d)^4*Sqrt[c + d]) + Sqrt[a - a*Sec[e + f*x]]/(2*a*(c - d)^3
*(1 + Sec[e + f*x])) + (d^3*Sqrt[a - a*Sec[e + f*x]])/(2*a*c*(c - d)^2*(c
+ d)*(c + d*Sec[e + f*x])^2) + ((3*c - d)*d^3*Sqrt[a - a*Sec[e + f*x]])/(a
*c^2*(c - d)^3*(c + d)*(c + d*Sec[e + f*x])) + (3*d^3*Sqrt[a - a*Sec[e + f
*x]])/(4*a*c*(c^2 - d^2)^2*(c + d*Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[a
- a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.177. $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx$

3.177.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.177.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 117746 vs. $2(694) = 1388$.

Time = 21.10 (sec) , antiderivative size = 117747, normalized size of antiderivative = 146.82

method	result	size
default	Expression too large to display	117747

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.177.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas
")
```

output
```
Timed out
```

3.177.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{3}{2}}(c+d\sec(e+fx))^3}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**3), x)
```

3.177.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima
")
```

output
```
Timed out
```

3.177.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = \text{Hanged}$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3),x)
```

output
```
\text{Hanged}
```

3.178 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.178.1 Optimal result

Integrand size = 27, antiderivative size = 480

$$\begin{aligned}\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = & -\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\ & -\frac{3(c-d)^3\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ & -\frac{(c-d)^2(c+2d)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ & +\frac{2c^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ & -\frac{3(c-d)^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ & -\frac{(c-d)^2(c+2d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ & -\frac{\sqrt{2}(c^3-d^3)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

output
```
-1/4*(c-d)^3*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-3/16
*(c-d)^3*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-1/2*(c-d)^
2*(c+2*d)*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^3*arc
tanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))
^(1/2)/(a+a*sec(f*x+e))^(1/2)-3/32*(c-d)^3*arctanh(1/2*(a-a*sec(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(
a+a*sec(f*x+e))^(1/2)-1/4*(c-d)^2*(c+2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a
+a*sec(f*x+e))^(1/2)-(c^3-d^3)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*2^(1/2)*tan(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+
e))^(1/2)
```

3.178.2 Mathematica [A] (verified)

Time = 9.68 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.91

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\left(\left(-43c^3+9c^2d+15cd^2+19d^3\right)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+32\sqrt{2}c^3\arctan\left(\right.\right.}{4f(d+c\cos(e+fx))^3\sqrt{\sec^2\left(\frac{1}{2}(e+\right.}}$$
$$+\frac{\cos^5\left(\frac{1}{2}(e+fx)\right)(c+d\sec(e+fx))^3\left(-\frac{3}{2}(-c+d)^2(5c+3d)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{1}{2}\sec^4\left(\frac{1}{2}(e+fx)\right)\left(-c^3\sin\right.\right.}{}$$

input
```
Integrate[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(((-43*c^3 + 9*c^2*d + 15*c*d^2 + 19*d^3)*ArcSin[Tan[(e + f*x)/2]] + 32*Sq
rt[2]*c^3*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])*
Cos[(e + f*x)/2]^4*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[1 + Sec[e +
f*x]]*(c + d*Sec[e + f*x])^3)/(4*f*(d + c*Cos[e + f*x])^3*Sqrt[Sec[(e + f*
x)/2]^2]*Sqrt[Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)) + (Cos[(e + f*x)
/2]^5*(c + d*Sec[e + f*x])^3*((-3*(-c + d)^2*(5*c + 3*d)*Sin[(e + f*x)/2])
/2 + (Sec[(e + f*x)/2]^4*(-(c^3*Sin[(e + f*x)/2]) + 3*c^2*d*Sin[(e + f*x)/
2] - 3*c*d^2*Sin[(e + f*x)/2] + d^3*Sin[(e + f*x)/2]))/2 + (Sec[(e + f*x)/
2]^2*(19*c^3*Sin[(e + f*x)/2] - 33*c^2*d*Sin[(e + f*x)/2] + 9*c*d^2*Sin[(e
 + f*x)/2] + 5*d^3*Sin[(e + f*x)/2]))/4))/(f*(d + c*Cos[e + f*x])^3*(a*(1
+ Sec[e + f*x]))^(5/2))
```

3.178.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^3}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^3}{a^3(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^3}{(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 198}$$

$$-\frac{\tan(e+fx)\int \left(\frac{\cos(e+fx)c^3}{\sqrt{a-a\sec(e+fx)}}+\frac{d^3-c^3}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^2(c+2d)}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^3}{(\sec(e+fx)+1)^3\sqrt{a-a\sec\ldots}}\right.}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\tan(e+fx)\left(\frac{\sqrt{2}(c^3-d^3)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{3(c-d)^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\sqrt{a}}+\ldots\right.}{af\sqrt{a-a\sec(e+f\ldots}}$$

input
```
Int[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((((-2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] + (3*(c - d
)^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*Sqrt[
a]) + ((c - d)^2*(c + 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[
a])])/(2*Sqrt[2]*Sqrt[a]) + (Sqrt[2]*(c^3 - d^3)*ArcTanh[Sqrt[a - a*Sec[e
+ f*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a] + ((c - d)^3*Sqrt[a - a*Sec[e + f*x]])
/(4*a*(1 + Sec[e + f*x])^2) + (3*(c - d)^3*Sqrt[a - a*Sec[e + f*x]])/(16*a
*(1 + Sec[e + f*x])) + ((c - d)^2*(c + 2*d)*Sqrt[a - a*Sec[e + f*x]])/(2*a
*(1 + Sec[e + f*x])))*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]))
```

3.178.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.178. $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.178.4 Maple [A] (warning: unable to verify)

Time = 4.86 (sec) , antiderivative size = 688, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,c^3(1-\cos(fx+e))^3\csc(f\right.}{}$
parts	Expression too large to display

input
```
int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/a^3/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))
^2*csc(f*x+e)^2-1)^(1/2)*(2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*c^3*(1
-cos(f*x+e))^3*csc(f*x+e)^3-6*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*c^2*
d*(1-cos(f*x+e))^3*csc(f*x+e)^3+6*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*
c*d^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1
/2)*d^3*(1-cos(f*x+e))^3*csc(f*x+e)^3-32*c^3*2^(1/2)*arctanh(2^(1/2)/((1-c
os(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-13*((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(1/2)*c^3*(-cot(f*x+e)+csc(f*x+e))+15*((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^(1/2)*c^2*d*(-cot(f*x+e)+csc(f*x+e))+9*((1-cos(f*x+e
))^2*csc(f*x+e)^2-1)^(1/2)*c*d^2*(-cot(f*x+e)+csc(f*x+e))-11*((1-cos(f*x+e
))^2*csc(f*x+e)^2-1)^(1/2)*d^3*(-cot(f*x+e)+csc(f*x+e))+43*c^3*ln(csc(f*x+
e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))-9*c^2*d*ln(csc(f*x+
e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))-15*c*d^2*ln(csc(f*x
+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))-19*d^3*ln(csc(f*x+
e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)))
```

3.178.5 Fricas [A] (verification not implemented)

Time = 30.82 (sec) , antiderivative size = 880, normalized size of antiderivative = 1.83

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[\frac{\sqrt{2}\left((43c^3-9c^2d-15cd^2-19d^3)\cos(fx+e)^3+43c^3-9c^2d-15cd^2-\right.}{}\right.$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(sqrt(2)*((43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)^3 + 43
*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d
^3)*cos(f*x + e)^2 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)
)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(c
os(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 64*(c^3*cos(f*x + e)^3 + 3*c^3*cos(
f*x + e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 +
2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(3*(5*c^3 - 7*c^2*d - c*d
^2 + 3*d^3)*cos(f*x + e)^2 + (11*c^3 - 9*c^2*d - 15*c*d^2 + 13*d^3)*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f
*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(
sqrt(2)*((43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)^3 + 43*c^3 -
9*c^2*d - 15*c*d^2 - 19*d^3 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos
(f*x + e)^2 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e))*sqrt(
a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sq
rt(a)*sin(f*x + e))) - 64*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c
^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(3*(5*c^3 - 7*c^2*d - c*d^2 +
 3*d^3)*cos(f*x + e)^2 + (11*c^3 - 9*c^2*d - 15*c*d^2 + 13*d^3)*cos(f*x...
```

3.178.6 Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{(c+d\sec{(e+fx)})^3}{(a(\sec{(e+fx)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**3/(a*(sec(e + f*x) + 1))**(5/2), x)
```

3.178.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.178.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2), x)
```

3.179 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.179.1 Optimal result

Integrand size = 27, antiderivative size = 468

$$\begin{aligned}\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx &= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\ &- \frac{3(c-d)^2\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ &- \frac{(c^2-d^2)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{2c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{\sqrt{2}c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{3(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{(c^2-d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

output
```
-1/4*(c-d)^2*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-3/16
*(c-d)^2*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-1/2*(c^2-d
^2)*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^2*arctanh((
a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)
/(a+a*sec(f*x+e))^(1/2)-3/32*(c-d)^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^
(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*se
c(f*x+e))^(1/2)-1/4*(c^2-d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*tan(f*x+e)/a^(3/2)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e
))^(1/2)-c^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*t
an(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.179.2 Mathematica [A] (verified)

Time = 6.25 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.56

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}(c+d\sec(e+fx))^2\left(\frac{\left((-43c^2+6cd+5d^2)\arcsin(\tan\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(Cos[(e + f*x)/2]^4*Sqrt[Sec[e + f*x]]*(c + d*Sec[e + f*x])^2*((((-43*c^2
+ 6*c*d + 5*d^2)*ArcSin[Tan[(e + f*x)/2]] + 32*Sqrt[2]*c^2*ArcTan[Tan[(e +
 f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])*Sqrt[Cos[e + f*x]/(1 + Co
s[e + f*x])]*Sqrt[1 + Sec[e + f*x]])/Sqrt[Sec[(e + f*x)/2]^2] + ((c - d)*(
11*c + 5*d + (15*c + d)*Cos[e + f*x])*Sec[(e + f*x)/2]^3*Sqrt[Sec[e + f*x]
]*(Sin[(e + f*x)/2] - Sin[(3*(e + f*x))/2]))/4))/(4*f*(d + c*Cos[e + f*x])
^2*(a*(1 + Sec[e + f*x]))^(5/2))
```

3.179.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^2}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^2}{a^3(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^2}{(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{\tan(e+fx)\int \left(\frac{\cos(e+fx)c^2}{\sqrt{a-a\sec(e+fx)}}-\frac{c^2}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}+\frac{d^2-c^2}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^2}{(\sec(e+fx)+1)^3\sqrt{a-a\,\text{se}}}\right.}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(\frac{(c^2-d^2)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}}-\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{2}c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}+\frac{3(c-d}{}\right.}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\,\text{sec}}}$$

input
```
Int[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((((-2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] + (Sqrt[2]*
c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a] + (3*(c -
 d)^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*Sqr
t[a]) + ((c^2 - d^2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/
(2*Sqrt[2]*Sqrt[a]) + ((c - d)^2*Sqrt[a - a*Sec[e + f*x]])/(4*a*(1 + Sec[e
 + f*x])^2) + (3*(c - d)^2*Sqrt[a - a*Sec[e + f*x]])/(16*a*(1 + Sec[e + f*
x])) + ((c^2 - d^2)*Sqrt[a - a*Sec[e + f*x]])/(2*a*(1 + Sec[e + f*x])))*Ta
n[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.179.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.179.4 Maple [A] (warning: unable to verify)

Time = 4.48 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.12

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}c^2(-\cot(fx+e)+\csc(f\right.}{}$
parts	$\frac{c^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-2(1-\cos(fx+e))^3\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\csc(f\right.}{}$

input
```
int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/a^3/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))
^2*csc(f*x+e)^2-1)^(1/2)*(2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*c^2*(-
cot(f*x+e)+csc(f*x+e))-4*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*c*d*(-cot
(f*x+e)+csc(f*x+e))+2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*d^2*(-cot(f*
x+e)+csc(f*x+e))-32*c^2*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+
e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-11*((1-cos(f*x+e))^2*csc(f*x+e)^2-
1)^(1/2)*c^2*(-cot(f*x+e)+csc(f*x+e))+6*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^
(1/2)*c*d*(-cot(f*x+e)+csc(f*x+e))+5*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/
2)*d^2*(-cot(f*x+e)+csc(f*x+e))+43*c^2*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(1/2))-6*c*d*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^(1/2))-5*d^2*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e)
)^2*csc(f*x+e)^2-1)^(1/2)))
```

3.179.5 Fricas [A] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.67

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[\frac{\sqrt{2}\left((43\,c^2-6\,cd-5\,d^2)\cos(fx+e)^3+3\,(43\,c^2-6\,cd-5\,d^2)\cos(fx+e)\right.}{}\right.$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(sqrt(2)*((43*c^2 - 6*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(43*c^2 - 6*c*
d - 5*d^2)*cos(f*x + e)^2 + 43*c^2 - 6*c*d - 5*d^2 + 3*(43*c^2 - 6*c*d - 5
*d^2)*cos(f*x + e))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*co
s(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 64*(c^2*cos(f*x +
 e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a
*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f
*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*((15*c^
2 - 14*c*d - d^2)*cos(f*x + e)^2 + (11*c^2 - 6*c*d - 5*d^2)*cos(f*x + e))*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^
3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*
((43*c^2 - 6*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(43*c^2 - 6*c*d - 5*d^2)*cos(
f*x + e)^2 + 43*c^2 - 6*c*d - 5*d^2 + 3*(43*c^2 - 6*c*d - 5*d^2)*cos(f*x +
 e))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)/(sqrt(a)*sin(f*x + e))) - 64*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x +
e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*((15*c^2 - 14*c*d -
 d^2)*cos(f*x + e)^2 + (11*c^2 - 6*c*d - 5*d^2)*cos(f*x + e))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*
cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

3.179.6 Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{(c+d\sec{(e+fx)})^2}{(a(\sec{(e+fx)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**2/(a*(sec(e + f*x) + 1))**(5/2), x)
```

3.179.7 Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e)+c)^2}{(a\sec(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)^2/(a*sec(f*x + e) + a)^(5/2), x)
```

3.179.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2), x)
```

3.180 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$

3.180.1 Optimal result

Integrand size $=25$, antiderivative size $=164$

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f}-\frac{(43c-3d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}-\frac{(11c-3d)\tan(e+fx)}{16af(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-1/32*(43*c
-3*d)*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2
)/f*2^(1/2)-1/4*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)-1/16*(11*c-3*d)*
tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(3/2)
```

3.180.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 343 vs. $2(164)=328$.

Time $=7.47$ (sec), antiderivative size $=343$, normalized size of antiderivative $=2.09$

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{\left(\left(-43c+3d\right)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+32\sqrt{2}c\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\right)\cos}{4f(d+c\cos(e+fx))\sqrt{\sec^2\left(\frac{1}{2}\right.}}+\frac{\cos^5\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)(c+d\sec(e+fx))\left(\frac{1}{2}(-15c+7d)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{1}{4}\sec^2\left(\frac{1}{2}(e+fx)\right)(19c\right.}{f(d+c\cos(e+fx))(a(}$$

input
```
Integrate[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(((-43*c + 3*d)*ArcSin[Tan[(e + f*x)/2]] + 32*Sqrt[2]*c*ArcTan[Tan[(e + f*
x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])*Cos[(e + f*x)/2]^4*Sqrt[Cos[
e + f*x]/(1 + Cos[e + f*x])]*Sec[e + f*x]^(3/2)*Sqrt[1 + Sec[e + f*x]]*(c
+ d*Sec[e + f*x]))/(4*f*(d + c*Cos[e + f*x])*Sqrt[Sec[(e + f*x)/2]^2]*(a*(
1 + Sec[e + f*x]))^(5/2)) + (Cos[(e + f*x)/2]^5*Sec[e + f*x]^2*(c + d*Sec[
e + f*x])*(((-15*c + 7*d)*Sin[(e + f*x)/2])/2 + (Sec[(e + f*x)/2]^2*(19*c*
Sin[(e + f*x)/2] - 11*d*Sin[(e + f*x)/2]))/4 + (Sec[(e + f*x)/2]^4*(-(c*Si
n[(e + f*x)/2]) + d*Sin[(e + f*x)/2]))/2))/(f*(d + c*Cos[e + f*x])*(a*(1 +
 Sec[e + f*x]))^(5/2))
```

3.180.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4410, 27, 3042, 4410, 27, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+d\sec(e+fx)}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \ 4410$$

$$-\frac{\int -\frac{8ac-3a(c-d)\sec(e+fx)}{2(\sec(e+fx)a+a)^{3/2}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{8ac-3a(c-d)\sec(e+fx)}{(\sec(e+fx)a+a)^{3/2}}dx}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{8ac-3a(c-d)\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 4410}$$

$$-\frac{\frac{\int -\frac{32a^2c-a^2(11c-3d)\sec(e+fx)}{2\sqrt{\sec(e+fx)a+a}}dx}{2a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{32a^2c-a^2(11c-3d)\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{32a^2c-a^2(11c-3d)\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 4408}$$

$$\frac{\frac{32ac\int \sqrt{\sec(e+fx)a+a}dx-a^2(43c-3d)\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{32ac\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-a^2(43c-3d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 4261}$$

$$\frac{\frac{-\left(a^2(43c-3d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx\right)-\frac{64a^2c\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{\frac{64a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-a^2(43c-3d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

3.180. $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$

$$\Big\downarrow \text{4282}$$

$$\frac{\frac{\frac{2a^2(43c-3d)\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+2a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}+\frac{64a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{64a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{\sqrt{2}a^{3/2}(43c-3d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{f}}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

input
```
Int[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-1/4*((c - d)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)) + (((64*a^(3/2)
*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (Sqrt[2]*a
^(3/2)*(43*c - 3*d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[
e + f*x]])])/f)/(4*a^2) - (a*(11*c - 3*d)*Tan[e + f*x])/(2*f*(a + a*Sec[e
+ f*x])^(3/2)))/(8*a^2)
```

3.180.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4410
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&
EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

3.180.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(139) = 278$.

Time = 2.89 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.29

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\Big(2c\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}(-\cot(fx+e)+\csc(f\cdots}{\cdots}$
parts	$\frac{c\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\Big(-2(1-\cos(fx+e))^3\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\csc(fx\cdots}{\cdots}$

input
```
int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/a^3/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))
^2*csc(f*x+e)^2-1)^(1/2)*(2*c*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(-co
t(f*x+e)+csc(f*x+e))-2*d*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(-cot(f*x
+e)+csc(f*x+e))-32*c*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^
2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-11*c*((1-cos(f*x+e))^2*csc(f*x+e)^2-1
)^(1/2)*(-cot(f*x+e)+csc(f*x+e))+3*d*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/
2)*(-cot(f*x+e)+csc(f*x+e))+43*c*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2))-3*d*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)^(1/2)))
```

3.180.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(139) = 278$.

Time = 3.61 (sec) , antiderivative size = 670, normalized size of antiderivative = 4.09

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[\frac{\sqrt{2}\left((43\,c-3\,d)\cos\left(fx+e\right)^{3}+3\,(43\,c-3\,d)\cos\left(fx+e\right)^{2}+3\,(43\,c-3\,d)\right.}{}\right.$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(sqrt(2)*((43*c - 3*d)*cos(f*x + e)^3 + 3*(43*c - 3*d)*cos(f*x + e)^
2 + 3*(43*c - 3*d)*cos(f*x + e) + 43*c - 3*d)*sqrt(-a)*log((2*sqrt(2)*sqrt
(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3
*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e)
 + 1)) - 64*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e) + c)
*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e)
 + 1)) - 4*((15*c - 7*d)*cos(f*x + e)^2 + (11*c - 3*d)*cos(f*x + e))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3
*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*((43*
c - 3*d)*cos(f*x + e)^3 + 3*(43*c - 3*d)*cos(f*x + e)^2 + 3*(43*c - 3*d)*c
os(f*x + e) + 43*c - 3*d)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 64*(c*cos(f*x + e)^3
 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*((15*c
 - 7*d)*cos(f*x + e)^2 + (11*c - 3*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x +
e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

3.180.6 Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{c+d\sec{(e+fx)}}{(a(\sec{(e+fx)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Integral((c + d*sec(e + f*x))/(a*(sec(e + f*x) + 1))**(5/2), x)
```

3.180.7 Maxima [F]

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{d\sec{(fx+e)}+c}{(a\sec{(fx+e)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)/(a*sec(f*x + e) + a)^(5/2), x)
```

3.180.8 Giac [F(-2)]

Exception generated.

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{c+\frac{d}{\cos(e+f\,x)}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2), x)
```

3.181 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx$

3.181.1 Optimal result

Integrand size = 27, antiderivative size = 592

$$\begin{aligned}
&\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \\
&-\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\
&-\frac{(c-2d)\tan(e+fx)}{2a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&-\frac{3\tan(e+fx)}{16a^2(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&-\frac{(c-2d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&-\frac{\sqrt{2}(c^2-3cd+3d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&+\frac{2d^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

output
```
-1/4*tan(f*x+e)/a^2/(c-d)/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/2*(c
-2*d)*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-3/16*
tan(f*x+e)/a^2/(c-d)/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh((a-
a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/c/f/(a-a*sec(f*x+e))^(1/2)
/(a+a*sec(f*x+e))^(1/2)-1/4*(c-2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/(c-d)^2/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/
(a+a*sec(f*x+e))^(1/2)-3/32*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*tan(f*x+e)/a^(3/2)/(c-d)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f
*x+e))^(1/2)-(c^2-3*c*d+3*d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*2^(1/2)*tan(f*x+e)/a^(3/2)/(c-d)^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*
sec(f*x+e))^(1/2)+2*d^(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)
/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c/(c-d)^3/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.181.2 Mathematica [A] (warning: unable to verify)

Time = 11.41 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \frac{\cos^4\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\sec^{\frac{7}{2}}(e+fx)\left(-\frac{4\left(\sqrt{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])),x]
```

output
```
(Cos[(e + f*x)/2]^4*(d + c*Cos[e + f*x])*Sec[e + f*x]^(7/2)*((-4*(Sqrt[-c
- d]*(c*(43*c^2 - 126*c*d + 115*d^2)*ArcSin[Tan[(e + f*x)/2]] - 32*Sqrt[2]
*(c - d)^3*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])
 + 32*Sqrt[2]*d^(7/2)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqr
t[Cos[e + f*x]/(1 + Cos[e + f*x])]])])*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])
]*Sqrt[1 + Sec[e + f*x]])/(c*Sqrt[-c - d]*Sqrt[Sec[(e + f*x)/2]^2]) + (c -
 d)*(11*c - 19*d + (15*c - 23*d)*Cos[e + f*x])*Sec[(e + f*x)/2]^3*Sqrt[Sec
[e + f*x]]*(Sin[(e + f*x)/2] - Sin[(3*(e + f*x))/2])))/(16*(c - d)^3*f*(a*
(1 + Sec[e + f*x]))^(5/2)*(c + d*Sec[e + f*x]))
```

3.181.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a^3(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 198}$$

$$-\frac{\tan(e+fx)\int \left(\frac{d^4}{c(c-d)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{\cos(e+fx)}{c\sqrt{a-a\sec(e+fx)}}+\frac{-c^2+3dc-3d^2}{(c-d)^3(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}+\frac{\cdots}{(c-d)^2(\mathrm{se}\cdots}\right.}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+\cdots}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\tan(e+fx)\left(\frac{\sqrt{2}(c^2-3cd+3d^2)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}(c-d)^3}-\frac{2d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c(c-d)^3\sqrt{c+d}}+\frac{3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\sqrt{a}(c-d)}\cdots\right.}{af\sqrt{a-a\,\mathrm{se}\cdots}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])),x]
```

output
```
-((((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c) + ((c - 2*d
)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*Sqrt[a]*
(c - d)^2) + (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(16*S
qrt[2]*Sqrt[a]*(c - d)) + (Sqrt[2]*(c^2 - 3*c*d + 3*d^2)*ArcTanh[Sqrt[a -
a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*(c - d)^3) - (2*d^(7/2)*ArcTa
nh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c*(
c - d)^3*Sqrt[c + d]) + Sqrt[a - a*Sec[e + f*x]]/(4*a*(c - d)*(1 + Sec[e +
 f*x])^2) + ((c - 2*d)*Sqrt[a - a*Sec[e + f*x]])/(2*a*(c - d)^2*(1 + Sec[e
 + f*x])) + (3*Sqrt[a - a*Sec[e + f*x]])/(16*a*(c - d)*(1 + Sec[e + f*x]))
)*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.181.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.181.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2344 vs. $2(509) = 1018$.

Time = 16.76 (sec) , antiderivative size = 2345, normalized size of antiderivative = 3.96

method	result	size
default	Expression too large to display	2345

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/192/f/(d/(c-d))^(1/2)/(c-d)^3/c/((c+d)*(c-d))^(1/2)/a^3*(576*((c+d)*(c-d
))^(1/2)*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(
-cot(f*x+e)+csc(f*x+e)))*(d/(c-d))^(1/2)*c*d^2+12*((c+d)*(c-d))^(1/2)*((1-
cos(f*x+e))^2*csc(f*x+e)^2-1)^(5/2)*(d/(c-d))^(1/2)*c^2*d*(-cot(f*x+e)+csc
(f*x+e))-576*((c+d)*(c-d))^(1/2)*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2
*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(d/(c-d))^(1/2)*c^2*d-96*
2^(1/2)*ln(-2*(-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^
(1/2)*c+2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+
((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))+c-d)/(c*(-cot(f*x+e)+csc(f*x+
e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))*d^4+96*2^(1/2)*ln(-2*
(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)
*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(
1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+
e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))*d^4-12*((c+d)*(c-d))^(1/2)*((1-cos(
f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*c^2*d*(1-cos(f*x+e))^5*csc
(f*x+e)^5+5*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(d
/(c-d))^(1/2)*c^3*(-cot(f*x+e)+csc(f*x+e))-17*((c+d)*(c-d))^(1/2)*((1-cos(
f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(d/(c-d))^(1/2)*d^3*(-cot(f*x+e)+csc(f*x+e
))+87*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d)
)^(1/2)*c^3*(-cot(f*x+e)+csc(f*x+e))+192*((c+d)*(c-d))^(1/2)*2^(1/2)*ar...
```

3.181.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.181.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{5}{2}}(c+d\sec(e+fx))}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))), x)
```

3.181.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c)), x)
```

3.181.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))), x)
```

3.182 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx$

3.182.1 Optimal result

Integrand size = 27, antiderivative size = 756

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-3d)\tan(e+fx)}{2a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{3\tan(e+fx)}{16a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-3d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c^2-4cd+6d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c(c-d)^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2(4c-d)d^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^2(c-d)^4\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d^4\tan(e+fx)}{a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
-1/4*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/2*
(c-3*d)*tan(f*x+e)/a^2/(c-d)^3/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-3/1
6*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+d^4*tan(f
*x+e)/a^2/c/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arct
anh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/c^2/f/(a-a*sec(f*x+
e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+d^(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^
(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c/(c-d)^3/(c+d)^(3/2)/f/(a-a
*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*(c-3*d)*arctanh(1/2*(a-a*sec
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/(c-d)^3/f*2^(1/2)/(a-a*
sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-3/32*arctanh(1/2*(a-a*sec(f*x+e))
^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/(c-d)^2/f*2^(1/2)/(a-a*sec(f*x+
e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(c^2-4*c*d+6*d^2)*arctanh(1/2*(a-a*sec(f*
x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*tan(f*x+e)/a^(3/2)/(c-d)^4/f/(a-a*sec
(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*(4*c-d)*d^(7/2)*arctanh(d^(1/2)*(a
-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c^2/(c-d)^4/f
/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.182.2 Mathematica [A] (warning: unable to verify)

Time = 13.65 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \frac{\cos^4\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))^2\sec^{\frac{9}{2}}(e+fx)\left(-\cdots\right.}{\cdots}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2),x]
```

output
```
(Cos[(e + f*x)/2]^4*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^(9/2)*(-(((c^2*(43
*c^3 - 123*c^2*d + 53*c*d^2 + 219*d^3)*ArcSin[Tan[(e + f*x)/2]] - 32*Sqrt[
2]*(c - d)^4*(c + d)*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e
+ f*x])]] + (16*Sqrt[2]*d^(7/2)*(9*c^2 + 5*c*d - 2*d^2)*ArcTanh[(Sqrt[d]*T
an[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])]])/Sq
rt[-c - d])*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*
Sec[e + f*x]])/(c^2*(c - d)^4*(c + d)*Sqrt[Sec[(e + f*x)/2]^2])) + (Cos[(e
 + f*x)/2]*Sqrt[Sec[e + f*x]]*((2*(15*c^2 - 16*c*d - 31*d^2 - (16*d^4*Cos[
e + f*x])/(c*(d + c*Cos[e + f*x])))*Sin[(e + f*x)/2])/(c + d) + 32*(c - d)
*Csc[e + f*x]^4*Sin[(e + f*x)/2]^5 + (-19*c + 35*d)*Sec[(e + f*x)/2]*Tan[(
e + f*x)/2]))/(-c + d)^3))/(4*f*(a*(1 + Sec[e + f*x]))^(5/2)*(c + d*Sec[e
+ f*x])^2)
```

3.182.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a^3(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{\tan(e+fx)\int\left(\frac{(4c-d)d^4}{c^2(c-d)^4\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{d^4}{c(c-d)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}+\frac{\cos(e+fx)}{c^2\sqrt{a-a\sec(e+fx)}}+\frac{}{(c-}\right.}{af\sqrt{a-a\sec(e+f}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(-\frac{2d^{7/2}(4c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2(c-d)^4\sqrt{c+d}}+\frac{\sqrt{2}(c^2-4cd+6d^2)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}(c-d)^4}-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec}}{\sqrt{}}\right)}{\sqrt{a}c^2}\right.}{}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2),x]
```

output
```
-((((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^2) + ((c - 3
*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*Sqrt[a
]*(c - d)^3) + (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(16
*Sqrt[2]*Sqrt[a]*(c - d)^2) + (Sqrt[2]*(c^2 - 4*c*d + 6*d^2)*ArcTanh[Sqrt[
a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*(c - d)^4) - (d^(7/2)*Arc
Tanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c
*(c - d)^3*(c + d)^(3/2)) - (2*(4*c - d)*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a -
 a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^2*(c - d)^4*Sqrt[c +
d]) + Sqrt[a - a*Sec[e + f*x]]/(4*a*(c - d)^2*(1 + Sec[e + f*x])^2) + ((c
- 3*d)*Sqrt[a - a*Sec[e + f*x]])/(2*a*(c - d)^3*(1 + Sec[e + f*x])) + (3*S
qrt[a - a*Sec[e + f*x]])/(16*a*(c - d)^2*(1 + Sec[e + f*x])) - (d^4*Sqrt[a
 - a*Sec[e + f*x]])/(a*c*(c - d)^3*(c + d)*(c + d*Sec[e + f*x])))*Tan[e +
f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.182.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.182.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 85543 vs. $2(653) = 1306$.

Time $= 20.18$ (sec) , antiderivative size $= 85544$, normalized size of antiderivative $= 113.15$

method	result	size
default	Expression too large to display	85544

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.182.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas
")
```

output
```
Timed out
```

3.182.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{(a\left(\sec\left(e+fx\right)+1\right))^{\frac{5}{2}}\left(c+d\sec\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**2), x)
```

3.182.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima
")
```

output
```
Timed out
```

3.182.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \text{Hanged}$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2),x)
```

output
```
\text{Hanged}
```

3.183 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3} dx$

3.183.1 Optimal result

Integrand size = 27, antiderivative size = 999

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-4d)\tan(e+fx)}{2a^2(c-d)^4f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{3\tan(e+fx)}{16a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-4d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c^2-5cd+10d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^5f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{3d^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4a^{3/2}c(c-d)^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(4c-d)d^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^2(c-d)^4(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2d^{7/2}(10c^2-5cd+d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^3(c-d)^5\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d^4\tan(e+fx)}{2a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{3d^4\tan(e+fx)}{4a^2c(c-d)^3(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(4c-d)d^4\tan(e+fx)}{a^2c^2(c-d)^4(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
-1/4*tan(f*x+e)/a^2/(c-d)^3/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/2*
(c-4*d)*tan(f*x+e)/a^2/(c-d)^4/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-3/1
6*tan(f*x+e)/a^2/(c-d)^3/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+1/2*d^4*t
an(f*x+e)/a^2/c/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)+
3/4*d^4*tan(f*x+e)/a^2/c/(c-d)^3/(c+d)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e
))^(1/2)+(4*c-d)*d^4*tan(f*x+e)/a^2/c^2/(c-d)^4/(c+d)/f/(c+d*sec(f*x+e))/(
a+a*sec(f*x+e))^(1/2)+2*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)
/a^(3/2)/c^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+3/4*d^(7/2)*a
rctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3
/2)/c/(c-d)^3/(c+d)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+
(4*c-d)*d^(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2)
)*tan(f*x+e)/a^(3/2)/c^2/(c-d)^4/(c+d)^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a
*sec(f*x+e))^(1/2)-1/4*(c-4*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*tan(f*x+e)/a^(3/2)/(c-d)^4/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*
sec(f*x+e))^(1/2)-3/32*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
*tan(f*x+e)/a^(3/2)/(c-d)^3/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+
e))^(1/2)-(c^2-5*c*d+10*d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*2^(1/2)*tan(f*x+e)/a^(3/2)/(c-d)^5/f/(a-a*sec(f*x+e))^(1/2)/(a+a*se
c(f*x+e))^(1/2)+2*d^(7/2)*(10*c^2-5*c*d+d^2)*arctanh(d^(1/2)*(a-a*sec(f*x+
e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c^3/(c-d)^5/f/(c+d)^(...
```

3.183.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2904 vs. 2(999) = 1998.

Time = 22.08 (sec) , antiderivative size = 2904, normalized size of antiderivative = 2.91

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3),x]
```

output
```
(Cos[(e + f*x)/2]^5*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^6*((-3*(5*c^6 - 3*
c^5*d - 21*c^4*d^2 - 13*c^3*d^3 - 28*c^2*d^4 - 12*c*d^5 + 8*d^6)*Sin[(e +
f*x)/2])/(2*c^3*(-c + d)^4*(c + d)^2) - (4*d^6*Sin[(e + f*x)/2])/(c^3*(-c
+ d)^3*(c + d)*(d + c*Cos[e + f*x])^2) + (Sec[(e + f*x)/2]^2*(19*c*Sin[(e
+ f*x)/2] - 43*d*Sin[(e + f*x)/2]))/(4*(-c + d)^4) + (2*(-23*c^2*d^5*Sin[(
e + f*x)/2] - 9*c*d^6*Sin[(e + f*x)/2] + 8*d^7*Sin[(e + f*x)/2]))/(c^3*(-c
 + d)^4*(c + d)^2*(d + c*Cos[e + f*x])) + (Sec[(e + f*x)/2]^3*Tan[(e + f*x
)/2])/(2*(-c + d)^3)))/(f*(a*(1 + Sec[e + f*x]))^(5/2)*(c + d*Sec[e + f*x]
)^3) - ((c^3*(c + d)^2*(43*c^2 - 206*c*d + 355*d^2)*ArcSin[Tan[(e + f*x)/2
]] - 32*Sqrt[2]*(c - d)^5*(c + d)^2*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f
*x]/(1 + Cos[e + f*x])]] + (4*Sqrt[2]*d^(7/2)*(99*c^4 + 110*c^3*d - 5*c^2*
d^2 - 20*c*d^3 + 8*d^4)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*S
qrt[Cos[e + f*x]/(1 + Cos[e + f*x])])])/Sqrt[-c - d])*Cos[(e + f*x)/2]^5*(
d + c*Cos[e + f*x])^3*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*((-11*c^4*Sec[
(e + f*x)/2])/(8*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*
x]]) + (45*c^3*d*Sec[(e + f*x)/2])/(8*(-c + d)^4*(c + d)^2*(d + c*Cos[e +
f*x])*Sqrt[Sec[e + f*x]]) - (5*c^2*d^2*Sec[(e + f*x)/2])/(8*(-c + d)^4*(c
+ d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (317*c*d^3*Sec[(e + f*x)
/2])/(8*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (6
9*d^4*Sec[(e + f*x)/2])/(2*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])*Sq...
```

3.183.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 729, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a^3(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 27

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 198

$$-\frac{\tan(e+fx)\int \left(\frac{(10c^2-5dc+d^2)d^4}{c^3(c-d)^5\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{(4c-d)d^4}{c^2(c-d)^4\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}+\frac{d^4}{c(c-d)^3\sqrt{a-a\sec(e+fx)}(\ldots}\right.}{a\ldots}$$

$\Big\downarrow$ 2009

$$-\frac{\tan(e+fx)\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^3}-\frac{d^{7/2}(4c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2(c-d)^4(c+d)^{3/2}}+\frac{\sqrt{2}(c^2-5cd+10d^2)\text{arctanh}\left(\frac{\sqrt{a-a\sec\ldots}}{\sqrt{2}\ldots}\right)}{\sqrt{a}(c-d)^5}\ldots\right.}{\ldots}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3),x]
```

output
```
-((((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^3) + ((c - 4
*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*Sqrt[a
]*(c - d)^4) + (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(16
*Sqrt[2]*Sqrt[a]*(c - d)^3) + (Sqrt[2]*(c^2 - 5*c*d + 10*d^2)*ArcTanh[Sqrt
[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*(c - d)^5) - (3*d^(7/2)*
ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(4*Sqrt
[a]*c*(c - d)^3*(c + d)^(5/2)) - ((4*c - d)*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[
a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^2*(c - d)^4*(c + d
)^(3/2)) - (2*d^(7/2)*(10*c^2 - 5*c*d + d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*S
ec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^3*(c - d)^5*Sqrt[c + d])
+ Sqrt[a - a*Sec[e + f*x]]/(4*a*(c - d)^3*(1 + Sec[e + f*x])^2) + ((c - 4*
d)*Sqrt[a - a*Sec[e + f*x]])/(2*a*(c - d)^4*(1 + Sec[e + f*x])) + (3*Sqrt[
a - a*Sec[e + f*x]])/(16*a*(c - d)^3*(1 + Sec[e + f*x])) - (d^4*Sqrt[a - a
*Sec[e + f*x]])/(2*a*c*(c - d)^3*(c + d)*(c + d*Sec[e + f*x])^2) - (3*d^4*
Sqrt[a - a*Sec[e + f*x]])/(4*a*c*(c - d)^3*(c + d)^2*(c + d*Sec[e + f*x]))
 - ((4*c - d)*d^4*Sqrt[a - a*Sec[e + f*x]])/(a*c^2*(c - d)^4*(c + d)*(c +
d*Sec[e + f*x])))*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]))
```

3.183. $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx$

3.183.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

3.183.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 139612 vs. $2(868) = 1736$.

Time = 23.72 (sec) , antiderivative size = 139613, normalized size of antiderivative = 139.75

method	result	size
default	Expression too large to display	139613

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.183.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas
")
```

output
```
Timed out
```

3.183.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{5}{2}}(c+d\sec(e+fx))^3}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**3), x)
```

3.183.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima
")
```

output
```
Timed out
```

3.183.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Hanged}$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3),x)
```

output
```
\text{Hanged}
```

3.184 $\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$

3.184.1 Optimal result

Integrand size = 29, antiderivative size = 123

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \frac{2\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} + \frac{2\sqrt{a}\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

output
```
2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)
)^(1/2))*a^(1/2)*c^(1/2)/f+2*arctanh(a^(1/2)*d^(1/2)*tan(f*x+e)/(a+a*sec(f
*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*a^(1/2)*d^(1/2)/f
```

3.184.2 Mathematica [A] (verified)

Time = 14.89 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.95

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c+d\sec(e+fx)}\left(-2\sqrt{c}\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+c\cos(e+fx)}}{\sqrt{d}\sqrt{c-c\cos(e+fx)}}\right)\sqrt{c(1+\cos}\right.}{f\sqrt{c(1+\cos(e+fx))}\sqrt{c-c}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(-2*Cot[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]]*(-2*S
qrt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d + c*Cos[e + f*x]])/(Sqrt[d]*Sqrt[c
- c*Cos[e + f*x]])]*Sqrt[c*(1 + Cos[e + f*x])]*Sin[(e + f*x)/2]^2 + ArcTan
[(Sqrt[c*(1 + Cos[e + f*x])]*Sqrt[d + c*Cos[e + f*x]])/Sqrt[c^2*Sin[e + f*
x]^2]]*Sqrt[c - c*Cos[e + f*x]]*Sqrt[c^2*Sin[e + f*x]^2]))/(f*Sqrt[c*(1 +
Cos[e + f*x])]*Sqrt[c - c*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])
```

3.184.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 4420, 3042, 4422, 216, 4468, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4420}$$

$$c\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c+d\sec(e+fx)}}dx+d\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$c\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4422}$$

$$d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2ac\int \frac{1}{\frac{ac\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \ 216$$

$$d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \ 4468$$

$$\frac{2\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}-\frac{2ad\int \frac{1}{1-\frac{ad\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{2\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}+\frac{2\sqrt{a}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/f + (2*Sqrt[a]*Sqrt[d]*ArcTanh[(Sqrt[
a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]
])])/f
```

3.184.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4420
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c +
 d*Csc[e + f*x]], x], x] + Simp[d   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*
x]]/Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Ne
Q[b*c - a*d, 0]
```

rule 4422
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(1 + a*c*x^2), x],
 x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 4468
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*(b/f)   Subs
t[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
 + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.184.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1495 vs. $2(99) = 198$.

Time = 5.00 (sec) , antiderivative size = 1496, normalized size of antiderivative = 12.16

method	result	size
default	Expression too large to display	1496

input
```
int((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(c-d)^(1/2)*2^(1/2)/(-d)^(1/2)/(c^2-2*c*d+d^2)*(a*(sec(f*x+e)+1))^(1/
2)*(c+d*sec(f*x+e))^(1/2)*(2^(1/2)*(-d)^(1/2)*ln(-(c*cot(f*x+e)-d*cot(f*x+
e)-c*csc(f*x+e)+d*csc(f*x+e)-(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(c
-d)^(1/2))/(c-d)^(1/2))*c^3-3*2^(1/2)*(-d)^(1/2)*ln(-(c*cot(f*x+e)-d*cot(f
*x+e)-c*csc(f*x+e)+d*csc(f*x+e)-(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
*(c-d)^(1/2))/(c-d)^(1/2))*c^2*d+3*2^(1/2)*(-d)^(1/2)*ln(-(c*cot(f*x+e)-d*
cot(f*x+e)-c*csc(f*x+e)+d*csc(f*x+e)-(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^
(1/2)*(c-d)^(1/2))/(c-d)^(1/2))*c*d^2-2^(1/2)*(-d)^(1/2)*ln(-(c*cot(f*x+e)
-d*cot(f*x+e)-c*csc(f*x+e)+d*csc(f*x+e)-(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1
))^(1/2)*(c-d)^(1/2))/(c-d)^(1/2))*d^3-2^(1/2)*(-d)^(1/2)*ln((-2*(d+c*cos(
f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*csc(f*x+e
))*c^3+3*2^(1/2)*(-d)^(1/2)*ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-
(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*csc(f*x+e))*c^2*d-3*2^(1/2)*(-d)^(1/2)*
ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)
^(1/2)*csc(f*x+e))*c*d^2+2^(1/2)*(-d)^(1/2)*ln((-2*(d+c*cos(f*x+e))/(cos(f
*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*csc(f*x+e))*d^3+(c-d)^(
1/2)*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)-sin(f*x+e)*c-sin(f*x+e)*d-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/(cos(f
*x+e)-1+sin(f*x+e)))*c^2*d-2*(c-d)^(1/2)*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+
c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-sin(f*x+e)*c-sin(f*x+e)*...
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 806, normalized size of antiderivative = 6.55

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{ad}\log\left(\frac{2\sqrt{ad}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac-ad)\cos(fx+e)^2+2\,ad+(ac+ad)\cos(fx+e)}{\cos(fx+e)^2+\cos(fx+e)}\right)+\sqrt{-ac}\,\mathrm{lo}}{f} \right.$$

$$-\frac{2\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{ac\sin(fx+e)}\right)-\sqrt{ad}\log\left(\frac{2\sqrt{ad}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\,\mathrm{si}}{\cos(fx+}\right.}{f}$$

$$-\frac{2\sqrt{-ad}\arctan\left(\frac{\sqrt{-ad}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{ad\sin(fx+e)}\right)-\sqrt{-ac}\log\left(\frac{2\,ac\cos(fx+e)^2-2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{}}{}\right.}{f}$$

$$-\frac{2\left(\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{ac\sin(fx+e)}\right)+\sqrt{-ad}\arctan\left(\frac{\sqrt{-ad}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\,\mathrm{co}}{ad\sin(fx+e)}\right.\right.}{f}$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[(sqrt(a*d)*log((2*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c - a*d)
*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)^2 + cos(
f*x + e))) + sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 - 2*sqrt(-a*c)*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos
(f*x + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x +
e) + 1)))/f, -(2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*
x + e))) - sqrt(a*d)*log((2*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a
*c - a*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)
^2 + cos(f*x + e))))/f, -(2*sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e
)/(a*d*sin(f*x + e))) - sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 - 2*sqrt(-a*c
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/
(cos(f*x + e) + 1)))/f, -2*(sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/
(a*c*sin(f*x + e))) + sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(...
```

3.184.6 Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{c+d\sec\left(e+fx\right)}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x)), x)
```

3.184.7 Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx=\int \sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

3.184.8 Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx=\int \sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx=\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2), x)
```

3.185 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

3.185.1 Optimal result

Integrand size = 29, antiderivative size = 61

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{c}f}$$

output
```
2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)
)^(1/2))*a^(1/2)/f/c^(1/2)
```

3.185.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\sqrt{d+c\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{\sqrt{c}f\sqrt{c+d\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]
]]*Sqrt[d + c*Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(
Sqrt[c]*f*Sqrt[c + d*Sec[e + f*x]])
```

3.185.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3042, 4422, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4422}$$

$$-\frac{2a\int \frac{1}{\frac{ac\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{c}f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[c]*f)
```

3.185.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4422
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(1 + a*c*x^2), x],
 x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

3.185.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(49) = 98.

Time = 2.90 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.72

method	result	size
default	$-\frac{2\sqrt{2}\,\sqrt{-(c-d)^4 c}\,\arctan\left(\frac{(c-d)^2 c\sqrt{2}\,\sin(fx+e)}{\sqrt{-(c-d)^4 c}\,(\cos(fx+e)+1)\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}}\right)\sqrt{c+d\sec(fx+e)}\,\sqrt{a(\sec(fx+e)+1)}\,\cos(fx+e)}{f(c^2-2cd+d^2)c(\cos(fx+e)+1)\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}}$	166

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f*2^(1/2)*(-(c-d)^4*c)^(1/2)/(c^2-2*c*d+d^2)/c*arctan((c-d)^2*c*2^(1/2)
/(-(c-d)^4*c)^(1/2)*sin(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*
x+e)+1))^(1/2))*(c+d*sec(f*x+e))^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*cos(f*x+e)
/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.38

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \left[\frac{\sqrt{-\frac{a}{c}}\log\left(-\frac{2\,c\sqrt{-\frac{a}{c}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-2\,ac\cos(fx+e)^2+ac-ad-(ac+ad)\cos(fx+e)}{\cos(fx+e)+1}\right)}{f},\; -\frac{2\sqrt{\frac{a}{c}}\arctan\left(\frac{\sqrt{\frac{a}{c}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{a\sin(fx+e)}\right)}{f}\right]$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[sqrt(-a/c)*log(-(2*c*sqrt(-a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*a*c*c
os(f*x + e)^2 + a*c - a*d - (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1))/
f, -2*sqrt(a/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e)))/f]
```

3.185.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/sqrt(c + d*sec(e + f*x)), x)
```

3.185.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-c>0)', see `assume?` for more
details)Is
```

3.185.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2), x)
```

3.186 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$

3.186.1 Optimal result

Integrand size = 29, antiderivative size = 111

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{2ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

output
```
2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)
)^(1/2))*a^(1/2)/c^(3/2)/f-2*a*d*tan(f*x+e)/c/(c+d)/f/(a+a*sec(f*x+e))^(1/
2)/(c+d*sec(f*x+e))^(1/2)
```

3.186.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = -\frac{\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}\left(-\sqrt{2}(c+d)^{3/2}\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)\sqrt{\frac{d+c\cos(e+fx)}{c+d}}+2\sqrt{c}d\sin\right.}{c^{3/2}(c+d)f\sqrt{c+d\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

output
```
-((Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(-(Sqrt[2]*(c + d)^(3/2)*Ar
cSin[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[c + d]]*Sqrt[(d + c*Cos[e + f
*x])/(c + d)]) + 2*Sqrt[c]*d*Sin[(e + f*x)/2]))/(c^(3/2)*(c + d)*f*Sqrt[c
+ d*Sec[e + f*x]]))
```

3.186.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 4427, 3042, 4422, 216, 4475, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c+d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4427$$

$$\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c+d\sec(e+fx)}}dx}{c}-\frac{d\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{(c+d\sec(e+fx))^{3/2}}dx}{c}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}$$

$$\Big\downarrow \; 4422$$

$$-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{2a\int \frac{1}{\frac{ac\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{cf}$$

$$\Big\downarrow \; 216$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{d\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}$$

$$\Big\downarrow \text{4475}$$

$$\frac{a^2d\tan(e+fx)\int\frac{1}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}d\sec(e+fx)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f}$$

$$\Big\downarrow \text{48}$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{2ad\tan(e+fx)}{cf(c+d)\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c + d*Sec[e + f*x]])])/(c^(3/2)*f) - (2*a*d*Tan[e + f*x])/(c*(c + d)
*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])
```

3.186.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4422
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(1 + a*c*x^2), x],
 x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 4427
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(3/2), x_Symbol] :> Simp[1/c   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt
[c + d*Csc[e + f*x]], x], x] - Simp[d/c   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[
e + f*x]]/(c + d*Csc[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[c^2 - d^2, 0]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

3.186.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(95) = 190$.

Time = 2.85 (sec) , antiderivative size = 609, normalized size of antiderivative = 5.49

method	result
default	$\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)\sqrt{\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2-d(1-\cos(fx+e))^2\csc(fx+e)^2-c-d}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\Big($

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(c+d)/c^2/(c^2-2*c*d+d^2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/
2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*((c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(
1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)
*(-2*c^3*d*(-cot(f*x+e)+csc(f*x+e))+4*c^2*d^2*(-cot(f*x+e)+csc(f*x+e))-2*c
*d^3*(-cot(f*x+e)+csc(f*x+e))+2^(1/2)*(-(c-d)^4*c)^(1/2)*arctan((c-d)^2*c*
2^(1/2)/(-(c-d)^4*c)^(1/2)/(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e
))^2*csc(f*x+e)^2-c-d)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(c*(1-cos(f*x+e))^2
*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)^(1/2)*c+2^(1/2)*(-(c-d)
^4*c)^(1/2)*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2)/(c*(1-cos(f*x+e))^
2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)^(1/2)*(-cot(f*x+e)+csc
(f*x+e)))*(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2
-c-d)^(1/2)*d)/(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x
+e)^2-c-d)
```

3.186.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(95) = 190$.

Time = 0.36 (sec) , antiderivative size = 517, normalized size of antiderivative = 4.66

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \left[-\frac{2\,d\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(\left(c^2+cd\right)\cos\left(\cdots\right.\right.}{(\cdots}\right.$$

$$-\frac{2\left(d\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(c^2+cd\right)\cos\left(fx+e\right)^2+cd+d^2+\left(c^2+2\,c\cdots\right.\right.\right.}{\left(c^3+c^2d\right)f\cos\left(fx+e\right)^2+\left(c^3+2\,c^2d+cd^2\right)f\cos\left(f\cdots\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
[-(2*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e) - ((c^2 + c*d)*cos(f*x + e)^2 + c*d
 + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a/c)*log(-(2*c*sqrt(-a/c)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) - 2*a*c*cos(f*x + e)^2 + a*c - a*d - (a*c
 + a*d)*cos(f*x + e))/(cos(f*x + e) + 1)))/((c^3 + c^2*d)*f*cos(f*x + e)^2
 + (c^3 + 2*c^2*d + c*d^2)*f*cos(f*x + e) + (c^2*d + c*d^2)*f), -2*(d*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e)
)*cos(f*x + e)*sin(f*x + e) + ((c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c
^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(a/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x +
 e)/(a*sin(f*x + e))))/((c^3 + c^2*d)*f*cos(f*x + e)^2 + (c^3 + 2*c^2*d +
c*d^2)*f*cos(f*x + e) + (c^2*d + c*d^2)*f)]
```

3.186.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(c+d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**(3/2), x)
```

3.186.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-c>0)', see `assume?` for more
details)Is
```

3.186.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{(d\sec\left(fx+e\right)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
sage0*x
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2), x)
```

3.187 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.187.1 Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)
)^(1/2))*c^(1/2)/f/a^(1/2)-arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/
2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*2^(1/2)*(c-d)^(1/2)/f/a^
(1/2)
```

3.187.2 Mathematica [A] (verified)

Time = 13.07 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{-c+d}\,\text{arctanh}\left(\frac{\sqrt{-c+d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)+\frac{\sqrt{2}\sqrt{c}\sqrt{c+d}\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)\sqrt{\frac{d+c\cos(e+fx)}{c+d}}}{\sqrt{d+c\cos(e+fx)}}\right)\sqrt{}}{f\sqrt{d+c\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Cos[(e + f*x)/2]*(Sqrt[-c + d]*ArcTanh[(Sqrt[-c + d]*Sin[(e + f*x)/2])/
Sqrt[d + c*Cos[e + f*x]]] + (Sqrt[2]*Sqrt[c]*Sqrt[c + d]*ArcSin[(Sqrt[2]*S
qrt[c]*Sin[(e + f*x)/2])/Sqrt[c + d]]*Sqrt[(d + c*Cos[e + f*x])/(c + d)])/
Sqrt[d + c*Cos[e + f*x]])*Sqrt[c + d*Sec[e + f*x]])/(f*Sqrt[d + c*Cos[e +
f*x]]*Sqrt[a*(1 + Sec[e + f*x])])
```

3.187.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 4423, 3042, 4422, 216, 4471, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4423}$$

$$\frac{c\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c+d\sec(e+fx)}}dx}{a}-(c-d)\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}-(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4422}$$

$$-\left((c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2c\int \frac{1}{\frac{ac\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$\Big\downarrow$ 216

$$\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}-(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$\Big\downarrow$ 4471

$$\frac{2(c-d)\int \frac{1}{\frac{a(c-d)\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}+\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

$\Big\downarrow$ 216

$$\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

input
```
Int[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*f) - (Sqrt[2]*Sqrt[c - d]*ArcTan[(Sq
rt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c +
 d*Sec[e + f*x]])])/(Sqrt[a]*f)
```

3.187. $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.187.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4422
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(1 + a*c*x^2), x],
 x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 4423
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[a/c   Int[Sqrt[c + d*Csc[e + f*x]]/Sqrt[a
 + b*Csc[e + f*x]], x], x] + Simp[(b*c - a*d)/c   Int[Csc[e + f*x]/(Sqrt[a
+ b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

rule 4471
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*(a/(b*f))
Subst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e +
f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.187.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(114) = 228$.

Time = 2.72 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.00

method	result
default	$-\frac{2\sqrt{c+d\sec(fx+e)}\,\sqrt{a(\sec(fx+e)+1)}\left(\sqrt{2}\,\sqrt{-(c-d)^4c}\,\arctan\left(\frac{(c-d)^2c\sqrt{2}\,\sin(fx+e)}{\sqrt{-(c-d)^4c}\,(\cos(fx+e)+1)\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}}\right)\sqrt{c-d}-\ln\left(\sqrt{-\frac{2}{}}\right.\right.}{}$

input
```
int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/a/(c-d)^(1/2)/(c^2-2*c*d+d^2)*(c+d*sec(f*x+e))^(1/2)*(a*(sec(f*x+e)+1
))^(1/2)*(2^(1/2)*(-(c-d)^4*c)^(1/2)*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)
^(1/2)*sin(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2
))*(c-d)^(1/2)-ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*c
ot(f*x+e)+(c-d)^(1/2)*csc(f*x+e))*c^3+3*ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e
)+1))^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*csc(f*x+e))*c^2*d-3*ln((-2*
(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*
csc(f*x+e))*c*d^2+ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2
)*cot(f*x+e)+(c-d)^(1/2)*csc(f*x+e))*d^3)*cos(f*x+e)/(cos(f*x+e)+1)/(-2*(d
+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 883, normalized size of antiderivative = 6.26

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$= \left[\frac{\sqrt{2}\sqrt{-\frac{c-d}{a}}\log\left(\frac{2\sqrt{2}\sqrt{-\frac{c-d}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(3c-d)\cos(fx+e)^2+2(c+d)\cos(fx+e)-c+3d}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)}{2f}\right.$$
$$-\frac{\sqrt{2}\sqrt{\frac{c-d}{a}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c-d}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{(c-d)\sin(fx+e)}\right)-\sqrt{-\frac{c}{a}}\log\left(-\frac{2\sqrt{-\frac{c}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}}{}\right.}{f}$$
$$-\frac{\sqrt{2}\sqrt{\frac{c-d}{a}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c-d}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{(c-d)\sin(fx+e)}\right)+2\sqrt{\frac{c}{a}}\arctan\left(\frac{\sqrt{\frac{c}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}}{c\sin(fx+e)}\right.}{f}$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[1/2*(sqrt(2)*sqrt(-(c - d)/a)*log((2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f
*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c + d)*cos(f*x + e) -
 c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-c/a)*log(-(2*sq
rt(-c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)
/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*c*cos(f*x + e)^2 - (c + d)*co
s(f*x + e) + c - d)/(cos(f*x + e) + 1)))/f, 1/2*(sqrt(2)*sqrt(-(c - d)/a)*
log((2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (3*c - d
)*cos(f*x + e)^2 + 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*c
os(f*x + e) + 1)) - 4*sqrt(c/a)*arctan(sqrt(c/a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(c*sin
(f*x + e))))/f, -(sqrt(2)*sqrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x
+ e))*cos(f*x + e)/((c - d)*sin(f*x + e))) - sqrt(-c/a)*log(-(2*sqrt(-c/a)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) - 2*c*cos(f*x + e)^2 - (c + d)*cos(f*x +
e) + c - d)/(cos(f*x + e) + 1)))/f, -(sqrt(2)*sqrt((c - d)/a)*arctan(-sqrt
(2)*sqrt((c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*
x + e) + d)/cos(f*x + e))*cos(f*x + e)/((c - d)*sin(f*x + e))) + 2*sqrt...
```

3.187.6 Sympy [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a\left(\sec(e+fx)+1\right)}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(c + d*sec(e + f*x))/sqrt(a*(sec(e + f*x) + 1)), x)
```

3.187.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-c>0)', see `assume?` for more
details)Is
```

3.187.8 Giac [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\sec(fx+e)+c}}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.188 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

3.188.1 Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}$$

output
```
2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)
)^(1/2))/f/a^(1/2)/c^(1/2)-arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/
2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)/(c-d)^
(1/2)
```

3.188.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\left(\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right) - \sqrt{c}\arctan\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{d+c\cos(e+}}{\sqrt{c}\sqrt{c-d}f\sqrt{a(1+\sec(e+fx))}\sqrt{c+d\sec(e+fx)}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
(2*(Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[d +
 c*Cos[e + f*x]]] - Sqrt[c]*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d +
 c*Cos[e + f*x]]])*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x])
/(Sqrt[c]*Sqrt[c - d]*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]
])
```

3.188.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 4426, 3042, 4422, 216, 4471, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4426}$$

$$\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c+d\sec(e+fx)}}dx}{a}-\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4422}$$

$$-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2\int \frac{1}{\frac{ac\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f}-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4471$$

$$\frac{2\int \frac{1}{\frac{a(c-d)\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}+\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f}$$

$$\Big\downarrow \; 216$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
(2*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
+ d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c]*f) - (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c
 - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f
*x]])])/(Sqrt[a]*Sqrt[c - d]*f)
```

3.188.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4422
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(1 + a*c*x^2), x],
 x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 4426
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x
_)]*(d_.) + (c_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sq
rt[c + d*Csc[e + f*x]], x], x] - Simp[b/a   Int[Csc[e + f*x]/(Sqrt[a + b*Cs
c[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0]
```

rule 4471
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*(a/(b*f))
Subst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e +
f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.188.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(114) = 228$.

Time = 2.50 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.60

method	result
default	$-\frac{2\sqrt{a(\sec(fx+e)+1)}\,\sqrt{c+d\sec(fx+e)}\left(-\ln\left(\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}-\sqrt{c-d}\,\cot(fx+e)+\sqrt{c-d}\,\csc(fx+e)\right)c^3+2\ln\left(\sqrt{-\frac{2(d+c\cos(}{\cos(fx+}}\right.\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/f/a/(c-d)^(1/2)/(c^2-2*c*d+d^2)/c*(a*(sec(f*x+e)+1))^(1/2)*(c+d*sec(f*x
+e))^(1/2)*(-ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot
(f*x+e)+(c-d)^(1/2)*csc(f*x+e))*c^3+2*ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+
1))^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*csc(f*x+e))*c^2*d-ln((-2*(d+c
*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*csc(
f*x+e))*c*d^2+2^(1/2)*(-(c-d)^4*c)^(1/2)*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^
4*c)^(1/2)*sin(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^
(1/2))*(c-d)^(1/2))*cos(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*
x+e)+1))^(1/2)
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 913, normalized size of antiderivative = 6.48

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$

$$= \left[\frac{\sqrt{2}ac\sqrt{-\frac{1}{ac-ad}}\log\left(\frac{2\sqrt{2}(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\sqrt{-\frac{1}{ac-ad}}\cos(fx+e)\sin(fx+e)+(3c-d)\cos(fx+e)^2+2(c+d)\cos(fx\cdots}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{2\cdots} \right.$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
[1/2*(sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-
1/(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c
 + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*s
qrt(-a*c)*log((2*a*c*cos(f*x + e)^2 + 2*sqrt(-a*c)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1)))/(a*c
*f), 1/2*(sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*
cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sq
rt(-1/(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 +
2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) -
 4*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*x + e))))/(a*c
*f), (sqrt(2)*a*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f
*x + e)))/sqrt(a*c - a*d) - sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 + 2*sqrt(
-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x +
e))/(cos(f*x + e) + 1)))/(a*c*f), (sqrt(2)*a*c*arctan(sqrt(2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos...
```

3.188.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x))), x)
```

3.188.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(1/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

3.188.8 Giac [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+fx)}}\sqrt{c+\frac{d}{\cos(e+fx)}}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)), x)
```

3.189 $\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx$

3.189.1 Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx=\frac{ax}{c}+\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}f}$$

output
```
a*x/c+2*(-a*d+b*c)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c/f
/(c-d)^(1/2)/(c+d)^(1/2)
```

3.189.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx=\frac{a(e+fx)+\frac{2(-bc+ad)\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}}{cf}$$

input
```
Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x]),x]
```

output
```
(a*(e + f*x) + (2*(-(b*c) + a*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/Sqrt[c^2 - d^2])/(c*f)
```

3.189.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4407}$$

$$\frac{(bc-ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{ 4318}$$

$$\frac{(bc-ad)\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{cd}+\frac{ax}{c}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(bc-ad)\int \frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{cd}+\frac{ax}{c}$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{2(bc-ad)\int \frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{cdf}+\frac{ax}{c}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\sqrt{c+d}}+\frac{ax}{c}$$

input
```
Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x]),x]
```

output
```
(a*x)/c + (2*(b*c - a*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d
]])/(c*Sqrt[c - d]*Sqrt[c + d]*f)
```

3.189.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

3.189.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{2(da-bc)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{c\sqrt{(c+d)(c-d)}}+\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}}{f}$
default	$\frac{-\frac{2(da-bc)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{c\sqrt{(c+d)(c-d)}}+\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}}{f}$
risch	$\frac{ax}{c}+\frac{\ln\left(\mathrm{e}^{i(fx+e)}-\frac{ic^2-id^2-\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)da}{\sqrt{c^2-d^2}\,fc}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}-\frac{ic^2-id^2-\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)b}{\sqrt{c^2-d^2}\,f}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d}}{\sqrt{c^2-d^2}\,c}\right.}{\sqrt{c^2-d^2}\,fc}$

input
```
int((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2*(a*d-b*c)/c/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/(
(c+d)*(c-d))^(1/2))+2*a/c*arctan(tan(1/2*f*x+1/2*e)))
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.73

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx$$
$$=\left[\frac{2\left(ac^2-ad^2\right)fx-(bc-ad)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2-2\,\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right.}{2\left(c^3-cd^2\right)f}\right.$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/2*(2*(a*c^2 - a*d^2)*f*x - (b*c - a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f
*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e)
 + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e)
 + d^2)))/((c^3 - c*d^2)*f), ((a*c^2 - a*d^2)*f*x + (b*c - a*d)*sqrt(-c^2
+ d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x
+ e))))/((c^3 - c*d^2)*f)]
```

3.189.6 Sympy [F]

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx = \int \frac{a+b\sec{(e+fx)}}{c+d\sec{(e+fx)}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

output
```
Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x)), x)
```

3.189.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

3.189.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(58) = 116$.

Time = 0.33 (sec) , antiderivative size = 274, normalized size of antiderivative = 4.09

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx = \frac{\frac{\left(\sqrt{-c^2+d^2}a(c-2d)|-c+d|+\sqrt{-c^2+d^2}bc|-c+d|-\sqrt{-c^2+d^2}a|c||-c+d|+\sqrt{-c^2+d^2}b|c||-c+d|\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-\frac{d+\sqrt{(c+d)(c-d)+d^2}}{c-d}}}\right.\right.}{(c^2-2cd+d^2)c^2+(c^2d-2cd^2+d^3)|c|}}{f}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
((sqrt(-c^2 + d^2)*a*(c - 2*d)*abs(-c + d) + sqrt(-c^2 + d^2)*b*c*abs(-c +
 d) - sqrt(-c^2 + d^2)*a*abs(c)*abs(-c + d) + sqrt(-c^2 + d^2)*b*abs(c)*ab
s(-c + d))*(pi*floor(1/2*(f*x + e)/pi + 1/2) + arctan(tan(1/2*f*x + 1/2*e)
/sqrt(-(d + sqrt((c + d)*(c - d) + d^2))/(c - d))))/((c^2 - 2*c*d + d^2)*c
^2 + (c^2*d - 2*c*d^2 + d^3)*abs(c)) + (a*c + b*c - 2*a*d + a*abs(c) - b*a
bs(c))*(pi*floor(1/2*(f*x + e)/pi + 1/2) + arctan(tan(1/2*f*x + 1/2*e)/sqr
t(-(d - sqrt((c + d)*(c - d) + d^2))/(c - d))))/(c^2 - d*abs(c)))/f
```

3.189.9 Mupad [B] (verification not implemented)

Time = 15.63 (sec) , antiderivative size = 573, normalized size of antiderivative = 8.55

$$
\begin{aligned}
\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx
&= \frac{b\,c^2\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(c^2-d^2\right)}^{3/2}} \\
&\quad - \frac{b\,d^2\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(c^2-d^2\right)}^{3/2}} + \frac{2\,a\,c\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(c^2-d^2\right)} \\
&\quad - \frac{b\,\ln\left(\frac{c\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\sqrt{\left(c+d\right)\,\left(c-d\right)}}{f\,\left(c^2-d^2\right)} \\
&\quad - \frac{a\,c\,d\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(c^2-d^2\right)}^{3/2}} \\
&\quad - \frac{2\,a\,d^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{c\,f\,\left(c^2-d^2\right)} + \frac{a\,d^3\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{c\,f\,{\left(c^2-d^2\right)}^{3/2}} \\
&\quad + \frac{a\,d\,\ln\left(\frac{c\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\sqrt{\left(c+d\right)\,\left(c-d\right)}}{c\,f\,\left(c^2-d^2\right)}
\end{aligned}
$$

input
```
int((a + b/cos(e + f*x))/(c + d/cos(e + f*x)),x)
```

output
```
(b*c^2*log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/
2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (b*d^2*
log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2
 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) + (2*a*c*atan(si
n(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)) - (b*log((c*cos(e/2
+ (f*x)/2) + d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/
cos(e/2 + (f*x)/2))*((c + d)*(c - d))^(1/2))/(f*(c^2 - d^2)) - (a*c*d*log(
(c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d
^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (2*a*d^2*atan(sin(
e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(c*f*(c^2 - d^2)) + (a*d^3*log((c*sin(
e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/
2))/cos(e/2 + (f*x)/2)))/(c*f*(c^2 - d^2)^(3/2)) + (a*d*log((c*cos(e/2 + (
f*x)/2) + d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos
(e/2 + (f*x)/2))*((c + d)*(c - d))^(1/2))/(c*f*(c^2 - d^2))
```

3.189. $\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx$

3.190 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$

3.190.1 Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \frac{ax}{c^2}+\frac{2(bc^3-2ac^2d+ad^3)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f} - \frac{d(bc-ad)\tan(e+fx)}{c\,(c^2-d^2)\,f(c+d\sec(e+fx))}$$

output
```
a*x/c^2+2*(-2*a*c^2*d+a*d^3+b*c^3)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/
(c+d)^(1/2))/c^2/(c-d)^(3/2)/(c+d)^(3/2)/f-d*(-a*d+b*c)*tan(f*x+e)/c/(c^2-
d^2)/f/(c+d*sec(f*x+e))
```

3.190.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.26

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \frac{-\frac{2(bc^3+ad(-2c^2+d^2))\,\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+\frac{ad(c^2-d^2)(e+fx)+ac(c^2-d^2)(e+fx)\cos(e+fx)-cd(bc-ad)\sin(e+fx)}{d+c\cos(e+fx)}}{c^2(c-d)(c+d)f}$$

input
```
Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^2,x]
```

output
```
((-2*(b*c^3 + a*d*(-2*c^2 + d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt
[c^2 - d^2]])/Sqrt[c^2 - d^2] + (a*d*(c^2 - d^2)*(e + f*x) + a*c*(c^2 - d^
2)*(e + f*x)*Cos[e + f*x] - c*d*(b*c - a*d)*Sin[e + f*x])/(d + c*Cos[e + f
*x]))/(c^2*(c - d)*(c + d)*f)
```

3.190.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4411, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \ 4411$$

$$-\frac{\int -\frac{a(c^2-d^2)+c(bc-ad)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{a(c^2-d^2)+c(bc-ad)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{a(c^2-d^2)+c(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}$$

$$\Big\downarrow \ 4407$$

$$\frac{\frac{(bc^3-ad(2c^2-d^2))\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{c}+\frac{ax(c^2-d^2)}{c}}{c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(bc^3-ad(2c^2-d^2))\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}+\frac{ax(c^2-d^2)}{c}}{c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{cf\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \textbf{4318}$$

$$\frac{\frac{(bc^3-ad(2c^2-d^2))\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{cd}+\frac{ax(c^2-d^2)}{c}}{c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{cf\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{(bc^3-ad(2c^2-d^2))\int \frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{cd}+\frac{ax(c^2-d^2)}{c}}{c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{cf\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \textbf{3138}$$

$$\frac{\frac{2(bc^3-ad(2c^2-d^2))\int \frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{cdf}+\frac{ax(c^2-d^2)}{c}}{c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{cf\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{\frac{2(bc^3-ad(2c^2-d^2))\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\sqrt{c+d}}+\frac{ax(c^2-d^2)}{c}}{c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{cf\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

input
```
Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^2,x]
```

output
```
((a*(c^2 - d^2)*x)/c + (2*(b*c^3 - a*d*(2*c^2 - d^2))*ArcTanh[(Sqrt[c - d]
*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c*Sqrt[c - d]*Sqrt[c + d]*f))/(c*(c^2 -
d^2)) - (d*(b*c - a*d)*Tan[e + f*x])/(c*(c^2 - d^2)*f*(c + d*Sec[e + f*x])
)
```

3.190.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4411
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2)
)   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
 a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && N
eQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

3.190.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{\frac{-\frac{2d(da-bc)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{2\left(2a\,c^2d-a\,d^3-b\,c^3\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{c^2}+\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{c^2}}{f}$
default	$\frac{\frac{-\frac{2d(da-bc)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{2\left(2a\,c^2d-a\,d^3-b\,c^3\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{c^2}+\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{c^2}}{f}$
risch	$\frac{ax}{c^2}-\frac{2id(-da+bc)\left(d\,e^{i(fx+e)}+c\right)}{c^2\left(c^2-d^2\right)f\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)}+\frac{2\ln\left(e^{i(fx+e)}+\frac{-ic^2+id^2+\sqrt{c^2-d^2}\,d}{c\sqrt{c^2-d^2}}\right)ad}{\sqrt{c^2-d^2}\,(c+d)(c-d)f}-\frac{\ln\left(e^{i(fx+e)}+\frac{-ic^2+id}{c\sqrt{}}\right.}{\sqrt{c^2-d^2}\,(c+d)}$

input
```
int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/c^2*(-d*(a*d-b*c)*c/(c^2-d^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e
)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)-(2*a*c^2*d-a*d^3-b*c^3)/(c+d)/(c-d)/((c+
d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))+2*a
/c^2*arctan(tan(1/2*f*x+1/2*e)))
```

3.190.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(114) = 228$.

Time = 0.31 (sec) , antiderivative size = 561, normalized size of antiderivative = 4.56

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$$
$$= \left[\frac{2\left(ac^5-2\,ac^3d^2+acd^4\right)fx\cos\left(fx+e\right)+2\left(ac^4d-2\,ac^2d^3+ad^5\right)fx-\left(bc^3d-2\,ac^2d^2+ad^4+\left(bc^4-\right.\right.}{2\left(\left(c^7\right.\right.}\right.$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(a*c^5 - 2*a*c^3*d^2 + a*c*d^4)*f*x*cos(f*x + e) + 2*(a*c^4*d - 2*
a*c^2*d^3 + a*d^5)*f*x - (b*c^3*d - 2*a*c^2*d^2 + a*d^4 + (b*c^4 - 2*a*c^3
*d + a*c*d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2
 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x
+ e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(
b*c^4*d - a*c^3*d^2 - b*c^2*d^3 + a*c*d^4)*sin(f*x + e))/((c^7 - 2*c^5*d^2
 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f), ((a*c^5 - 2
*a*c^3*d^2 + a*c*d^4)*f*x*cos(f*x + e) + (a*c^4*d - 2*a*c^2*d^3 + a*d^5)*f
*x + (b*c^3*d - 2*a*c^2*d^2 + a*d^4 + (b*c^4 - 2*a*c^3*d + a*c*d^3)*cos(f*
x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c
^2 - d^2)*sin(f*x + e))) - (b*c^4*d - a*c^3*d^2 - b*c^2*d^3 + a*c*d^4)*sin
(f*x + e))/((c^7 - 2*c^5*d^2 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^
3 + c^2*d^5)*f)]
```

3.190.6 Sympy [F]

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx=\int \frac{a+b\sec{(e+fx)}}{(c+d\sec{(e+fx)})^2}\,dx$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x))**2, x)
```

3.190.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.63

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$$
$$=\frac{\frac{2\left(bc^3-2\,ac^2d+ad^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^4-c^2d^2)\sqrt{-c^2+d^2}}+\frac{(fx+e)a}{c^2}+\frac{2\left(bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.}{(c^3-cd^2)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.}}{f}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
(2*(b*c^3 - 2*a*c^2*d + a*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c
+ 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c
^2 + d^2)))/((c^4 - c^2*d^2)*sqrt(-c^2 + d^2)) + (f*x + e)*a/c^2 + 2*(b*c*
d*tan(1/2*f*x + 1/2*e) - a*d^2*tan(1/2*f*x + 1/2*e))/((c^3 - c*d^2)*(c*tan
(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)))/f
```

3.190.9 Mupad [B] (verification not implemented)

Time = 22.21 (sec) , antiderivative size = 3763, normalized size of antiderivative = 30.59

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx=\text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))/(c + d/cos(e + f*x))^2,x)
```

output
```
(2*a*atan(((a*((a*((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^
2 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 -
 c^4*d^2) - (a*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6
*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2))
)*1i)/c^2 + (32*tan(e/2 + (f*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*
c*d^5 - 2*a^2*c^5*d - 5*a^2*c^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*
b*c^5*d + 2*a*b*c^3*d^3))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))/c^2 - (a*((a
*((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2 - b*c^6*d^3 + b
*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a*ta
n(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3
 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2 - (32*t
an(e/2 + (f*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5
*d - 5*a^2*c^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c
^3*d^3))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))/c^2)/((64*(a^3*d^5 + a*b^2*c^
5 - a^2*b*c^5 - a^3*c*d^4 + 2*a^3*c^4*d - 3*a^3*c^2*d^3 + 2*a^3*c^3*d^2 +
a^2*b*c^2*d^3 + a^2*b*c^3*d^2 - 3*a^2*b*c^4*d))/(c^5*d + c^6 - c^3*d^3 - c
^4*d^2) + (a*((a*((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2
 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 -
c^4*d^2) - (a*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*
d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2...
```

3.190. $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$

3.191 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx$

3.191.1 Optimal result

Integrand size = 23, antiderivative size = 204

$$\begin{aligned}\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx &= \frac{ax}{c^3}+\frac{(bc^3(2c^2+d^2)-ad(6c^4-5c^2d^2+2d^4))\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f}\\ &\quad-\frac{d(bc-ad)\tan(e+fx)}{2c(c^2-d^2)f(c+d\sec(e+fx))^2}-\frac{d(3bc^3-5ac^2d+2ad^3)\tan(e+fx)}{2c^2(c^2-d^2)^2f(c+d\sec(e+fx))}\end{aligned}$$

output
```
a*x/c^3+(b*c^3*(2*c^2+d^2)-a*d*(6*c^4-5*c^2*d^2+2*d^4))*arctanh((c-d)^(1/2
)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^3/(c-d)^(5/2)/(c+d)^(5/2)/f-1/2*d*(-a*
d+b*c)*tan(f*x+e)/c/(c^2-d^2)/f/(c+d*sec(f*x+e))^2-1/2*d*(-5*a*c^2*d+2*a*d
^3+3*b*c^3)*tan(f*x+e)/c^2/(c^2-d^2)^2/f/(c+d*sec(f*x+e))
```

3.191.2 Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.31

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = \frac{(d+c\cos(e+fx))\sec^2(e+fx)(a+b\sec(e+fx))\left(2a(e+fx)(d+c\cos(e+fx))^2-\frac{2(bc^3(2c^2+d^2)+ad(-6\cdots}{\cdots}\right.}{2c^3f(b+a\cos(\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^3,x]
```

output
```
((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])*(2*a*(e + f*x)*(
d + c*Cos[e + f*x])^2 - (2*(b*c^3*(2*c^2 + d^2) + a*d*(-6*c^4 + 5*c^2*d^2
- 2*d^4))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[
e + f*x])^2)/(c^2 - d^2)^(5/2) + (c*d^2*(b*c - a*d)*Sin[e + f*x])/((c - d)
*(c + d)) - (c*d*(4*b*c^3 - 6*a*c^2*d - b*c*d^2 + 3*a*d^3)*(d + c*Cos[e +
f*x])*Sin[e + f*x])/((c - d)^2*(c + d)^2)))/(2*c^3*f*(b + a*Cos[e + f*x])*
(c + d*Sec[e + f*x])^3)
```

3.191.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.23, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4411, 25, 3042, 4548, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4411}$$

$$-\frac{\int -\frac{-d(bc-ad)\sec^2(e+fx)+2c(bc-ad)\sec(e+fx)+2a(c^2-d^2)}{(c+d\sec(e+fx))^2}dx}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-d(bc-ad)\sec^2(e+fx)+2c(bc-ad)\sec(e+fx)+2a(c^2-d^2)}{(c+d\sec(e+fx))^2}dx}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-d(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)^2+2c(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)+2a(c^2-d^2)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{4548}$$

$$\frac{-\frac{\int -\frac{2a(c^2-d^2)^2-c\left(ad(4c^2-d^2)-bc(2c^2+d^2)\right)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2a(c^2-d^2)^2-c\left(ad(4c^2-d^2)-bc(2c^2+d^2)\right)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{2a(c^2-d^2)^2-c\left(ad(4c^2-d^2)-bc(2c^2+d^2)\right)\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \; 4407$$

$$\frac{\frac{\frac{\left(bc^3(2c^2+d^2)-ad(6c^4-5c^2d^2+2d^4)\right)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{c}+\frac{2ax(c^2-d^2)^2}{c}}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(bc^3(2c^2+d^2)-ad(6c^4-5c^2d^2+2d^4)\right)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}+\frac{2ax(c^2-d^2)^2}{c}}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \; 4318$$

$$\frac{\frac{\frac{\left(bc^3(2c^2+d^2)-ad(6c^4-5c^2d^2+2d^4)\right)\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{cd}+\frac{2ax(c^2-d^2)^2}{c}}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

3.191. $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx$

$$\frac{\frac{\frac{\left(bc^3\left(2c^2+d^2\right)-ad\left(6c^4-5c^2d^2+2d^4\right)\right)\int \frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{cd}+\frac{2ax\left(c^2-d^2\right)^2}{c}}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{2cf\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\frac{2\left(bc^3\left(2c^2+d^2\right)-ad\left(6c^4-5c^2d^2+2d^4\right)\right)\int \frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{cdf}+\frac{2ax\left(c^2-d^2\right)^2}{c}}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{2cf\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{2\left(bc^3\left(2c^2+d^2\right)-ad\left(6c^4-5c^2d^2+2d^4\right)\right)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\sqrt{c+d}}+\frac{2ax\left(c^2-d^2\right)^2}{c}}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{2cf\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

input
```
Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^3,x]
```

output
```
-1/2*(d*(b*c - a*d)*Tan[e + f*x])/(c*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^2)
 + (((2*a*(c^2 - d^2)^2*x)/c + (2*(b*c^3*(2*c^2 + d^2) - a*d*(6*c^4 - 5*c^
2*d^2 + 2*d^4))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c*Sq
rt[c - d]*Sqrt[c + d]*f))/(c*(c^2 - d^2)) - (d*(3*b*c^3 - 5*a*c^2*d + 2*a*
d^3)*Tan[e + f*x])/(c*(c^2 - d^2)*f*(c + d*Sec[e + f*x])))/(2*c*(c^2 - d^2
))
```

3.191.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4411
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2)
)   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
 a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && N
eQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.191. $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx$

3.191.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a\,c^2d+ac\,d^2-2a\,d^3-4b\,c^3-b\,c^2d\right)cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{dc\left(6a\,c^2d-ac\,d^2-2a\,d^3-4b\,c^3+b\,c^2d\right)\tan(\cdots}{2(c+d)(c-d)^2}\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}\cdots}{f}$
default	$\frac{\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a\,c^2d+ac\,d^2-2a\,d^3-4b\,c^3-b\,c^2d\right)cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{dc\left(6a\,c^2d-ac\,d^2-2a\,d^3-4b\,c^3+b\,c^2d\right)\tan(\cdots}{2(c+d)(c-d)^2}\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}\cdots}{f}$
risch	Expression too large to display

input
```
int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a/c^3*arctan(tan(1/2*f*x+1/2*e))+2/c^3*((-1/2*(6*a*c^2*d+a*c*d^2-2*
a*d^3-4*b*c^3-b*c^2*d)*c*d/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*
d*c*(6*a*c^2*d-a*c*d^2-2*a*d^3-4*b*c^3+b*c^2*d)/(c+d)/(c-d)^2*tan(1/2*f*x+
1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-1/2*(6*a*c^4
*d-5*a*c^2*d^3+2*a*d^5-2*b*c^5-b*c^3*d^2)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d)
)^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

3.191.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(191) = 382.

Time = 0.34 (sec) , antiderivative size = 1152, normalized size of antiderivative = 5.65

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/4*(4*(a*c^8 - 3*a*c^6*d^2 + 3*a*c^4*d^4 - a*c^2*d^6)*f*x*cos(f*x + e)^2
 + 8*(a*c^7*d - 3*a*c^5*d^3 + 3*a*c^3*d^5 - a*c*d^7)*f*x*cos(f*x + e) + 4*
(a*c^6*d^2 - 3*a*c^4*d^4 + 3*a*c^2*d^6 - a*d^8)*f*x - (2*b*c^5*d^2 - 6*a*c
^4*d^3 + b*c^3*d^4 + 5*a*c^2*d^5 - 2*a*d^7 + (2*b*c^7 - 6*a*c^6*d + b*c^5*
d^2 + 5*a*c^4*d^3 - 2*a*c^2*d^5)*cos(f*x + e)^2 + 2*(2*b*c^6*d - 6*a*c^5*d
^2 + b*c^4*d^3 + 5*a*c^3*d^4 - 2*a*c*d^6)*cos(f*x + e))*sqrt(c^2 - d^2)*lo
g((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(
d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*
d*cos(f*x + e) + d^2)) - 2*(3*b*c^6*d^2 - 5*a*c^5*d^3 - 3*b*c^4*d^4 + 7*a*
c^3*d^5 - 2*a*c*d^7 + (4*b*c^7*d - 6*a*c^6*d^2 - 5*b*c^5*d^3 + 9*a*c^4*d^4
 + b*c^3*d^5 - 3*a*c^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^2
 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d
^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)
*f), 1/2*(2*(a*c^8 - 3*a*c^6*d^2 + 3*a*c^4*d^4 - a*c^2*d^6)*f*x*cos(f*x +
e)^2 + 4*(a*c^7*d - 3*a*c^5*d^3 + 3*a*c^3*d^5 - a*c*d^7)*f*x*cos(f*x + e)
+ 2*(a*c^6*d^2 - 3*a*c^4*d^4 + 3*a*c^2*d^6 - a*d^8)*f*x + (2*b*c^5*d^2 - 6
*a*c^4*d^3 + b*c^3*d^4 + 5*a*c^2*d^5 - 2*a*d^7 + (2*b*c^7 - 6*a*c^6*d + b*
c^5*d^2 + 5*a*c^4*d^3 - 2*a*c^2*d^5)*cos(f*x + e)^2 + 2*(2*b*c^6*d - 6*a*c
^5*d^2 + b*c^4*d^3 + 5*a*c^3*d^4 - 2*a*c*d^6)*cos(f*x + e))*sqrt(-c^2 + d^
2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x +...
```

3.191.6 Sympy [F]

$$\int \frac{a + b\sec(e + fx)}{(c + d\sec(e + fx))^3}\, dx = \int \frac{a + b\sec{(e + fx)}}{(c + d\sec{(e + fx)})^3}\, dx$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x))**3, x)
```

3.191.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec(e + fx)}{(c + d\sec(e + fx))^3}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

3.191.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(191) = 382$.

Time = 0.37 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.24

$$\int \frac{a + b\sec(e + fx)}{(c + d\sec(e + fx))^3}\, dx$$
$$= \frac{\frac{(2\,bc^5-6\,ac^4d+bc^3d^2+5\,ac^2d^3-2\,ad^5)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^7-2\,c^5d^2+c^3d^4)\sqrt{-c^2+d^2}} + \frac{(fx+e)a}{c^3} + \frac{4\,bc^4d\tan}{}}{}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
((2*b*c^5 - 6*a*c^4*d + b*c^3*d^2 + 5*a*c^2*d^3 - 2*a*d^5)*(pi*floor(1/2*(
f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*t
an(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 - 2*c^5*d^2 + c^3*d^4)*sqrt(
-c^2 + d^2)) + (f*x + e)*a/c^3 + (4*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 6*a*c
^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 5*a*c
^2*d^3*tan(1/2*f*x + 1/2*e)^3 - b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 3*a*c*d
^4*tan(1/2*f*x + 1/2*e)^3 - 2*a*d^5*tan(1/2*f*x + 1/2*e)^3 - 4*b*c^4*d*tan
(1/2*f*x + 1/2*e) + 6*a*c^3*d^2*tan(1/2*f*x + 1/2*e) - 3*b*c^3*d^2*tan(1/2
*f*x + 1/2*e) + 5*a*c^2*d^3*tan(1/2*f*x + 1/2*e) + b*c^2*d^3*tan(1/2*f*x +
 1/2*e) - 3*a*c*d^4*tan(1/2*f*x + 1/2*e) - 2*a*d^5*tan(1/2*f*x + 1/2*e))/(
(c^6 - 2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/
2*e)^2 - c - d)^2))/f
```

3.191.9 Mupad [B] (verification not implemented)

Time = 24.05 (sec) , antiderivative size = 6909, normalized size of antiderivative = 33.87

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))/(c + d/cos(e + f*x))^3,x)
```

output
```
(2*a*atan(((a*((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^10
 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^
4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2
+ b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d
^5 + 8*a*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^
7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) + (a*((8*(4*a*c^15 + 4*b*c^15 - 4*a*c^6*d^9
 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4
 + 34*a*c^12*d^3 - 8*a*c^13*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3
 - 6*b*c^13*d^2 - 12*a*c^14*d - 4*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^
7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (a*tan(e/2 + (f
*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c
^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(
c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*
c^9*d^2)))*1i)/c^3))/c^3 + (a*((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8*a^2*d
^10 + 4*b^2*c^10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3
*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 +
 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d
^7 + 2*a*b*c^5*d^5 + 8*a*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 +
3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) - (a*((8*(4*a*c^15 + 4*b*c^
15 - 4*a*c^6*d^9 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10...
```

3.192 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$

3.192.1 Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \frac{a^2x}{c^2}+\frac{2(bc-ad)\left(2ac^2-bcd-ad^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f}+\frac{(bc-ad)^2\sin(e+fx)}{c\left(c^2-d^2\right)f(d+c\cos(e+fx))}$$

output
```
a^2*x/c^2+2*(-a*d+b*c)*(2*a*c^2-a*d^2-b*c*d)*arctanh((c-d)^(1/2)*tan(1/2*f
*x+1/2*e)/(c+d)^(1/2))/c^2/(c-d)^(3/2)/(c+d)^(3/2)/f+(-a*d+b*c)^2*sin(f*x+
e)/c/(c^2-d^2)/f/(d+c*cos(f*x+e))
```

3.192.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \frac{a^2(e+fx)+\frac{2\left(-2abc^3+b^2c^2d+a^2\left(2c^2d-d^3\right)\right)\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}}+\frac{c(bc-ad)^2\sin(e+fx)}{(c-d)(c+d)(d+c\cos(e+fx))}}{c^2f}$$

input
```
Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^2,x]
```

output
```
(a^2*(e + f*x) + (2*(-2*a*b*c^3 + b^2*c^2*d + a^2*(2*c^2*d - d^3))*ArcTanh
[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) + (c*(b*c
 - a*d)^2*Sin[e + f*x])/((c - d)*(c + d)*(d + c*Cos[e + f*x]))/(c^2*f)
```

3.192.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4429, 3042, 3269, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4429}$$

$$\int \frac{(a\cos(e+fx)+b)^2}{(c\cos(e+fx)+d)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^2}{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^2}dx$$

$$\Big\downarrow \text{3269}$$

$$\frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)(c\cos(e+fx)+d)}-\frac{\int -\frac{\left(c^2-d^2\right)\cos(e+fx)a^2+c\left(2abc-\left(a^2+b^2\right)d\right)}{d+c\cos(e+fx)}dx}{c\left(c^2-d^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(c^2-d^2\right)\cos(e+fx)a^2+c\left(2abc-\left(a^2+b^2\right)d\right)}{d+c\cos(e+fx)}dx}{c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)(c\cos(e+fx)+d)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)a^2+c\left(2abc-\left(a^2+b^2\right)d\right)}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)(c\cos(e+fx)+d)}$$

$$\Big\downarrow \; 3214$$

$$\frac{\frac{(bc-ad)(2ac^2-ad^2-bcd)\int \frac{1}{d+c\cos(e+fx)}dx}{c}+\frac{a^2x(c^2-d^2)}{c}}{c(c^2-d^2)}+\frac{(bc-ad)^2\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(bc-ad)(2ac^2-ad^2-bcd)\int \frac{1}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{c}+\frac{a^2x(c^2-d^2)}{c}}{c(c^2-d^2)}+\frac{(bc-ad)^2\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{2(bc-ad)(2ac^2-ad^2-bcd)\int \frac{1}{-\left((c-d)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+c+d}d\tan\left(\frac{1}{2}(e+fx)\right)}{cf}+\frac{a^2x(c^2-d^2)}{c}}{c(c^2-d^2)}+\frac{(bc-ad)^2\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{a^2x(c^2-d^2)}{c}+\frac{2(bc-ad)(2ac^2-ad^2-bcd)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\sqrt{c+d}}}{c(c^2-d^2)}+\frac{(bc-ad)^2\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}$$

input
```
Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^2,x]
```

output
```
((a^2*(c^2 - d^2)*x)/c + (2*(b*c - a*d)*(2*a*c^2 - b*c*d - a*d^2)*ArcTanh[
(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c*Sqrt[c - d]*Sqrt[c + d]*f)
)/(c*(c^2 - d^2)) + ((b*c - a*d)^2*Sin[e + f*x])/(c*(c^2 - d^2)*f*(d + c*C
os[e + f*x]))
```

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3269
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[
1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)
*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1
) + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4429
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

3.192.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^2}+\frac{-\frac{2\left(a^2d^2-2dabc+b^2c^2\right)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{2\left(2a^2c^2d-a^2d^3-2ab\,c^3+b^2c^2d\right)\operatorname{arctanh}\left(\frac{(c-d)\cdots}{\sqrt{(\cdots}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{c^2}}{f}$
default	$\frac{\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^2}+\frac{-\frac{2\left(a^2d^2-2dabc+b^2c^2\right)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{2\left(2a^2c^2d-a^2d^3-2ab\,c^3+b^2c^2d\right)\operatorname{arctanh}\left(\frac{(c-d)\cdots}{\sqrt{(\cdots}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{c^2}}{f}$
risch	$\frac{a^2x}{c^2}+\frac{2i\left(a^2d^2-2dabc+b^2c^2\right)\left(d\,e^{i(fx+e)}+c\right)}{c^2(c^2-d^2)f\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)}+\frac{2\ln\left(e^{i(fx+e)}+\frac{-ic^2+id^2+\sqrt{c^2-d^2}\,d}{c\sqrt{c^2-d^2}}\right)a^2d}{\sqrt{c^2-d^2}\,(c+d)(c-d)f}-\frac{\ln\left(e^{i(fx+e)}+\frac{-ic^2+\cdots}{\cdots}\right)\cdots}{\sqrt{c^2-d^2}\,(c-\cdots}$

3.192. $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$

input
```
int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a^2/c^2*arctan(tan(1/2*f*x+1/2*e))+2/c^2*(-(a^2*d^2-2*a*b*c*d+b^2*c
^2)*c/(c^2-d^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2
*e)^2*d-c-d)-(2*a^2*c^2*d-a^2*d^3-2*a*b*c^3+b^2*c^2*d)/(c+d)/(c-d)/((c+d)*
(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

3.192.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(124) = 248$.

Time = 0.32 (sec) , antiderivative size = 671, normalized size of antiderivative = 5.05

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$$
$$= \left[\frac{2\,(a^2c^5 - 2\,a^2c^3d^2 + a^2cd^4)fx\cos{(fx+e)} + 2\,(a^2c^4d - 2\,a^2c^2d^3 + a^2d^5)fx + (2\,abc^3d + a^2d^4 - (2\,a^2 + \right.$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(a^2*c^5 - 2*a^2*c^3*d^2 + a^2*c*d^4)*f*x*cos(f*x + e) + 2*(a^2*c^
4*d - 2*a^2*c^2*d^3 + a^2*d^5)*f*x + (2*a*b*c^3*d + a^2*d^4 - (2*a^2 + b^2
)*c^2*d^2 + (2*a*b*c^4 + a^2*c*d^3 - (2*a^2 + b^2)*c^3*d)*cos(f*x + e))*sq
rt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*s
qrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f
*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^2*c^5 - 2*a*b*c^4*d + 2*a*b*
c^2*d^3 - a^2*c*d^4 + (a^2 - b^2)*c^3*d^2)*sin(f*x + e))/((c^7 - 2*c^5*d^2
 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f), ((a^2*c^5 -
 2*a^2*c^3*d^2 + a^2*c*d^4)*f*x*cos(f*x + e) + (a^2*c^4*d - 2*a^2*c^2*d^3
+ a^2*d^5)*f*x + (2*a*b*c^3*d + a^2*d^4 - (2*a^2 + b^2)*c^2*d^2 + (2*a*b*c
^4 + a^2*c*d^3 - (2*a^2 + b^2)*c^3*d)*cos(f*x + e))*sqrt(-c^2 + d^2)*arcta
n(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b^
2*c^5 - 2*a*b*c^4*d + 2*a*b*c^2*d^3 - a^2*c*d^4 + (a^2 - b^2)*c^3*d^2)*sin
(f*x + e))/((c^7 - 2*c^5*d^2 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^
3 + c^2*d^5)*f)]
```

3.192.6 Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**2, x)
```

3.192.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.78

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \frac{\frac{(fx+e)a^2}{c^2} + \frac{2\left(2abc^3-2a^2c^2d-b^2c^2d+a^2d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^4-c^2d^2)\sqrt{-c^2+d^2}} - \frac{2\left(b^2c^2\tan\left(\frac{1}{2}\right.\right.}{(c^3-cd}}{f}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
((f*x + e)*a^2/c^2 + 2*(2*a*b*c^3 - 2*a^2*c^2*d - b^2*c^2*d + a^2*d^3)*(pi
*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x +
1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^4 - c^2*d^2)*sqrt(
-c^2 + d^2)) - 2*(b^2*c^2*tan(1/2*f*x + 1/2*e) - 2*a*b*c*d*tan(1/2*f*x + 1
/2*e) + a^2*d^2*tan(1/2*f*x + 1/2*e))/((c^3 - c*d^2)*(c*tan(1/2*f*x + 1/2*
e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)))/f
```

3.192.9 Mupad [B] (verification not implemented)

Time = 22.90 (sec) , antiderivative size = 4934, normalized size of antiderivative = 37.10

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^2,x)
```

output
```
(2*a^2*atan(((a^2*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d
^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c
^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4
*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) +
 (a^2*((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^
3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*
a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d
^2) - (a^2*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4
 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i
)/c^2))/c^2 + (a^2*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*
d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*
c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^
4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2)
- (a^2*((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d
^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2
*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*
d^2) + (a^2*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^
4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1
i)/c^2))/c^2)/((64*(a^6*d^5 - 2*a^5*b*c^5 - a^6*c*d^4 + 2*a^6*c^4*d + 4*a^
4*b^2*c^5 - 3*a^6*c^2*d^3 + 2*a^6*c^3*d^2 - 4*a^3*b^3*c^4*d - a^4*b^2*c...
```

3.193 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$

3.193.1 Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \frac{a^2x}{c^3} - \frac{(3b^2c^4d-2abc^3(2c^2+d^2)+a^2(6c^4d-5c^2d^3+2d^5))\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} - \frac{d(bc-ad)^2\sin(e+fx)}{2c^2\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} - \frac{(bc-ad)\left(3ad(2c^2-d^2)-bc(2c^2+d^2)\right)\sin(e+fx)}{2c^2\left(c^2-d^2\right)^2 f(d+c\cos(e+fx))}$$

output
```
a^2*x/c^3-(3*b^2*c^4*d-2*a*b*c^3*(2*c^2+d^2)+a^2*(6*c^4*d-5*c^2*d^3+2*d^5)
)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^3/(c-d)^(5/2)/(c+d
)^(5/2)/f-1/2*d*(-a*d+b*c)^2*sin(f*x+e)/c^2/(c^2-d^2)/f/(d+c*cos(f*x+e))^2
-1/2*(-a*d+b*c)*(3*a*d*(2*c^2-d^2)-b*c*(2*c^2+d^2))*sin(f*x+e)/c^2/(c^2-d^
2)^2/f/(d+c*cos(f*x+e))
```

3.193.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 493 vs. $2(237) = 474$.

Time = 2.69 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.08

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{(d+c\cos(e+fx))\sec(e+fx)(a+b\sec(e+fx))^2\left(\frac{4(3b^2c^4d-2abc^3(2c^2+d^2)+a^2(6c^4d-5c^2d^3+2d^5))\text{arctanh}\left(\frac{(-c+d)}{\cdots}\right.}{(c^2-d^2)^{5/2}}\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^3,x]
```

output
```
((d + c*Cos[e + f*x])*Sec[e + f*x]*(a + b*Sec[e + f*x])^2*((4*(3*b^2*c^4*d
 - 2*a*b*c^3*(2*c^2 + d^2) + a^2*(6*c^4*d - 5*c^2*d^3 + 2*d^5))*ArcTanh[((
-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^2)/(c^2 -
d^2)^(5/2) + (2*a^2*c^6*e - 6*a^2*c^2*d^4*e + 4*a^2*d^6*e + 2*a^2*c^6*f*x
- 6*a^2*c^2*d^4*f*x + 4*a^2*d^6*f*x + 8*a^2*c*d*(c^2 - d^2)^2*(e + f*x)*Co
s[e + f*x] + 2*a^2*c^2*(c^2 - d^2)^2*(e + f*x)*Cos[2*(e + f*x)] + 2*b^2*c^
5*d*Sin[e + f*x] - 12*a*b*c^4*d^2*Sin[e + f*x] + 10*a^2*c^3*d^3*Sin[e + f*
x] + 4*b^2*c^3*d^3*Sin[e + f*x] - 4*a^2*c*d^5*Sin[e + f*x] + 2*b^2*c^6*Sin
[2*(e + f*x)] - 8*a*b*c^5*d*Sin[2*(e + f*x)] + 6*a^2*c^4*d^2*Sin[2*(e + f*
x)] + b^2*c^4*d^2*Sin[2*(e + f*x)] + 2*a*b*c^3*d^3*Sin[2*(e + f*x)] - 3*a^
2*c^2*d^4*Sin[2*(e + f*x)])/(c^2 - d^2)^2))/(4*c^3*f*(b + a*Cos[e + f*x])^
2*(c + d*Sec[e + f*x])^3)
```

3.193.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4429, 3042, 3467, 25, 3042, 3500, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4429}$$

$$\int \frac{\cos(e+fx)(a\cos(e+fx)+b)^2}{(c\cos(e+fx)+d)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^2}{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^3}dx$$

$$\Big\downarrow \text{3467}$$

$$-\frac{\int -\frac{2c(bc-ad)^2+2a^2c(c^2-d^2)\cos^2(e+fx)-((2c^2d-d^3)a^2-2bc(2c^2-d^2)a+b^2c^2d)\cos(e+fx)}{(d+c\cos(e+fx))^2}dx}{2c^2(c^2-d^2)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2c(bc-ad)^2+2a^2c(c^2-d^2)\cos^2(e+fx)-((2c^2d-d^3)a^2-2bc(2c^2-d^2)a+b^2c^2d)\cos(e+fx)}{(d+c\cos(e+fx))^2}dx}{2c^2(c^2-d^2)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2c(bc-ad)^2+2a^2c(c^2-d^2)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(-((2c^2d-d^3)a^2)+2bc(2c^2-d^2)a-b^2c^2d\right)\sin\left(e+fx+\frac{\pi}{2}\right)}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{2c^2(c^2-d^2)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \text{3500}$$

$$\frac{\frac{\int \frac{(bc-ad)\left(4ac^2-3bdc-ad^2\right)c^2+2a^2\left(c^2-d^2\right)^2\cos(e+fx)c}{d+c\cos(e+fx)}dx}{c(c^2-d^2)}+\frac{(bc-ad)(-6ac^2d+3ad^3+2bc^3+bcd^2)\sin(e+fx)}{f(c^2-d^2)(c\cos(e+fx)+d)}}{2c^2(c^2-d^2)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \text{3042}$$

3.193. $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}dx$

$$\frac{\frac{\int \frac{(bc-ad)\left(4ac^2-3bdc-ad^2\right)c^2+2a^2\left(c^2-d^2\right)^2\sin\left(e+fx+\frac{\pi}{2}\right)c}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{c(c^2-d^2)}+\frac{(bc-ad)\left(-6ac^2d+3ad^3+2bc^3+bcd^2\right)\sin(e+fx)}{f(c^2-d^2)(c\cos(e+fx)+d)}}{2c^2\left(c^2-d^2\right)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$\big\downarrow$ 3214

$$\frac{\frac{2a^2x\left(c^2-d^2\right)^2-\left(a^2\left(6c^4d-5c^2d^3+2d^5\right)-2abc^3\left(2c^2+d^2\right)+3b^2c^4d\right)\int\frac{1}{d+c\cos(e+fx)}dx}{c(c^2-d^2)}+\frac{(bc-ad)\left(-6ac^2d+3ad^3+2bc^3+bcd^2\right)\sin(e+fx)}{f(c^2-d^2)(c\cos(e+fx)+d)}}{2c^2\left(c^2-d^2\right)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{2a^2x\left(c^2-d^2\right)^2-\left(a^2\left(6c^4d-5c^2d^3+2d^5\right)-2abc^3\left(2c^2+d^2\right)+3b^2c^4d\right)\int\frac{1}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{c(c^2-d^2)}+\frac{(bc-ad)\left(-6ac^2d+3ad^3+2bc^3+bcd^2\right)\sin(e+fx)}{f(c^2-d^2)(c\cos(e+fx)+d)}}{2c^2\left(c^2-d^2\right)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$\big\downarrow$ 3138

$$\frac{\frac{2a^2x\left(c^2-d^2\right)^2-\frac{2\left(a^2\left(6c^4d-5c^2d^3+2d^5\right)-2abc^3\left(2c^2+d^2\right)+3b^2c^4d\right)\int\frac{1}{-\left((c-d)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+c+d}d\tan\left(\frac{1}{2}(e+fx)\right)}{f}}{c(c^2-d^2)}+\frac{(bc-ad)\left(-6ac^2d+3ad^3+2bc^3+bcd^2\right)\sin(e+fx)}{f(c^2-d^2)(c\cos(e+fx)+d)}}{2c^2\left(c^2-d^2\right)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$\big\downarrow$ 221

$$\frac{\frac{2a^2x\left(c^2-d^2\right)^2-\frac{2\left(a^2\left(6c^4d-5c^2d^3+2d^5\right)-2abc^3\left(2c^2+d^2\right)+3b^2c^4d\right)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}}}{c(c^2-d^2)}+\frac{(bc-ad)\left(-6ac^2d+3ad^3+2bc^3+bcd^2\right)\sin(e+fx)}{f(c^2-d^2)(c\cos(e+fx)+d)}}{2c^2\left(c^2-d^2\right)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

input
```
Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^3,x]
```

3.193. $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$

output
```
-1/2*(d*(b*c - a*d)^2*Sin[e + f*x])/(c^2*(c^2 - d^2)*f*(d + c*Cos[e + f*x]
)^2) + ((2*a^2*(c^2 - d^2)^2*x - (2*(3*b^2*c^4*d - 2*a*b*c^3*(2*c^2 + d^2)
 + a^2*(6*c^4*d - 5*c^2*d^3 + 2*d^5))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2
])/Sqrt[c + d]])/(Sqrt[c - d]*Sqrt[c + d]*f))/(c*(c^2 - d^2)) + ((b*c - a*
d)*(2*b*c^3 - 6*a*c^2*d + b*c*d^2 + 3*a*d^3)*Sin[e + f*x])/((c^2 - d^2)*f*
(d + c*Cos[e + f*x])))/(2*c^2*(c^2 - d^2))
```

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

3.193. $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4429
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

3.193.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.63

method	result
derivativedivides	$\frac{\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a^2c^2d^2+a^2d^3c-2a^2d^4-8ab\,c^3d-2ab\,c^2d^2+2b^2c^4+b^2c^3d+2b^2c^2d^2\right)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{c\left(6a^2c^2\cdots\right.}{\cdots}\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\cdots\right.}}{\cdots}$
default	$\frac{\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a^2c^2d^2+a^2d^3c-2a^2d^4-8ab\,c^3d-2ab\,c^2d^2+2b^2c^4+b^2c^3d+2b^2c^2d^2\right)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{c\left(6a^2c^2\cdots\right.}{\cdots}\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\cdots\right.}}{\cdots}$
risch	Expression too large to display

input
```
int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a^2/c^3*arctan(tan(1/2*f*x+1/2*e))+2/c^3*((-1/2*(6*a^2*c^2*d^2+a^2*
c*d^3-2*a^2*d^4-8*a*b*c^3*d-2*a*b*c^2*d^2+2*b^2*c^4+b^2*c^3*d+2*b^2*c^2*d^
2)*c/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*c*(6*a^2*c^2*d^2-a^2*c
*d^3-2*a^2*d^4-8*a*b*c^3*d+2*a*b*c^2*d^2+2*b^2*c^4-b^2*c^3*d+2*b^2*c^2*d^2
)/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/
2*e)^2*d-c-d)^2-1/2*(6*a^2*c^4*d-5*a^2*c^2*d^3+2*a^2*d^5-4*a*b*c^5-2*a*b*c
^3*d^2+3*b^2*c^4*d)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*
tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

3.193.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. 2(223) = 446.

Time = 0.36 (sec) , antiderivative size = 1409, normalized size of antiderivative = 5.95

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2*c^8 - 3*a^2*c^6*d^2 + 3*a^2*c^4*d^4 - a^2*c^2*d^6)*f*x*cos(f*
x + e)^2 + 8*(a^2*c^7*d - 3*a^2*c^5*d^3 + 3*a^2*c^3*d^5 - a^2*c*d^7)*f*x*c
os(f*x + e) + 4*(a^2*c^6*d^2 - 3*a^2*c^4*d^4 + 3*a^2*c^2*d^6 - a^2*d^8)*f*
x - (4*a*b*c^5*d^2 + 2*a*b*c^3*d^4 + 5*a^2*c^2*d^5 - 2*a^2*d^7 - 3*(2*a^2
+ b^2)*c^4*d^3 + (4*a*b*c^7 + 2*a*b*c^5*d^2 + 5*a^2*c^4*d^3 - 2*a^2*c^2*d^
5 - 3*(2*a^2 + b^2)*c^6*d)*cos(f*x + e)^2 + 2*(4*a*b*c^6*d + 2*a*b*c^4*d^3
 + 5*a^2*c^3*d^4 - 2*a^2*c*d^6 - 3*(2*a^2 + b^2)*c^5*d^2)*cos(f*x + e))*sq
rt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*s
qrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f
*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^2*c^7*d - 6*a*b*c^6*d^2 + 6*
a*b*c^4*d^4 + 2*a^2*c*d^7 + (5*a^2 + b^2)*c^5*d^3 - (7*a^2 + 2*b^2)*c^3*d^
5 + (2*b^2*c^8 - 8*a*b*c^7*d + 10*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + 3*a^2*c^2*
d^6 + (6*a^2 - b^2)*c^6*d^2 - (9*a^2 + b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x
 + e))/((c^11 - 3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^1
0*d - 3*c^8*d^3 + 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d
^4 + 3*c^5*d^6 - c^3*d^8)*f), 1/2*(2*(a^2*c^8 - 3*a^2*c^6*d^2 + 3*a^2*c^4*
d^4 - a^2*c^2*d^6)*f*x*cos(f*x + e)^2 + 4*(a^2*c^7*d - 3*a^2*c^5*d^3 + 3*a
^2*c^3*d^5 - a^2*c*d^7)*f*x*cos(f*x + e) + 2*(a^2*c^6*d^2 - 3*a^2*c^4*d^4
+ 3*a^2*c^2*d^6 - a^2*d^8)*f*x + (4*a*b*c^5*d^2 + 2*a*b*c^3*d^4 + 5*a^2*c^
2*d^5 - 2*a^2*d^7 - 3*(2*a^2 + b^2)*c^4*d^3 + (4*a*b*c^7 + 2*a*b*c^5*d^...
```

3.193.6 Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \int \frac{(a+b\sec{(e+fx)})^2}{(c+d\sec{(e+fx)})^3}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**3, x)
```

3.193.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\sec(e + fx))^2}{(c + d\sec(e + fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

3.193.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(223) = 446$.

Time = 0.40 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.78

$$\int \frac{(a + b\sec(e + fx))^2}{(c + d\sec(e + fx))^3}\,dx$$
$$= \frac{\frac{(4\,abc^5 - 6\,a^2c^4d - 3\,b^2c^4d + 2\,abc^3d^2 + 5\,a^2c^2d^3 - 2\,a^2d^5)\left(\pi\left\lfloor \frac{fx+e}{2\,\pi} + \frac{1}{2}\right\rfloor \operatorname{sgn}(-2\,c + 2\,d) + \arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - d\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)}{\sqrt{-c^2 + d^2}}\right)\right)}{(c^7 - 2\,c^5d^2 + c^3d^4)\sqrt{-c^2 + d^2}} + \frac{(fx+e}{c^3}}{}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
((4*a*b*c^5 - 6*a^2*c^4*d - 3*b^2*c^4*d + 2*a*b*c^3*d^2 + 5*a^2*c^2*d^3 -
2*a^2*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*
tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 -
2*c^5*d^2 + c^3*d^4)*sqrt(-c^2 + d^2)) + (f*x + e)*a^2/c^3 - (2*b^2*c^5*ta
n(1/2*f*x + 1/2*e)^3 - 8*a*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - b^2*c^4*d*tan(
1/2*f*x + 1/2*e)^3 + 6*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*a*b*c^3*d^2*
tan(1/2*f*x + 1/2*e)^3 + b^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 5*a^2*c^2*d^
3*tan(1/2*f*x + 1/2*e)^3 + 2*a*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b^2*c^
2*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*a^2*
d^5*tan(1/2*f*x + 1/2*e)^3 - 2*b^2*c^5*tan(1/2*f*x + 1/2*e) + 8*a*b*c^4*d*
tan(1/2*f*x + 1/2*e) - b^2*c^4*d*tan(1/2*f*x + 1/2*e) - 6*a^2*c^3*d^2*tan(
1/2*f*x + 1/2*e) + 6*a*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - b^2*c^3*d^2*tan(1/
2*f*x + 1/2*e) - 5*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 2*a*b*c^2*d^3*tan(1/
2*f*x + 1/2*e) - 2*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) + 3*a^2*c*d^4*tan(1/2*
f*x + 1/2*e) + 2*a^2*d^5*tan(1/2*f*x + 1/2*e))/((c^6 - 2*c^4*d^2 + c^2*d^4
)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

3.193.9 Mupad [B] (verification not implemented)

Time = 24.42 (sec) , antiderivative size = 8682, normalized size of antiderivative = 36.63

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^3,x)
```

output
```
((tan(e/2 + (f*x)/2)^3*(2*b^2*c^4 - 2*a^2*d^4 + a^2*c*d^3 + b^2*c^3*d + 6*
a^2*c^2*d^2 + 2*b^2*c^2*d^2 - 8*a*b*c^3*d - 2*a*b*c^2*d^2))/((c^2*d - c^3)
*(c + d)^2) - (tan(e/2 + (f*x)/2)*(2*a^2*d^4 - 2*b^2*c^4 + a^2*c*d^3 + b^2
*c^3*d - 6*a^2*c^2*d^2 - 2*b^2*c^2*d^2 + 8*a*b*c^3*d - 2*a*b*c^2*d^2))/((c
 + d)*(c^4 - 2*c^3*d + c^2*d^2)))/(f*(2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2
- 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d + d^2) + c^2 + d^2)) - (2*a^2
*atan(((a^2*((a^2*((8*(4*a^2*c^15 - 12*a^2*c^14*d - 6*b^2*c^14*d - 4*a^2*c
^6*d^9 + 2*a^2*c^7*d^8 + 18*a^2*c^8*d^7 - 4*a^2*c^9*d^6 - 36*a^2*c^10*d^5
+ 6*a^2*c^11*d^4 + 34*a^2*c^12*d^3 - 8*a^2*c^13*d^2 + 6*b^2*c^9*d^6 - 6*b^
2*c^10*d^5 - 12*b^2*c^11*d^4 + 12*b^2*c^12*d^3 + 6*b^2*c^13*d^2 + 8*a*b*c^
15 - 8*a*b*c^14*d - 4*a*b*c^8*d^7 + 4*a*b*c^9*d^6 + 12*a*b*c^12*d^3 - 12*a
*b*c^13*d^2))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 -
 3*c^10*d^3 - 3*c^11*d^2) - (a^2*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10
 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^
12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5
*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3 + (8*tan(e
/2 + (f*x)/2)*(4*a^4*c^10 + 8*a^4*d^10 - 8*a^4*c*d^9 - 8*a^4*c^9*d + 16*a^
2*b^2*c^10 - 32*a^4*c^2*d^8 + 32*a^4*c^3*d^7 + 57*a^4*c^4*d^6 - 48*a^4*c^5
*d^5 - 52*a^4*c^6*d^4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^2 -
12*a*b^3*c^7*d^3 - 8*a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3...
```

3.193. $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$

3.194 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

3.194.1 Optimal result

Integrand size = 25, antiderivative size = 377

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \frac{a^2x}{c^4} - \frac{(b^2c^4d(4c^2+d^2)-ab(4c^7+6c^5d^2)+a^2(8c^6d-8c^4d^3+7c^2d^5-2d^7))\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^4(c-d)^{7/2}(c+d)^{7/2}f} + \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)(6bc^3-8ac^2d-bcd^2+3ad^3)\sin(e+fx)}{6c^3(c^2-d^2)^2f(d+c\cos(e+fx))^2} - \frac{(2abcd(18c^4-5c^2d^2+2d^4)-a^2d^2(34c^4-28c^2d^2+9d^4)-b^2(6c^6+10c^4d^2-c^2d^4))\sin(e+fx)}{6c^3(c^2-d^2)^3f(d+c\cos(e+fx))}$$

output
```
a^2*x/c^4-(b^2*c^4*d*(4*c^2+d^2)-a*b*(4*c^7+6*c^5*d^2)+a^2*(8*c^6*d-8*c^4*
d^3+7*c^2*d^5-2*d^7))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/
c^4/(c-d)^(7/2)/(c+d)^(7/2)/f+1/3*d^2*(b+a*cos(f*x+e))^2*sin(f*x+e)/c/(c^2
-d^2)/f/(d+c*cos(f*x+e))^3-1/6*d*(-a*d+b*c)*(-8*a*c^2*d+3*a*d^3+6*b*c^3-b*
c*d^2)*sin(f*x+e)/c^3/(c^2-d^2)^2/f/(d+c*cos(f*x+e))^2-1/6*(2*a*b*c*d*(18*
c^4-5*c^2*d^2+2*d^4)-a^2*d^2*(34*c^4-28*c^2*d^2+9*d^4)-b^2*(6*c^6+10*c^4*d
^2-c^2*d^4))*sin(f*x+e)/c^3/(c^2-d^2)^3/f/(d+c*cos(f*x+e))
```

3.194.2 Mathematica [A] (verified)

Time = 4.42 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{(d+c\cos(e+fx))\sec^2(e+fx)(a+b\sec(e+fx))^2\left(6a^2(e+fx)(d+c\cos(e+fx))^3+\frac{6(b^2c^4d(4c^2+d^2)-2\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^4,x]
```

output
```
((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^2*(6*a^2*(e + f*
x)*(d + c*Cos[e + f*x])^3 + (6*(b^2*c^4*d*(4*c^2 + d^2) - 2*a*b*c^5*(2*c^2
 + 3*d^2) + a^2*(8*c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTanh[((-c +
d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^3)/(c^2 - d^2)^
(7/2) + (2*c*d^2*(b*c - a*d)^2*Sin[e + f*x])/(c^2 - d^2) - (c*d*(a^2*d^2*(
12*c^2 - 7*d^2) + b^2*(6*c^4 - c^2*d^2) + a*b*(-18*c^3*d + 8*c*d^3))*(d +
c*Cos[e + f*x])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(-2*a*b*c*d*(18*c^4 - 5*c
^2*d^2 + 2*d^4) + a^2*d^2*(36*c^4 - 32*c^2*d^2 + 11*d^4) + b^2*(6*c^6 + 10
*c^4*d^2 - c^2*d^4))*(d + c*Cos[e + f*x])^2*Sin[e + f*x])/(c^2 - d^2)^3))/
(6*c^4*f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^4)
```

3.194.3 Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.15, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4429, 3042, 3527, 25, 3042, 3510, 3042, 3500, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \; 4429$$

$$\int \frac{\cos^2(e+fx)(a\cos(e+fx)+b)^2}{(c\cos(e+fx)+d)^4}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^2\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^2}{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^4}dx$$

$$\Big\downarrow \; 3527$$

$$\frac{\int -\frac{(b+a\cos(e+fx))\left(-3a\left(c^2-d^2\right)\cos^2(e+fx)-\left(3bc^2-3adc-bd^2\right)\cos(e+fx)+d(3bc-2ad)\right)}{(d+c\cos(e+fx))^3}dx}{3c\left(c^2-d^2\right)}+\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\int \frac{(b+a\cos(e+fx))\left(-3a\left(c^2-d^2\right)\cos^2(e+fx)+\left(3acd-b\left(3c^2-d^2\right)\right)\cos(e+fx)+d(3bc-2ad)\right)}{(d+c\cos(e+fx))^3}dx}{3c\left(c^2-d^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\int \frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(-3a\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(3acd-b\left(3c^2-d^2\right)\right)\sin\left(e+fx+\frac{\pi}{2}\right)+d(3bc-2ad)\right)}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx}{3c\left(c^2-d^2\right)}$$

$$\Big\downarrow \; 3510$$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\int \frac{6a^2c\left(c^2-d^2\right)^2\cos^2(e+fx)-\left(\left(3d^5-10c^2d^3+12c^4d\right)a^2-2bc\left(6c^4-3d^2c^2+2d^4\right)a+b^2c^2d\left(6c^2-d^2\right)\right) \cdots}{(d+c\cos(e+fx))^2} \cdots}{2c^2(c^2-d^2)} \cdots}{3c\left(c^2-d^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\int \frac{6a^2c\left(c^2-d^2\right)^2\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(-\left(\left(3d^5-10c^2d^3+12c^4d\right)a^2\right)+2bc\left(6c^4-3d^2c^2+2d^4\right)a-b^2c^2d\left(6c \cdots\right)\right)}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right) \cdots} \cdots}{2c^2(c^2-d^2)} \cdots}{3c\left(c^2-d^2\right)}$$

3.194. $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

$\downarrow$ 3500

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\frac{\int -\frac{3\left(c^2\left(-2ab\left(2c^2+3d^2\right)c^3+b^2d\left(4c^2+d^2\right)c^2+a^2\left(d^5-2c^2d^3+6c^4d\right)\right)-2a^2c\left(c^2-d^2\right)^3\cos(e+fx)\right)}{d+c\cos(e+fx)}dx}{c\left(c^2-d^2\right)}}{2c^2(c^2}}{3c\left(c^2-d^2\right)}$$

$\downarrow$ 27

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{-\frac{3\int\frac{c^2\left(-2ab\left(2c^2+3d^2\right)c^3+b^2d\left(4c^2+d^2\right)c^2+a^2d\left(6c^4-2d^2c^2+d^4\right)\right)-2a^2c\left(c^2-d^2\right)^3\cos(e+fx)}{d+c\cos(e+fx)}dx}{c\left(c^2-d^2\right)}-}{2c^2(c^2-}}{3c\left(c^2-d^2\right)}$$

$\downarrow$ 3042

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{-\frac{3\int\frac{c^2\left(-2ab\left(2c^2+3d^2\right)c^3+b^2d\left(4c^2+d^2\right)c^2+a^2d\left(6c^4-2d^2c^2+d^4\right)\right)-2a^2c\left(c^2-d^2\right)^3\sin\left(e+fx+\frac{\pi}{2}\right)}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}{c\left(c^2-d^2\right)}}{2c^2(c^2}}{3c\left(c^2-d^2\right)}$$

$\downarrow$ 3214

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{-\frac{3\left(\left(a^2\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-2abc^5\left(2c^2+3d^2\right)+b^2c^4d\left(4c^2+d^2\right)\right)\int\frac{1}{d+c\cos(e+fx)}dx-2a^2x\left(\right.}{c\left(c^2-d^2\right)}}{2c^}}{3c\left(c^2-d^2\right)}$$

$\downarrow$ 3042

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{-\frac{3\left(\left(a^2\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-2abc^5\left(2c^2+3d^2\right)+b^2c^4d\left(4c^2+d^2\right)\right)\int\frac{1}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx-2a\right.}{c\left(c^2-d^2\right)}}{}}{3c\left(c^2-d^2\right)}$$

$\downarrow$ 3138

3.194. $\int\frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{-\frac{3\left(\frac{2\left(a^2\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-2abc^5\left(2c^2+3d^2\right)+b^2c^4d\left(4c^2+d^2\right)\right)\int\frac{1}{-\left((c-d)\tan^2\left(\frac{1}{2}(e+fx)\right)\cdots\right.}\cdots}{f}\cdots\right.}{c\left(c^2-d^2\right)}\cdots}{3c\left(c^2-d^2\right)\cdots}$$

$\downarrow$ 221

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{-\frac{3\left(\frac{2\left(a^2\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-2abc^5\left(2c^2+3d^2\right)+b^2c^4d\left(4c^2+d^2\right)\right)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}\cdots\right)}{\sqrt{c+d}}\cdots\right)}{f\sqrt{c-d}\sqrt{c+d}}\cdots\right.}{c\left(c^2-d^2\right)}\cdots}{3c\left(c^2-d^2\right)}$$

input
```
Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^4,x]
```

output
```
(d^2*(b + a*Cos[e + f*x])^2*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e
+ f*x])^3) - ((d*(b*c - a*d)*(6*b*c^3 - 8*a*c^2*d - b*c*d^2 + 3*a*d^3)*Sin
[e + f*x])/(2*c^2*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^2) - ((-3*(-2*a^2*(c^
2 - d^2)^3*x + (2*(b^2*c^4*d*(4*c^2 + d^2) - 2*a*b*c^5*(2*c^2 + 3*d^2) + a
^2*(8*c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTanh[(Sqrt[c - d]*Tan[(e
+ f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*Sqrt[c + d]*f)))/(c*(c^2 - d^2)) - (
(2*a*b*c*d*(18*c^4 - 5*c^2*d^2 + 2*d^4) - a^2*d^2*(34*c^4 - 28*c^2*d^2 + 9
*d^4) - b^2*(6*c^6 + 10*c^4*d^2 - c^2*d^4))*Sin[e + f*x])/((c^2 - d^2)*f*(
d + c*Cos[e + f*x])))/(2*c^2*(c^2 - d^2)))/(3*c*(c^2 - d^2))
```

3.194.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

3.194. $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4429
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

3.194.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.68

method	result
derivativedivides	$\frac{\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^2c^4d^2+4a^2c^3d^3-6a^2d^4c^2-a^2c\,d^5+2a^2d^6-12dab\,c^5-6ab\,c^4d^2-4ab\,d^3c^3+2b^2c^6+2b^2c^5d+6b\right.}{2(c-d)\left(c^3+3c^2d+3c\,d^2+d^3\right)}\right.}{}}{}$
default	$\frac{\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^2c^4d^2+4a^2c^3d^3-6a^2d^4c^2-a^2c\,d^5+2a^2d^6-12dab\,c^5-6ab\,c^4d^2-4ab\,d^3c^3+2b^2c^6+2b^2c^5d+6b\right.}{2(c-d)\left(c^3+3c^2d+3c\,d^2+d^3\right)}\right.}{}}{}$
risch	Expression too large to display

input
```
int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a^2/c^4*arctan(tan(1/2*f*x+1/2*e))+2/c^4*((-1/2*(12*a^2*c^4*d^2+4*a
^2*c^3*d^3-6*a^2*c^2*d^4-a^2*c*d^5+2*a^2*d^6-12*a*b*c^5*d-6*a*b*c^4*d^2-4*
a*b*c^3*d^3+2*b^2*c^6+2*b^2*c^5*d+6*b^2*c^4*d^2+b^2*c^3*d^3)*c/(c-d)/(c^3+
3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5+2/3*(18*a^2*c^4*d^2-11*a^2*c^2*d
^4+3*a^2*d^6-18*a*b*c^5*d-2*a*b*c^3*d^3+3*b^2*c^6+7*b^2*c^4*d^2)*c/(c^2-2*
c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1/2*(12*a^2*c^4*d^2-4*a^2*c^
3*d^3-6*a^2*c^2*d^4+a^2*c*d^5+2*a^2*d^6-12*a*b*c^5*d+6*a*b*c^4*d^2-4*a*b*c
^3*d^3+2*b^2*c^6-2*b^2*c^5*d+6*b^2*c^4*d^2-b^2*c^3*d^3)*c/(c+d)/(c^3-3*c^2
*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/
2*e)^2*d-c-d)^3-1/2*(8*a^2*c^6*d-8*a^2*c^4*d^3+7*a^2*c^2*d^5-2*a^2*d^7-4*a
*b*c^7-6*a*b*c^5*d^2+4*b^2*c^6*d+b^2*c^4*d^3)/(c^6-3*c^4*d^2+3*c^2*d^4-d^6
)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)
)))
```

3.194.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1152 vs. $2(362) = 724$.

Time = 0.45 (sec) , antiderivative size = 2362, normalized size of antiderivative = 6.27

$$\int \frac{(a + b\sec(e + fx))^2}{(c + d\sec(e + fx))^4} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
[1/12*(12*(a^2*c^11 - 4*a^2*c^9*d^2 + 6*a^2*c^7*d^4 - 4*a^2*c^5*d^6 + a^2*
c^3*d^8)*f*x*cos(f*x + e)^3 + 36*(a^2*c^10*d - 4*a^2*c^8*d^3 + 6*a^2*c^6*d
^5 - 4*a^2*c^4*d^7 + a^2*c^2*d^9)*f*x*cos(f*x + e)^2 + 36*(a^2*c^9*d^2 - 4
*a^2*c^7*d^4 + 6*a^2*c^5*d^6 - 4*a^2*c^3*d^8 + a^2*c*d^10)*f*x*cos(f*x + e
) + 12*(a^2*c^8*d^3 - 4*a^2*c^6*d^5 + 6*a^2*c^4*d^7 - 4*a^2*c^2*d^9 + a^2*
d^11)*f*x - 3*(4*a*b*c^7*d^3 + 6*a*b*c^5*d^5 - 7*a^2*c^2*d^8 + 2*a^2*d^10
- 4*(2*a^2 + b^2)*c^6*d^4 + (8*a^2 - b^2)*c^4*d^6 + (4*a*b*c^10 + 6*a*b*c^
8*d^2 - 7*a^2*c^5*d^5 + 2*a^2*c^3*d^7 - 4*(2*a^2 + b^2)*c^9*d + (8*a^2 - b
^2)*c^7*d^3)*cos(f*x + e)^3 + 3*(4*a*b*c^9*d + 6*a*b*c^7*d^3 - 7*a^2*c^4*d
^6 + 2*a^2*c^2*d^8 - 4*(2*a^2 + b^2)*c^8*d^2 + (8*a^2 - b^2)*c^6*d^4)*cos(
f*x + e)^2 + 3*(4*a*b*c^8*d^2 + 6*a*b*c^6*d^4 - 7*a^2*c^3*d^7 + 2*a^2*c*d^
9 - 4*(2*a^2 + b^2)*c^7*d^3 + (8*a^2 - b^2)*c^5*d^5)*cos(f*x + e))*sqrt(c^
2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c
^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x +
e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*b^2*c^9*d^2 - 22*a*b*c^8*d^3 + 14
*a*b*c^6*d^5 + 8*a*b*c^4*d^7 + 23*a^2*c^3*d^8 - 6*a^2*c*d^10 + (26*a^2 + 1
1*b^2)*c^7*d^4 - (43*a^2 + 13*b^2)*c^5*d^6 + (6*b^2*c^11 - 36*a*b*c^10*d +
 46*a*b*c^8*d^3 - 14*a*b*c^6*d^5 + 4*a*b*c^4*d^7 - 11*a^2*c^3*d^8 + 4*(9*a
^2 + b^2)*c^9*d^2 - (68*a^2 + 11*b^2)*c^7*d^4 + (43*a^2 + b^2)*c^5*d^6)*co
s(f*x + e)^2 + 3*(2*b^2*c^10*d - 18*a*b*c^9*d^2 + 16*a*b*c^7*d^4 + 2*a*...
```

3.194.6 Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx=\int \frac{(a+b\sec{(e+fx)})^2}{(c+d\sec{(e+fx)})^4}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**4,x)
```

output
```
Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**4, x)
```

3.194.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

3.194.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. $2(362) = 724$.

Time = 0.40 (sec) , antiderivative size = 1201, normalized size of antiderivative = 3.19

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/3*(3*(4*a*b*c^7 - 8*a^2*c^6*d - 4*b^2*c^6*d + 6*a*b*c^5*d^2 + 8*a^2*c^4*
d^3 - b^2*c^4*d^3 - 7*a^2*c^2*d^5 + 2*a^2*d^7)*(pi*floor(1/2*(f*x + e)/pi
+ 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x +
 1/2*e))/sqrt(-c^2 + d^2)))/((c^10 - 3*c^8*d^2 + 3*c^6*d^4 - c^4*d^6)*sqrt
(-c^2 + d^2)) + 3*(f*x + e)*a^2/c^4 - (6*b^2*c^8*tan(1/2*f*x + 1/2*e)^5 -
36*a*b*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 6*b^2*c^7*d*tan(1/2*f*x + 1/2*e)^5 +
 36*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 54*a*b*c^6*d^2*tan(1/2*f*x + 1/2*
e)^5 + 12*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^2*c^5*d^3*tan(1/2*f*x
+ 1/2*e)^5 - 12*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 27*b^2*c^5*d^3*tan(1/
2*f*x + 1/2*e)^5 - 6*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*c^4*d^4*ta
n(1/2*f*x + 1/2*e)^5 + 12*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 45*a^2*c^3*
d^5*tan(1/2*f*x + 1/2*e)^5 - 12*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 + 3*b^2
*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 - 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 1
5*a^2*c*d^7*tan(1/2*f*x + 1/2*e)^5 + 6*a^2*d^8*tan(1/2*f*x + 1/2*e)^5 - 12
*b^2*c^8*tan(1/2*f*x + 1/2*e)^3 + 72*a*b*c^7*d*tan(1/2*f*x + 1/2*e)^3 - 72
*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 16*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^
3 - 64*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 + 116*a^2*c^4*d^4*tan(1/2*f*x +
1/2*e)^3 + 28*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*a*b*c^3*d^5*tan(1/2*f
*x + 1/2*e)^3 - 56*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 12*a^2*d^8*tan(1/2
*f*x + 1/2*e)^3 + 6*b^2*c^8*tan(1/2*f*x + 1/2*e) - 36*a*b*c^7*d*tan(1/2...
```

3.194.9 Mupad [B] (verification not implemented)

Time = 27.00 (sec) , antiderivative size = 12818, normalized size of antiderivative = 34.00

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^4,x)
```

output
```
(2*a^2*atan(((a^2*((8*tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*
c*d^13 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^1
1 + 117*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7
 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 +
 44*a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b
^3*c^9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*
a^3*b*c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c^6*d^8
 + 40*a^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*
b^3*c^13*d - 64*a^3*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^
8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4
- 5*c^14*d^3 - 5*c^15*d^2) + (a^2*((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*
c^20*d - 4*a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*
d^10 - 70*a^2*c^12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*
d^6 - 110*a^2*c^16*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d
^2 - 2*b^2*c^11*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 +
18*b^2*c^15*d^6 - 18*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*
b^2*c^19*d^2 + 8*a*b*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d
^8 - 28*a*b*c^14*d^7 + 28*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*c^17*d^4
 + 12*a*b*c^18*d^3 - 12*a*b*c^19*d^2))/(c^19*d + c^20 - c^9*d^11 - c^10*d^
10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 ...
```

3.194. $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

3.195 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$

3.195.1 Optimal result

Integrand size = 25, antiderivative size = 254

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \frac{a^3x}{c^3} - \frac{(bc-ad)\left(2abcd(4c^2-d^2)-b^2c^2(c^2+2d^2)-a^2(6c^4-5c^2d^2+2d^4)\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} + \frac{(bc-ad)^2(b+a\cos(e+fx))\sin(e+fx)}{2c\,(c^2-d^2)\,f(d+c\cos(e+fx))^2} + \frac{(bc-ad)^2\,(5ac^2-3bcd-2ad^2)\sin(e+fx)}{2c^2\,(c^2-d^2)^2\,f(d+c\cos(e+fx))}$$

output
```
a^3*x/c^3-(-a*d+b*c)*(2*a*b*c*d*(4*c^2-d^2)-b^2*c^2*(c^2+2*d^2)-a^2*(6*c^4
-5*c^2*d^2+2*d^4))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^3
/(c-d)^(5/2)/(c+d)^(5/2)/f+1/2*(-a*d+b*c)^2*(b+a*cos(f*x+e))*sin(f*x+e)/c/
(c^2-d^2)/f/(d+c*cos(f*x+e))^2+1/2*(-a*d+b*c)^2*(5*a*c^2-2*a*d^2-3*b*c*d)*
sin(f*x+e)/c^2/(c^2-d^2)^2/f/(d+c*cos(f*x+e))
```

3.195.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 517 vs. $2(254) = 508$.

Time = 2.25 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \frac{-\frac{4\left(-9ab^2c^4d+3a^2bc^3(2c^2+d^2)+b^3c^3(c^2+2d^2)+a^3(-6c^4d+5c^2d^3-2d^5)\right)\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{5/2}} + \frac{2a^3c^6e-6a^3c^2d^4e+4a^3d^6e+\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^3,x]
```

output
```
((-4*(-9*a*b^2*c^4*d + 3*a^2*b*c^3*(2*c^2 + d^2) + b^3*c^3*(c^2 + 2*d^2) +
 a^3*(-6*c^4*d + 5*c^2*d^3 - 2*d^5))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/S
qrt[c^2 - d^2]])/(c^2 - d^2)^(5/2) + (2*a^3*c^6*e - 6*a^3*c^2*d^4*e + 4*a^
3*d^6*e + 2*a^3*c^6*f*x - 6*a^3*c^2*d^4*f*x + 4*a^3*d^6*f*x + 8*a^3*c*d*(c
^2 - d^2)^2*(e + f*x)*Cos[e + f*x] + 2*a^3*(c^3 - c*d^2)^2*(e + f*x)*Cos[2
*(e + f*x)] + 2*b^3*c^6*Sin[e + f*x] + 6*a*b^2*c^5*d*Sin[e + f*x] - 18*a^2
*b*c^4*d^2*Sin[e + f*x] - 8*b^3*c^4*d^2*Sin[e + f*x] + 10*a^3*c^3*d^3*Sin[
e + f*x] + 12*a*b^2*c^3*d^3*Sin[e + f*x] - 4*a^3*c*d^5*Sin[e + f*x] + 6*a*
b^2*c^6*Sin[2*(e + f*x)] - 12*a^2*b*c^5*d*Sin[2*(e + f*x)] - 3*b^3*c^5*d*S
in[2*(e + f*x)] + 6*a^3*c^4*d^2*Sin[2*(e + f*x)] + 3*a*b^2*c^4*d^2*Sin[2*(
e + f*x)] + 3*a^2*b*c^3*d^3*Sin[2*(e + f*x)] - 3*a^3*c^2*d^4*Sin[2*(e + f*
x)])/((c^2 - d^2)^2*(d + c*Cos[e + f*x])^2))/(4*c^3*f)
```

3.195.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4429, 3042, 3271, 3042, 3500, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4429}$$

$$\int \frac{(a\cos(e+fx)+b)^3}{(c\cos(e+fx)+d)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^3}{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^3}dx$$

$$\Big\downarrow \text{3271}$$

$$\frac{\int \frac{d^2a^3+2(c^2-d^2)\cos^2(e+fx)a^3-4bcda^2+5b^2c^2a-2b^3cd+(-2cda^3+b(6c^2-d^2)a^2-4b^2cda+b^3c^2)\cos(e+fx)}{(d+c\cos(e+fx))^2}dx}{2c(c^2-d^2)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf(c^2-d^2)(c\cos(e+fx)+d)^2}$$

$\downarrow$ 3042

$$\frac{\int \frac{d^2a^3+2(c^2-d^2)\sin\left(e+fx+\frac{\pi}{2}\right)^2a^3-4bcda^2+5b^2c^2a-2b^3cd+(-2cda^3+b(6c^2-d^2)a^2-4b^2cda+b^3c^2)\sin\left(e+fx+\frac{\pi}{2}\right)}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{2c(c^2-d^2)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf(c^2-d^2)(c\cos(e+fx)+d)^2}$$

$\downarrow$ 3500

$$\frac{\frac{\int -\frac{c\left(\left(4c^2d-d^3\right)a^3-3bc\left(2c^2+d^2\right)a^2+9b^2c^2da-b^3c\left(c^2+2d^2\right)\right)-2a^3\left(c^2-d^2\right)^2\cos(e+fx)}{d+c\cos(e+fx)}dx}{c(c^2-d^2)}+\frac{(5ac^2-2ad^2-3bcd)(bc-ad)^2\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}}{2c(c^2-d^2)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf(c^2-d^2)(c\cos(e+fx)+d)^2}$$

$\downarrow$ 25

$$\frac{\frac{(bc-ad)^2(5ac^2-2ad^2-3bcd)\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}-\frac{\int \frac{c\left(\left(4c^2d-d^3\right)a^3-3bc\left(2c^2+d^2\right)a^2+9b^2c^2da-b^3c\left(c^2+2d^2\right)\right)-2a^3\left(c^2-d^2\right)^2\cos(e+fx)}{d+c\cos(e+fx)}dx}{c(c^2-d^2)}}{2c(c^2-d^2)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf(c^2-d^2)(c\cos(e+fx)+d)^2}$$

$\downarrow$ 3042

$$\frac{\frac{(bc-ad)^2(5ac^2-2ad^2-3bcd)\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}-\frac{\int \frac{c\left(\left(4c^2d-d^3\right)a^3-3bc\left(2c^2+d^2\right)a^2+9b^2c^2da-b^3c\left(c^2+2d^2\right)\right)-2a^3\left(c^2-d^2\right)^2\sin\left(e+fx+\frac{\pi}{2}\right)}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{c(c^2-d^2)}}{2c(c^2-d^2)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf(c^2-d^2)(c\cos(e+fx)+d)^2}$$

$\downarrow$ 3214

$$\frac{\frac{(bc-ad)^2(5ac^2-2ad^2-3bcd)\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}-\frac{\frac{\left(a^3\left(6c^4d-5c^2d^3+2d^5\right)-3a^2bc^3\left(2c^2+d^2\right)+9ab^2c^4d-b^3c^3\left(c^2+2d^2\right)\right)\int \frac{1}{d+c\cos(e+fx)}dx}{c}-\frac{2a^3x\left(c^2-d^2\right)^2}{c}}{c(c^2-d^2)}}{2c(c^2-d^2)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf(c^2-d^2)(c\cos(e+fx)+d)^2}$$

3.195. $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}dx$

$\big\downarrow$ 3042

$$\frac{\frac{(bc-ad)^2(5ac^2-2ad^2-3bcd)\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}-\frac{\frac{\left(a^3\left(6c^4d-5c^2d^3+2d^5\right)-3a^2bc^3\left(2c^2+d^2\right)+9ab^2c^4d-b^3c^3\left(c^2+2d^2\right)\right)\int\frac{1}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{c}-\frac{2a^3x\left(c^2-d^2\right)^2}{c}}{c(c^2-d^2)}}{2c\left(c^2-d^2\right)}$$
$$\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$\big\downarrow$ 3138

$$\frac{\frac{(bc-ad)^2(5ac^2-2ad^2-3bcd)\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}-\frac{\frac{2\left(a^3\left(6c^4d-5c^2d^3+2d^5\right)-3a^2bc^3\left(2c^2+d^2\right)+9ab^2c^4d-b^3c^3\left(c^2+2d^2\right)\right)\int\frac{1}{-\left((c-d)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+c+d}d\tan(}{cf}}{c(c^2-d^2)}}{2c\left(c^2-d^2\right)}$$
$$\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$\big\downarrow$ 221

$$\frac{\frac{(bc-ad)^2(5ac^2-2ad^2-3bcd)\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}-\frac{\frac{2\left(a^3\left(6c^4d-5c^2d^3+2d^5\right)-3a^2bc^3\left(2c^2+d^2\right)+9ab^2c^4d-b^3c^3\left(c^2+2d^2\right)\right)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\sqrt{c+d}}-2}{c(c^2-d^2)}}{2c\left(c^2-d^2\right)}$$
$$\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

input
```
Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^3,x]
```

output
```
((b*c - a*d)^2*(b + a*Cos[e + f*x])*Sin[e + f*x])/(2*c*(c^2 - d^2)*f*(d +
c*Cos[e + f*x])^2) + (-(((-2*a^3*(c^2 - d^2)^2*x)/c + (2*(9*a*b^2*c^4*d -
3*a^2*b*c^3*(2*c^2 + d^2) - b^3*c^3*(c^2 + 2*d^2) + a^3*(6*c^4*d - 5*c^2*d
^3 + 2*d^5))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c*Sqrt[
c - d]*Sqrt[c + d]*f))/(c*(c^2 - d^2))) + ((b*c - a*d)^2*(5*a*c^2 - 3*b*c*
d - 2*a*d^2)*Sin[e + f*x])/(c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])))/(2*c*(c
^2 - d^2))
```

3.195. $\int\frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$

3.195.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

3.195. $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4429
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

3.195.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\dfrac{\frac{2a^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\dfrac{2\left(-\frac{\left(6a^3c^2d^2+a^3d^3c-2a^3d^4-12a^2b\,c^3d-3a^2b\,c^2d^2+6a\,b^2c^4+3a\,b^2c^3d+6a\,b^2c^2d^2-b^3c^4-4b^3c^3d\right)c\,t\cdots}{2(c-d)\left(c^2+2cd+d^2\right)}\cdots\right)}{\left(\tan\left(\frac{fx}{2}\cdots\right)\cdots\right)\cdots}}{\cdots}$
default	$\dfrac{\frac{2a^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\dfrac{2\left(-\frac{\left(6a^3c^2d^2+a^3d^3c-2a^3d^4-12a^2b\,c^3d-3a^2b\,c^2d^2+6a\,b^2c^4+3a\,b^2c^3d+6a\,b^2c^2d^2-b^3c^4-4b^3c^3d\right)c\,t\cdots}{2(c-d)\left(c^2+2cd+d^2\right)}\cdots\right)}{\left(\tan\left(\frac{fx}{2}\cdots\right)\cdots\right)\cdots}}{\cdots}$
risch	Expression too large to display

input
```
int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a^3/c^3*arctan(tan(1/2*f*x+1/2*e))+2/c^3*((-1/2*(6*a^3*c^2*d^2+a^3*
c*d^3-2*a^3*d^4-12*a^2*b*c^3*d-3*a^2*b*c^2*d^2+6*a*b^2*c^4+3*a*b^2*c^3*d+6
*a*b^2*c^2*d^2-b^3*c^4-4*b^3*c^3*d)*c/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/
2*e)^3+1/2*c*(6*a^3*c^2*d^2-a^3*c*d^3-2*a^3*d^4-12*a^2*b*c^3*d+3*a^2*b*c^2
*d^2+6*a*b^2*c^4-3*a*b^2*c^3*d+6*a*b^2*c^2*d^2+b^3*c^4-4*b^3*c^3*d)/(c+d)/
(c-d)^2*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d
-c-d)^2-1/2*(6*a^3*c^4*d-5*a^3*c^2*d^3+2*a^3*d^5-6*a^2*b*c^5-3*a^2*b*c^3*d
^2+9*a*b^2*c^4*d-b^3*c^5-2*b^3*c^3*d^2)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^
(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

3.195.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(238) = 476$.

Time = 0.40 (sec) , antiderivative size = 1629, normalized size of antiderivative = 6.41

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^3*c^8 - 3*a^3*c^6*d^2 + 3*a^3*c^4*d^4 - a^3*c^2*d^6)*f*x*cos(f*
x + e)^2 + 8*(a^3*c^7*d - 3*a^3*c^5*d^3 + 3*a^3*c^3*d^5 - a^3*c*d^7)*f*x*c
os(f*x + e) + 4*(a^3*c^6*d^2 - 3*a^3*c^4*d^4 + 3*a^3*c^2*d^6 - a^3*d^8)*f*
x - (5*a^3*c^2*d^5 - 2*a^3*d^7 + (6*a^2*b + b^3)*c^5*d^2 - 3*(2*a^3 + 3*a*
b^2)*c^4*d^3 + (3*a^2*b + 2*b^3)*c^3*d^4 + (5*a^3*c^4*d^3 - 2*a^3*c^2*d^5
+ (6*a^2*b + b^3)*c^7 - 3*(2*a^3 + 3*a*b^2)*c^6*d + (3*a^2*b + 2*b^3)*c^5*
d^2)*cos(f*x + e)^2 + 2*(5*a^3*c^3*d^4 - 2*a^3*c*d^6 + (6*a^2*b + b^3)*c^6
*d - 3*(2*a^3 + 3*a*b^2)*c^5*d^2 + (3*a^2*b + 2*b^3)*c^4*d^3)*cos(f*x + e)
)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 -
 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*c
os(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^3*c^8 + 3*a*b^2*c^7*d +
2*a^3*c*d^7 - (9*a^2*b + 5*b^3)*c^6*d^2 + (5*a^3 + 3*a*b^2)*c^5*d^3 + (9*a
^2*b + 4*b^3)*c^4*d^4 - (7*a^3 + 6*a*b^2)*c^3*d^5 + 3*(2*a*b^2*c^8 - a^2*b
*c^3*d^5 + a^3*c^2*d^6 - (4*a^2*b + b^3)*c^7*d + (2*a^3 - a*b^2)*c^6*d^2 +
 (5*a^2*b + b^3)*c^5*d^3 - (3*a^3 + a*b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x
+ e))/((c^11 - 3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10
*d - 3*c^8*d^3 + 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^
4 + 3*c^5*d^6 - c^3*d^8)*f), 1/2*(2*(a^3*c^8 - 3*a^3*c^6*d^2 + 3*a^3*c^4*d
^4 - a^3*c^2*d^6)*f*x*cos(f*x + e)^2 + 4*(a^3*c^7*d - 3*a^3*c^5*d^3 + 3*a^
3*c^3*d^5 - a^3*c*d^7)*f*x*cos(f*x + e) + 2*(a^3*c^6*d^2 - 3*a^3*c^4*d^...
```

3.195.6 Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**3, x)
```

3.195.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

3.195.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(238) = 476.

Time = 0.41 (sec) , antiderivative size = 818, normalized size of antiderivative = 3.22

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
((f*x + e)*a^3/c^3 + (6*a^2*b*c^5 + b^3*c^5 - 6*a^3*c^4*d - 9*a*b^2*c^4*d
+ 3*a^2*b*c^3*d^2 + 2*b^3*c^3*d^2 + 5*a^3*c^2*d^3 - 2*a^3*d^5)*(pi*floor(1
/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) -
 d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 - 2*c^5*d^2 + c^3*d^4)*s
qrt(-c^2 + d^2)) - (6*a*b^2*c^5*tan(1/2*f*x + 1/2*e)^3 - b^3*c^5*tan(1/2*f
*x + 1/2*e)^3 - 12*a^2*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^2*c^4*d*tan(
1/2*f*x + 1/2*e)^3 - 3*b^3*c^4*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^3*c^3*d^2*ta
n(1/2*f*x + 1/2*e)^3 + 9*a^2*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 3*a*b^2*c^
3*d^2*tan(1/2*f*x + 1/2*e)^3 + 4*b^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 5*a^
3*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3
- 6*a*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^3*c*d^4*tan(1/2*f*x + 1/2*e
)^3 + 2*a^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 6*a*b^2*c^5*tan(1/2*f*x + 1/2*e)
- b^3*c^5*tan(1/2*f*x + 1/2*e) + 12*a^2*b*c^4*d*tan(1/2*f*x + 1/2*e) - 3*a
*b^2*c^4*d*tan(1/2*f*x + 1/2*e) + 3*b^3*c^4*d*tan(1/2*f*x + 1/2*e) - 6*a^3
*c^3*d^2*tan(1/2*f*x + 1/2*e) + 9*a^2*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - 3*a
*b^2*c^3*d^2*tan(1/2*f*x + 1/2*e) + 4*b^3*c^3*d^2*tan(1/2*f*x + 1/2*e) - 5
*a^3*c^2*d^3*tan(1/2*f*x + 1/2*e) - 3*a^2*b*c^2*d^3*tan(1/2*f*x + 1/2*e) -
 6*a*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) + 3*a^3*c*d^4*tan(1/2*f*x + 1/2*e) +
 2*a^3*d^5*tan(1/2*f*x + 1/2*e))/((c^6 - 2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f
*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

3.195.9 Mupad [B] (verification not implemented)

Time = 27.04 (sec) , antiderivative size = 10759, normalized size of antiderivative = 42.36

$$\int \frac{(a + b\sec(e + fx))^3}{(c + d\sec(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^3,x)
```

output
```
(atan(((((8*tan(e/2 + (f*x)/2)*(4*a^6*c^10 + 8*a^6*d^10 + b^6*c^10 - 8*a^6
*c*d^9 - 8*a^6*c^9*d + 12*a^2*b^4*c^10 + 36*a^4*b^2*c^10 - 32*a^6*c^2*d^8
+ 32*a^6*c^3*d^7 + 57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52*a^6*c^6*d^4 + 32*a
^6*c^7*d^3 + 24*a^6*c^8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 36*a*b^5*c^7
*d^3 - 120*a^3*b^3*c^9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5*d^5 + 24*a^5*b*c
^7*d^3 + 12*a^2*b^4*c^6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*b^3*c^3*d^7 + 16
*a^3*b^3*c^5*d^5 - 68*a^3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6 - 81*a^4*b^2*c^
6*d^4 + 144*a^4*b^2*c^8*d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*d))/(c^10*d +
c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)
+ ((a*d - b*c)*((8*(4*a^3*c^15 + 2*b^3*c^15 + 12*a^2*b*c^15 - 12*a^3*c^14*
d - 2*b^3*c^14*d - 4*a^3*c^6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^3*
c^9*d^6 - 36*a^3*c^10*d^5 + 6*a^3*c^11*d^4 + 34*a^3*c^12*d^3 - 8*a^3*c^13*
d^2 - 4*b^3*c^8*d^7 + 4*b^3*c^9*d^6 + 6*b^3*c^10*d^5 - 6*b^3*c^11*d^4 + 18
*a*b^2*c^9*d^6 - 18*a*b^2*c^10*d^5 - 36*a*b^2*c^11*d^4 + 36*a*b^2*c^12*d^3
 + 18*a*b^2*c^13*d^2 - 6*a^2*b*c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^12*d
^3 - 18*a^2*b*c^13*d^2 - 18*a*b^2*c^14*d - 12*a^2*b*c^14*d))/(c^12*d + c^1
3 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) -
 (4*tan(e/2 + (f*x)/2)*(a*d - b*c)*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4
+ 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*
b*c^3*d)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 -...
```

3.195. $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$

3.196 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

3.196.1 Optimal result

Integrand size = 25, antiderivative size = 412

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \frac{a^3x}{c^4} - \frac{(3ab^2c^4d(4c^2+d^2)-b^3c^5(c^2+4d^2)-a^2b(6c^7+9c^5d^2)+a^3(8c^6d-8c^4d^3+7c^2d^5-2d^7))\operatorname{arctanh}\left(\frac{\sqrt{c-}}{}\right.}{c^4\sqrt{c-d}\sqrt{c+d}\,(c^2-d^2)^3 f} - \frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))^3} + \frac{(bc-ad)^2(3bc^3-8ac^2d+2bcd^2+3ad^3)\sin(e+fx)}{6c^3(c^2-d^2)^2f(d+c\cos(e+fx))^2} - \frac{(bc-ad)(b^2c^2d(13c^2+2d^2)-abc(18c^4+17c^2d^2-5d^4)+a^2(34c^4d-28c^2d^3+9d^5))\sin(e+fx)}{6c^3(c^2-d^2)^3f(d+c\cos(e+fx))}$$

output
```
a^3*x/c^4-1/3*d*(-a*d+b*c)*(b+a*cos(f*x+e))^2*sin(f*x+e)/c/(c^2-d^2)/f/(d+
c*cos(f*x+e))^3+1/6*(-a*d+b*c)^2*(-8*a*c^2*d+3*a*d^3+3*b*c^3+2*b*c*d^2)*si
n(f*x+e)/c^3/(c^2-d^2)^2/f/(d+c*cos(f*x+e))^2-1/6*(-a*d+b*c)*(b^2*c^2*d*(1
3*c^2+2*d^2)-a*b*c*(18*c^4+17*c^2*d^2-5*d^4)+a^2*(34*c^4*d-28*c^2*d^3+9*d^
5))*sin(f*x+e)/c^3/(c^2-d^2)^3/f/(d+c*cos(f*x+e))-(3*a*b^2*c^4*d*(4*c^2+d^
2)-b^3*c^5*(c^2+4*d^2)-a^2*b*(6*c^7+9*c^5*d^2)+a^3*(8*c^6*d-8*c^4*d^3+7*c^
2*d^5-2*d^7))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^4/(c^2
-d^2)^3/f/(c-d)^(1/2)/(c+d)^(1/2)
```

3.196.2 Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{(d+c\cos(e+fx))\sec(e+fx)(a+b\sec(e+fx))^3\left(6a^3(e+fx)(d+c\cos(e+fx))^3 - \frac{6(-3ab^2c^4d(4c^2+d^2)\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^4,x]
```

output
```
((d + c*Cos[e + f*x])*Sec[e + f*x]*(a + b*Sec[e + f*x])^3*(6*a^3*(e + f*x)
*(d + c*Cos[e + f*x])^3 - (6*(-3*a*b^2*c^4*d*(4*c^2 + d^2) + b^3*c^5*(c^2
+ 4*d^2) + a^2*b*(6*c^7 + 9*c^5*d^2) + a^3*(-8*c^6*d + 8*c^4*d^3 - 7*c^2*d
^5 + 2*d^7))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*C
os[e + f*x])^3)/(c^2 - d^2)^(7/2) - (2*c*d*(b*c - a*d)^3*Sin[e + f*x])/(c^
2 - d^2) + (c*(b*c - a*d)^2*(3*b*c^3 - 12*a*c^2*d + 2*b*c*d^2 + 7*a*d^3)*(
d + c*Cos[e + f*x])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(-(b^3*c^3*d*(13*c^2
+ 2*d^2)) + 3*a*b^2*c^2*(6*c^4 + 10*c^2*d^2 - d^4) - 3*a^2*b*c*d*(18*c^4 -
 5*c^2*d^2 + 2*d^4) + a^3*(36*c^4*d^2 - 32*c^2*d^4 + 11*d^6))*(d + c*Cos[e
 + f*x])^2*Sin[e + f*x])/(c^2 - d^2)^3))/(6*c^4*f*(b + a*Cos[e + f*x])^3*(
c + d*Sec[e + f*x])^4)
```

3.196.3 Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4429, 3042, 3468, 3042, 3510, 3042, 3500, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4429}$$

$$\int \frac{\cos(e+fx)(a\cos(e+fx)+b)^3}{(c\cos(e+fx)+d)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^3}{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^4}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{\int \frac{(b+a\cos(e+fx))(3a^2(c^2-d^2)\cos^2(e+fx)-(3cda^2-b(6c^2-d^2)a+2b^2cd)\cos(e+fx)+(3bc-2ad)(bc-ad))}{(d+c\cos(e+fx))^3}dx}{3c(c^2-d^2)}-\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(3a^2(c^2-d^2)\sin\left(e+fx+\frac{\pi}{2}\right)^2+(-3cda^2+b(6c^2-d^2)a-2b^2cd)\sin\left(e+fx+\frac{\pi}{2}\right)+(3bc-2ad)(bc-ad)\right)}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx}{3c(c^2-d^2)}-\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}$$

$$\Big\downarrow \text{3510}$$

$$\frac{\frac{(bc-ad)^2(-8ac^2d+3ad^3+3bc^3+2bcd^2)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\int \frac{-6c\left(c^2-d^2\right)^2\cos^2(e+fx)a^3+2c(bc-ad)\left(\left(8c^2d-3d^3\right)a^2-bc\left(9c^2+d^2\right)a+5b^2c^2d\right)+\left(\left(3d^5-10c^2d^3\right.\right.}{(d+c\cos(e+fx}}{2c^2(c^2-d^2}}{3c(c^2-d^2)}-\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(bc-ad)^2(-8ac^2d+3ad^3+3bc^3+2bcd^2)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\int \frac{-6c\left(c^2-d^2\right)^2\sin\left(e+fx+\frac{\pi}{2}\right)^2a^3+2c(bc-ad)\left(\left(8c^2d-3d^3\right)a^2-bc\left(9c^2+d^2\right)a+5b^2c^2d\right)+\left(\left(3d^5-10\right.\right.}{\left(d+c\sin\left(e+\right.\right.}}{2c^2(c^2}}{3c(c^2-d^2)}-\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}$$

$$\Big\downarrow \text{3500}$$

3.196. $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\frac{\int\frac{3\left(c^2(bc-ad)\left(-\left(\left(6c^4-2d^2c^2+d^4\right)a^2\right)+bcd\left(11c^2-d^2\right)a-b^2c^2\left(c^2+4d^2\right)\right)-2a^3c\left(c^2-d^2\right)^3\cos(e+\cdots}{d+c\cos(e+fx)}}{c\left(c^2-d^2\right)}}{2c\cdots}}{3c\left(c^2-d^2\right)}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$\downarrow$ 27

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\frac{3\int\frac{c^2(bc-ad)\left(-\left(\left(6c^4-2d^2c^2+d^4\right)a^2\right)+bcd\left(11c^2-d^2\right)a-b^2c^2\left(c^2+4d^2\right)\right)-2a^3c\left(c^2-d^2\right)^3\cos(e+f\cdots}{d+c\cos(e+fx)}}{c\left(c^2-d^2\right)}}{2c^2\cdots}}{3c\left(c^2-d^2\right)}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$\downarrow$ 3042

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\frac{3\int\frac{c^2(bc-ad)\left(-\left(\left(6c^4-2d^2c^2+d^4\right)a^2\right)+bcd\left(11c^2-d^2\right)a-b^2c^2\left(c^2+4d^2\right)\right)-2a^3c\left(c^2-d^2\right)^3\sin\left(e+f\cdots\right.}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}{c\left(c^2-d^2\right)}}{2c\cdots}}{3c\left(c^2-d^2\right)}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$\downarrow$ 3214

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{3\left(\left(a^3\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-3a^2bc^5\left(2c^2+3d^2\right)+3ab^2c^4d\left(4c^2+d^2\right)-b^3c^5\left(c^2+4d^2\right)\right)\int\frac{\cdots}{d+c\cos\cdots}\right.}{c\left(c^2-d^2\right)}}{3c\left(c^2-d^2\right)}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$\downarrow$ 3042

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{3\left(\left(a^3\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-3a^2bc^5\left(2c^2+3d^2\right)+3ab^2c^4d\left(4c^2+d^2\right)-b^3c^5\left(c^2+4d^2\right)\right)\int\frac{\cdots}{d+c\sin\cdots}\right.}{c\left(c^2-d^2\right)}}{3c\left(c^2-d^2\right)}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$\downarrow$ 3138

3.196. $\int\frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{3\left(\frac{2\left(a^3\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-3a^2bc^5\left(2c^2+3d^2\right)+3ab^2c^4d\left(4c^2+d^2\right)-b^3c^5\left(c^2+4d^2\right)\right)\int \frac{\cdots}{-((\cdots}}{f}\cdots\right)}{c\left(c^2-d^2\right)}\cdots}{3c\,(c\cdots}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^3}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{3\left(\frac{2\left(a^3\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-3a^2bc^5\left(2c^2+3d^2\right)+3ab^2c^4d\left(4c^2+d^2\right)-b^3c^5\left(c^2+4d^2\right)\right)\text{arcta}\cdots}{f\sqrt{c-d}\sqrt{c+d}}\cdots\right)}{c\left(c^2-d^2\right)}\cdots}{3c\left(c^2-d\cdots\right.}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^3}$$

input
```
Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^4,x]
```

output
```
-1/3*(d*(b*c - a*d)*(b + a*Cos[e + f*x])^2*Sin[e + f*x])/(c*(c^2 - d^2)*f*
(d + c*Cos[e + f*x])^3) + (((b*c - a*d)^2*(3*b*c^3 - 8*a*c^2*d + 2*b*c*d^2
 + 3*a*d^3)*Sin[e + f*x])/(2*c^2*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^2) - (
(3*(-2*a^3*(c^2 - d^2)^3*x + (2*(3*a*b^2*c^4*d*(4*c^2 + d^2) - 3*a^2*b*c^5
*(2*c^2 + 3*d^2) - b^3*c^5*(c^2 + 4*d^2) + a^3*(8*c^6*d - 8*c^4*d^3 + 7*c^
2*d^5 - 2*d^7))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt
[c - d]*Sqrt[c + d]*f)))/(c*(c^2 - d^2)) - ((b*c - a*d)*(18*a*b*c^5 - 34*a
^2*c^4*d - 13*b^2*c^4*d + 17*a*b*c^3*d^2 + 28*a^2*c^2*d^3 - 2*b^2*c^2*d^3
- 5*a*b*c*d^4 - 9*a^2*d^5)*Sin[e + f*x])/((c^2 - d^2)*f*(d + c*Cos[e + f*x
])))/(2*c^2*(c^2 - d^2)))/(3*c*(c^2 - d^2))
```

3.196.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.196. $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

3.196. $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 4429
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

3.196.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.91

method	result
derivativedivides	$\frac{\frac{2a^3 \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^3c^4d^2+4a^3c^3d^3-6a^3c^2d^4-a^3c\,d^5+2a^3d^6-18a^2b\,c^5d-9a^2b\,c^4d^2-6a^2b\,c^3d^3+6a\,b^2c^6+6a\,b^2 \cdots\right)}{2(c-d)\left(c^3+3c^2d+3c\,d^2 \cdots\right)} \cdots\right)}{\cdots}}{\cdots}$
default	$\frac{\frac{2a^3 \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^3c^4d^2+4a^3c^3d^3-6a^3c^2d^4-a^3c\,d^5+2a^3d^6-18a^2b\,c^5d-9a^2b\,c^4d^2-6a^2b\,c^3d^3+6a\,b^2c^6+6a\,b^2 \cdots\right)}{2(c-d)\left(c^3+3c^2d+3c\,d^2 \cdots\right)} \cdots\right)}{\cdots}}{\cdots}$
risch	Expression too large to display

input
```
int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a^3/c^4*arctan(tan(1/2*f*x+1/2*e))+2/c^4*((-1/2*(12*a^3*c^4*d^2+4*a
^3*c^3*d^3-6*a^3*c^2*d^4-a^3*c*d^5+2*a^3*d^6-18*a^2*b*c^5*d-9*a^2*b*c^4*d^
2-6*a^2*b*c^3*d^3+6*a*b^2*c^6+6*a*b^2*c^5*d+18*a*b^2*c^4*d^2+3*a*b^2*c^3*d
^3-b^3*c^6-6*b^3*c^5*d-2*b^3*c^4*d^2-2*b^3*c^3*d^3)*c/(c-d)/(c^3+3*c^2*d+3
*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5+2/3*(18*a^3*c^4*d^2-11*a^3*c^2*d^4+3*a^3*
d^6-27*a^2*b*c^5*d-3*a^2*b*c^3*d^3+9*a*b^2*c^6+21*a*b^2*c^4*d^2-7*b^3*c^5*
d-3*b^3*c^3*d^3)*c/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1/
2*(12*a^3*c^4*d^2-4*a^3*c^3*d^3-6*a^3*c^2*d^4+a^3*c*d^5+2*a^3*d^6-18*a^2*b
*c^5*d+9*a^2*b*c^4*d^2-6*a^2*b*c^3*d^3+6*a*b^2*c^6-6*a*b^2*c^5*d+18*a*b^2*
c^4*d^2-3*a*b^2*c^3*d^3+b^3*c^6-6*b^3*c^5*d+2*b^3*c^4*d^2-2*b^3*c^3*d^3)*c
/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2
*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3-1/2*(8*a^3*c^6*d-8*a^3*c^4*d^3+7*a^3*c^2*
d^5-2*a^3*d^7-6*a^2*b*c^7-9*a^2*b*c^5*d^2+12*a*b^2*c^6*d+3*a*b^2*c^4*d^3-b
^3*c^7-4*b^3*c^5*d^2)/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*ar
ctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1359 vs. $2(396) = 792$.

Time = 0.48 (sec) , antiderivative size = 2776, normalized size of antiderivative = 6.74

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
[1/12*(12*(a^3*c^11 - 4*a^3*c^9*d^2 + 6*a^3*c^7*d^4 - 4*a^3*c^5*d^6 + a^3*
c^3*d^8)*f*x*cos(f*x + e)^3 + 36*(a^3*c^10*d - 4*a^3*c^8*d^3 + 6*a^3*c^6*d
^5 - 4*a^3*c^4*d^7 + a^3*c^2*d^9)*f*x*cos(f*x + e)^2 + 36*(a^3*c^9*d^2 - 4
*a^3*c^7*d^4 + 6*a^3*c^5*d^6 - 4*a^3*c^3*d^8 + a^3*c*d^10)*f*x*cos(f*x + e
) + 12*(a^3*c^8*d^3 - 4*a^3*c^6*d^5 + 6*a^3*c^4*d^7 - 4*a^3*c^2*d^9 + a^3*
d^11)*f*x + 3*(7*a^3*c^2*d^8 - 2*a^3*d^10 - (6*a^2*b + b^3)*c^7*d^3 + 4*(2
*a^3 + 3*a*b^2)*c^6*d^4 - (9*a^2*b + 4*b^3)*c^5*d^5 - (8*a^3 - 3*a*b^2)*c^
4*d^6 + (7*a^3*c^5*d^5 - 2*a^3*c^3*d^7 - (6*a^2*b + b^3)*c^10 + 4*(2*a^3 +
 3*a*b^2)*c^9*d - (9*a^2*b + 4*b^3)*c^8*d^2 - (8*a^3 - 3*a*b^2)*c^7*d^3)*c
os(f*x + e)^3 + 3*(7*a^3*c^4*d^6 - 2*a^3*c^2*d^8 - (6*a^2*b + b^3)*c^9*d +
 4*(2*a^3 + 3*a*b^2)*c^8*d^2 - (9*a^2*b + 4*b^3)*c^7*d^3 - (8*a^3 - 3*a*b^
2)*c^6*d^4)*cos(f*x + e)^2 + 3*(7*a^3*c^3*d^7 - 2*a^3*c*d^9 - (6*a^2*b + b
^3)*c^8*d^2 + 4*(2*a^3 + 3*a*b^2)*c^7*d^3 - (9*a^2*b + 4*b^3)*c^6*d^4 - (8
*a^3 - 3*a*b^2)*c^5*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x
+ e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) +
c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) +
d^2)) + 2*(b^3*c^10*d + 6*a*b^2*c^9*d^2 + 23*a^3*c^3*d^8 - 6*a^3*c*d^10 -
11*(3*a^2*b + b^3)*c^8*d^3 + (26*a^3 + 33*a*b^2)*c^7*d^4 + (21*a^2*b + 4*b
^3)*c^6*d^5 - (43*a^3 + 39*a*b^2)*c^5*d^6 + 6*(2*a^2*b + b^3)*c^4*d^7 + (1
8*a*b^2*c^11 + 6*a^2*b*c^4*d^7 - 11*a^3*c^3*d^8 - (54*a^2*b + 13*b^3)*c...
```

3.196.6 Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \int \frac{(a+b\sec{(e+fx)})^3}{(c+d\sec{(e+fx)})^4}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**4,x)
```

output
```
Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**4, x)
```

3.196.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

3.196.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1572 vs. $2(396) = 792$.

Time = 0.43 (sec) , antiderivative size = 1572, normalized size of antiderivative = 3.82

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/3*(3*(6*a^2*b*c^7 + b^3*c^7 - 8*a^3*c^6*d - 12*a*b^2*c^6*d + 9*a^2*b*c^5
*d^2 + 4*b^3*c^5*d^2 + 8*a^3*c^4*d^3 - 3*a*b^2*c^4*d^3 - 7*a^3*c^2*d^5 + 2
*a^3*d^7)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*t
an(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^10 -
3*c^8*d^2 + 3*c^6*d^4 - c^4*d^6)*sqrt(-c^2 + d^2)) + 3*(f*x + e)*a^3/c^4 -
 (18*a*b^2*c^8*tan(1/2*f*x + 1/2*e)^5 - 3*b^3*c^8*tan(1/2*f*x + 1/2*e)^5 -
 54*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 18*a*b^2*c^7*d*tan(1/2*f*x + 1/2*
e)^5 - 12*b^3*c^7*d*tan(1/2*f*x + 1/2*e)^5 + 36*a^3*c^6*d^2*tan(1/2*f*x +
1/2*e)^5 + 81*a^2*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 36*a*b^2*c^6*d^2*tan(
1/2*f*x + 1/2*e)^5 + 27*b^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^3*c^5*d^
3*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 81*a*
b^2*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 12*b^3*c^5*d^3*tan(1/2*f*x + 1/2*e)^5
 - 6*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 9*a^2*b*c^4*d^4*tan(1/2*f*x + 1/
2*e)^5 + 36*a*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*b^3*c^4*d^4*tan(1/2*f
*x + 1/2*e)^5 + 45*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*b*c^3*d^5*t
an(1/2*f*x + 1/2*e)^5 + 9*a*b^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 - 6*b^3*c^3
*d^5*tan(1/2*f*x + 1/2*e)^5 - 6*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 15*a^
3*c*d^7*tan(1/2*f*x + 1/2*e)^5 + 6*a^3*d^8*tan(1/2*f*x + 1/2*e)^5 - 36*a*b
^2*c^8*tan(1/2*f*x + 1/2*e)^3 + 108*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e)^3 + 2
8*b^3*c^7*d*tan(1/2*f*x + 1/2*e)^3 - 72*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e...
```

3.196.9 Mupad [B] (verification not implemented)

Time = 28.48 (sec) , antiderivative size = 15647, normalized size of antiderivative = 37.98

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^4,x)
```

output
```
((tan(e/2 + (f*x)/2)^5*(b^3*c^6 - 2*a^3*d^6 - 6*a*b^2*c^6 + a^3*c*d^5 + 6*
b^3*c^5*d + 6*a^3*c^2*d^4 - 4*a^3*c^3*d^3 - 12*a^3*c^4*d^2 + 2*b^3*c^3*d^3
 + 2*b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 - 18*a*b^2*c^4*d^2 + 6*a^2*b*c^3*d^3 +
9*a^2*b*c^4*d^2 - 6*a*b^2*c^5*d + 18*a^2*b*c^5*d))/((c^3*d - c^4)*(c + d)^
3) + (4*tan(e/2 + (f*x)/2)^3*(7*b^3*c^5*d - 9*a*b^2*c^6 - 3*a^3*d^6 + 11*a
^3*c^2*d^4 - 18*a^3*c^4*d^2 + 3*b^3*c^3*d^3 - 21*a*b^2*c^4*d^2 + 3*a^2*b*c
^3*d^3 + 27*a^2*b*c^5*d))/(3*(c + d)^2*(c^5 - 2*c^4*d + c^3*d^2)) - (tan(e
/2 + (f*x)/2)*(2*a^3*d^6 + b^3*c^6 + 6*a*b^2*c^6 + a^3*c*d^5 - 6*b^3*c^5*d
 - 6*a^3*c^2*d^4 - 4*a^3*c^3*d^3 + 12*a^3*c^4*d^2 - 2*b^3*c^3*d^3 + 2*b^3*
c^4*d^2 - 3*a*b^2*c^3*d^3 + 18*a*b^2*c^4*d^2 - 6*a^2*b*c^3*d^3 + 9*a^2*b*c
^4*d^2 - 6*a*b^2*c^5*d - 18*a^2*b*c^5*d))/((c + d)*(3*c^5*d - c^6 + c^3*d^
3 - 3*c^4*d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d
^3) - tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 +
 3*c^2*d + c^3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3
))) - (2*a^3*atan(((a^3*((a^3*((8*(4*a^3*c^21 + 2*b^3*c^21 + 12*a^2*b*c^21
 - 16*a^3*c^20*d - 2*b^3*c^20*d - 4*a^3*c^8*d^13 + 2*a^3*c^9*d^12 + 26*a^3
*c^10*d^11 - 14*a^3*c^11*d^10 - 70*a^3*c^12*d^9 + 30*a^3*c^13*d^8 + 110*a^
3*c^14*d^7 - 30*a^3*c^15*d^6 - 110*a^3*c^16*d^5 + 20*a^3*c^17*d^4 + 64*a^3
*c^18*d^3 - 12*a^3*c^19*d^2 + 8*b^3*c^12*d^9 - 8*b^3*c^13*d^8 - 22*b^3*c^1
4*d^7 + 22*b^3*c^15*d^6 + 18*b^3*c^16*d^5 - 18*b^3*c^17*d^4 - 2*b^3*c^1...
```

3.196. $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

3.197 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

3.197.1 Optimal result

Integrand size = 25, antiderivative size = 622

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \frac{a^3x}{c^5}$$
$$-\frac{(15ab^2c^6d(4c^2+3d^2)-3a^2bc^5(8c^4+24c^2d^2+3d^4)-b^3c^5(4c^4+27c^2d^2+4d^4)+a^3(40c^8d-40c^6d^3+\cdots}{4c^5\sqrt{c-d}\sqrt{c+d}\,(c^2-d^2)^4 f}$$
$$+\frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\,(c^2-d^2)\,f(d+c\cos(e+fx))^4}$$
$$-\frac{d(8bc^3-11ac^2d-bcd^2+4ad^3)\,(b+a\cos(e+fx))^2\sin(e+fx)}{12c^2\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^3}$$
$$-\frac{(bc-ad)\,(2abcd(32c^4+c^2d^2+2d^4)-a^2d^2(58c^4-35c^2d^2+12d^4)-b^2(12c^6+25c^4d^2-2c^2d^4))\sin(e+\cdots}{24c^4\,(c^2-d^2)^3\,f(d+c\cos(e+fx))^2}$$
$$-\frac{(b^3c^3d(68c^4+39c^2d^2-2d^4)+a^2bcd(272c^6+10c^4d^2+49c^2d^4-16d^6)-3ab^2c^2(24c^6+84c^4d^2-5c^2d^4\cdots}{24c^4\,(c^2-d^2)^4\,f(d+c\cos(e+fx))}$$

output
```
a^3*x/c^5+1/4*d^2*(b+a*cos(f*x+e))^3*sin(f*x+e)/c/(c^2-d^2)/f/(d+c*cos(f*x
+e))^4-1/12*d*(-11*a*c^2*d+4*a*d^3+8*b*c^3-b*c*d^2)*(b+a*cos(f*x+e))^2*sin
(f*x+e)/c^2/(c^2-d^2)^2/f/(d+c*cos(f*x+e))^3-1/24*(-a*d+b*c)*(2*a*b*c*d*(3
2*c^4+c^2*d^2+2*d^4)-a^2*d^2*(58*c^4-35*c^2*d^2+12*d^4)-b^2*(12*c^6+25*c^4
*d^2-2*c^2*d^4))*sin(f*x+e)/c^4/(c^2-d^2)^3/f/(d+c*cos(f*x+e))^2-1/24*(b^3
*c^3*d*(68*c^4+39*c^2*d^2-2*d^4)+a^2*b*c*d*(272*c^6+10*c^4*d^2+49*c^2*d^4-
16*d^6)-3*a*b^2*c^2*(24*c^6+84*c^4*d^2-5*c^2*d^4+2*d^6)-a^3*(212*c^6*d^2-2
10*c^4*d^4+139*c^2*d^6-36*d^8))*sin(f*x+e)/c^4/(c^2-d^2)^4/f/(d+c*cos(f*x+
e))-1/4*(15*a*b^2*c^6*d*(4*c^2+3*d^2)-3*a^2*b*c^5*(8*c^4+24*c^2*d^2+3*d^4)
-b^3*c^5*(4*c^4+27*c^2*d^2+4*d^4)+a^3*(40*c^8*d-40*c^6*d^3+63*c^4*d^5-36*c
^2*d^7+8*d^9))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^5/(c^
2-d^2)^4/f/(c-d)^(1/2)/(c+d)^(1/2)
```

3.197.2 Mathematica [A] (verified)

Time = 9.72 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$$

$$= \frac{(d+c\cos(e+fx))\sec^2(e+fx)(a+b\sec(e+fx))^3\left(24a^3(e+fx)(d+c\cos(e+fx))^4 - \frac{6(-15ab^2c^6d(4c^2+\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^5,x]
```

output
```
((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^3*(24*a^3*(e + f
*x)*(d + c*Cos[e + f*x])^4 - (6*(-15*a*b^2*c^6*d*(4*c^2 + 3*d^2) + 3*a^2*b
*c^5*(8*c^4 + 24*c^2*d^2 + 3*d^4) + b^3*c^5*(4*c^4 + 27*c^2*d^2 + 4*d^4) +
 a^3*(-40*c^8*d + 40*c^6*d^3 - 63*c^4*d^5 + 36*c^2*d^7 - 8*d^9))*ArcTanh[(
(-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^4)/(c^2 -
 d^2)^(9/2) + (6*c*d^2*(b*c - a*d)^3*Sin[e + f*x])/(c^2 - d^2) - (2*c*d*(b
*c - a*d)^2*(8*b*c^3 - 20*a*c^2*d - b*c*d^2 + 13*a*d^3)*(d + c*Cos[e + f*x
])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(a^3*d^3*(-120*c^4 + 131*c^2*d^2 - 46*
d^4) - 3*a*b^2*c^2*d*(36*c^4 - 3*c^2*d^2 + 2*d^4) + 3*a^2*b*c*d^2*(72*c^4
- 55*c^2*d^2 + 18*d^4) + b^3*(12*c^7 + 25*c^5*d^2 - 2*c^3*d^4))*(d + c*Cos
[e + f*x])^2*Sin[e + f*x])/(c^2 - d^2)^3 + (c*(b^3*c^3*d*(-68*c^4 - 39*c^2
*d^2 + 2*d^4) - 3*a^2*b*c*d*(96*c^6 - 8*c^4*d^2 + 23*c^2*d^4 - 6*d^6) + 3*
a*b^2*c^2*(24*c^6 + 84*c^4*d^2 - 5*c^2*d^4 + 2*d^6) + 5*a^3*(48*c^6*d^2 -
56*c^4*d^4 + 39*c^2*d^6 - 10*d^8))*(d + c*Cos[e + f*x])^3*Sin[e + f*x])/(c
^2 - d^2)^4))/(24*c^5*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[e + f*x])^5)
```

3.197.3 Rubi [A] (verified)

Time = 3.20 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.10, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4429, 3042, 3527, 25, 3042, 3526, 25, 3042, 3510, 3042, 3500, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4429}$$

$$\int \frac{\cos^2(e+fx)(a\cos(e+fx)+b)^3}{(c\cos(e+fx)+d)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^2\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^3}{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^5}dx$$

$$\Big\downarrow \; 3527$$

$$\frac{\int -\frac{(b+a\cos(e+fx))^2\left(-4a\left(c^2-d^2\right)\cos^2(e+fx)-\left(4bc^2-4adc-bd^2\right)\cos(e+fx)+d(4bc-3ad)\right)}{(d+c\cos(e+fx))^4}dx}{4c\left(c^2-d^2\right)}+\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}$$

$$\Big\downarrow \; 25$$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-\frac{\int \frac{(b+a\cos(e+fx))^2\left(-4a\left(c^2-d^2\right)\cos^2(e+fx)+\left(4acd-b\left(4c^2-d^2\right)\right)\cos(e+fx)+d(4bc-3ad)\right)}{(d+c\cos(e+fx))^4}dx}{4c\left(c^2-d^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-\frac{\int \frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(-4a\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(4acd-b\left(4c^2-d^2\right)\right)\sin\left(e+fx+\frac{\pi}{2}\right)+d(4bc-3ad)\right)}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx}{4c\left(c^2-d^2\right)}$$

$$\Big\downarrow \; 3526$$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-\frac{\frac{\int -\frac{(b+a\cos(e+fx))\left(12b^2c^4-40abdc^3+22a^2d^2c^2+9b^2d^2c^2+5abd^3c-8a^2d^4+12a^2\left(c^2-d^2\right)^2\cos^2(e+fx)-\left(3\left(8c^3d-cd^3\right)a^2-b\left(24c^4+7d^2c^2+4d^4\right)a+2b^2cd\left(8c^2-\right.\right.}{(d+c\cos(e+fx))^3}}{3c\left(c^2-d^2\right)}}{4c\left(c^2-d^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-\frac{\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\int \frac{(b+a\cos(e+fx))\left(3\left(4c^4+3d^2c^2\right)b^2-5acd\left(8c^2-d^2\right)b+12a^2\left(c^2-d^2\right)^2\cos^2(e+fx)+2a^2d\right.}{(d+c\cos}}{3c(c^2}}{4c\left(c^2-d^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-\frac{\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\int \frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(3\left(4c^4+3d^2c^2\right)b^2-5acd\left(8c^2-d^2\right)b+12a^2\left(c^2-d^2\right)^2\sin\left(e+fx+\frac{\pi}{2}\right.\right.}{(d+}}{}}{4c\left(c^2-d^2\right)}$$

$$\Big\downarrow \; 3510$$

3.197. $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf(c^2-d^2)(c\cos(e+fx)+d)^4}-\frac{\frac{d(-11ac^2d+4ad^3+8bc^3-bcd^2)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{-\frac{\int\frac{-24a^3c\cos^2(e+fx)\left(c^2-d^2\right)^3+2c\left(-\left(\left(12d^6-35c^2d^4+58c^4d^2\right)a^3\right)+bcd\left(100c^4-3d^2c^2\ldots\right.\right.}{}}{}}{}}{}$$

$\downarrow$ 3042

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf(c^2-d^2)(c\cos(e+fx)+d)^4}-\frac{\frac{d(-11ac^2d+4ad^3+8bc^3-bcd^2)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{-\frac{\int\frac{-24a^3c\sin\left(e+fx+\frac{\pi}{2}\right)^2\left(c^2-d^2\right)^3+2c\left(-\left(\left(12d^6-35c^2d^4+58c^4d^2\right)a^3\right)+bcd\left(100c^4-3\ldots\right.\right.}{}}{}}{}}{}$$

$\downarrow$ 3500

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf(c^2-d^2)(c\cos(e+fx)+d)^4}-\frac{\frac{d(-11ac^2d+4ad^3+8bc^3-bcd^2)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{-\frac{\int\frac{3\left(c\left(15ab^2d\left(4c^2+3d^2\right)c^5-3a^2b\left(8c^4+24d^2c^2+3d^4\right)c^4-b^3\left(4c^4+27d^2c^2+4d^4\right)c^4+a^3\ldots\right.\right.}{d+c\cos(e+fx)}}{c\left(c^2-d^2\right)}}{}}{}$$

$\downarrow$ 27

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf(c^2-d^2)(c\cos(e+fx)+d)^4}-\frac{\frac{d(-11ac^2d+4ad^3+8bc^3-bcd^2)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{-\frac{3\int\frac{c\left(15ab^2d\left(4c^2+3d^2\right)c^5-3a^2b\left(8c^4+24d^2c^2+3d^4\right)c^4-b^3\left(4c^4+27d^2c^2+4d^4\right)c^4+a^3\left(\ldots\right.\right.}{d+c\cos(e+fx)}}{c\left(c^2-d^2\right)}}{}}{}$$

$\downarrow$ 3042

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf(c^2-d^2)(c\cos(e+fx)+d)^4}-\frac{\frac{d(-11ac^2d+4ad^3+8bc^3-bcd^2)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{-\frac{3\int\frac{c\left(15ab^2d\left(4c^2+3d^2\right)c^5-3a^2b\left(8c^4+24d^2c^2+3d^4\right)c^4-b^3\left(4c^4+27d^2c^2+4d^4\right)c^4+a^3\left(\ldots\right.\right.}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}{c\left(c^2-d^2\right)}}{}}{}$$

$\downarrow$ 3214

3.197. $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-$$

$$\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-$$

$$-\frac{3\left(\left(a^3\left(40c^8d-40c^6d^3+63c^4d^5-36c^2d^7+8d^9\right)-3a^2bc^5\left(8c^4+24c^2d^2+3d^4\right)+15ab^2c^6d\left(\right.\right.\right.}{c\left(c^2-d^2\right)}$$

$\downarrow$ 3042

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-$$

$$\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-$$

$$-\frac{3\left(\left(a^3\left(40c^8d-40c^6d^3+63c^4d^5-36c^2d^7+8d^9\right)-3a^2bc^5\left(8c^4+24c^2d^2+3d^4\right)+15ab^2c^6d\left(\right.\right.\right.}{c\left(c^2-d^2\right.}$$

$\downarrow$ 3138

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-$$

$$\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-$$

$$-3\left(\frac{2\left(a^3\left(40c^8d-40c^6d^3+63c^4d^5-36c^2d^7+8d^9\right)-3a^2bc^5\left(8c^4+24c^2d^2+3d^4\right)+15ab^2c^6d\right.}{}\right.$$

$\downarrow$ 221

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-$$

$$\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-$$

$$-\frac{(bc-ad)\left(-a^2d^2\left(58c^4-35c^2d^2+12d^4\right)+2abcd\left(32c^4+c^2d^2+2d^4\right)-\left(b^2\left(12c^6+25c^4d^2-2\right.\right.\right.}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

input
```
Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^5,x]
```

3.197. $\int\frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

output
```
(d^2*(b + a*Cos[e + f*x])^3*Sin[e + f*x])/(4*c*(c^2 - d^2)*f*(d + c*Cos[e
+ f*x])^4) - ((d*(8*b*c^3 - 11*a*c^2*d - b*c*d^2 + 4*a*d^3)*(b + a*Cos[e +
 f*x])^2*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3) - (-1/2*
((b*c - a*d)*(2*a*b*c*d*(32*c^4 + c^2*d^2 + 2*d^4) - a^2*d^2*(58*c^4 - 35*
c^2*d^2 + 12*d^4) - b^2*(12*c^6 + 25*c^4*d^2 - 2*c^2*d^4))*Sin[e + f*x])/(
c^2*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^2) - ((3*(-8*a^3*(c^2 - d^2)^4*x +
(2*(15*a*b^2*c^6*d*(4*c^2 + 3*d^2) - 3*a^2*b*c^5*(8*c^4 + 24*c^2*d^2 + 3*d
^4) - b^3*c^5*(4*c^4 + 27*c^2*d^2 + 4*d^4) + a^3*(40*c^8*d - 40*c^6*d^3 +
63*c^4*d^5 - 36*c^2*d^7 + 8*d^9))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/S
qrt[c + d]])/(Sqrt[c - d]*Sqrt[c + d]*f)))/(c*(c^2 - d^2)) + ((b^3*c^3*d*(
68*c^4 + 39*c^2*d^2 - 2*d^4) + a^2*b*c*d*(272*c^6 + 10*c^4*d^2 + 49*c^2*d^
4 - 16*d^6) - 3*a*b^2*c^2*(24*c^6 + 84*c^4*d^2 - 5*c^2*d^4 + 2*d^6) - a^3*
(212*c^6*d^2 - 210*c^4*d^4 + 139*c^2*d^6 - 36*d^8))*Sin[e + f*x])/((c^2 -
d^2)*f*(d + c*Cos[e + f*x])))/(2*c^2*(c^2 - d^2)))/(3*c*(c^2 - d^2)))/(4*c
*(c^2 - d^2))
```

3.197.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.197. $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.197. $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4429
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

3.197.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(603) = 1206$.

Time = 2.38 (sec) , antiderivative size = 1345, normalized size of antiderivative = 2.16

method	result	size
derivativedivides	Expression too large to display	1345
default	Expression too large to display	1345
risch	Expression too large to display	5129

input
```
int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a^3/c^5*arctan(tan(1/2*f*x+1/2*e))+2/c^5*((-1/8*(80*a^3*c^6*d^2+40*
a^3*c^5*d^3-40*a^3*c^4*d^4-15*a^3*c^3*d^5+32*a^3*c^2*d^6+4*a^3*c*d^7-8*a^3
*d^8-96*a^2*b*c^7*d-72*a^2*b*c^6*d^2-96*a^2*b*c^5*d^3-15*a^2*b*c^4*d^4+24*
a*b^2*c^8+36*a*b^2*c^7*d+144*a*b^2*c^6*d^2+51*a*b^2*c^5*d^3+24*a*b^2*c^4*d
^4-4*b^3*c^8-32*b^3*c^7*d-21*b^3*c^6*d^2-32*b^3*c^5*d^3-4*b^3*c^4*d^4)*c/(
c-d)/(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3+d^4)*tan(1/2*f*x+1/2*e)^7+1/24*c*(720*
a^3*c^6*d^2+120*a^3*c^5*d^3-520*a^3*c^4*d^4-69*a^3*c^3*d^5+320*a^3*c^2*d^6
+12*a^3*c*d^7-72*a^3*d^8-864*a^2*b*c^7*d-216*a^2*b*c^6*d^2-480*a^2*b*c^5*d
^3+27*a^2*b*c^4*d^4+216*a*b^2*c^8+108*a*b^2*c^7*d+1008*a*b^2*c^6*d^2+81*a*
b^2*c^5*d^3+120*a*b^2*c^4*d^4-12*b^3*c^8-224*b^3*c^7*d-39*b^3*c^6*d^2-224*
b^3*c^5*d^3-12*b^3*c^4*d^4)/(c^3+3*c^2*d+3*c*d^2+d^3)/(c-d)^2*tan(1/2*f*x+
1/2*e)^5-1/24*c*(720*a^3*c^6*d^2-120*a^3*c^5*d^3-520*a^3*c^4*d^4+69*a^3*c^
3*d^5+320*a^3*c^2*d^6-12*a^3*c*d^7-72*a^3*d^8-864*a^2*b*c^7*d+216*a^2*b*c^
6*d^2-480*a^2*b*c^5*d^3-27*a^2*b*c^4*d^4+216*a*b^2*c^8-108*a*b^2*c^7*d+100
8*a*b^2*c^6*d^2-81*a*b^2*c^5*d^3+120*a*b^2*c^4*d^4+12*b^3*c^8-224*b^3*c^7*
d+39*b^3*c^6*d^2-224*b^3*c^5*d^3+12*b^3*c^4*d^4)/(c-d)^3/(c^2+2*c*d+d^2)*t
an(1/2*f*x+1/2*e)^3+1/8*(80*a^3*c^6*d^2-40*a^3*c^5*d^3-40*a^3*c^4*d^4+15*a
^3*c^3*d^5+32*a^3*c^2*d^6-4*a^3*c*d^7-8*a^3*d^8-96*a^2*b*c^7*d+72*a^2*b*c^
6*d^2-96*a^2*b*c^5*d^3+15*a^2*b*c^4*d^4+24*a*b^2*c^8-36*a*b^2*c^7*d+144*a*
b^2*c^6*d^2-51*a*b^2*c^5*d^3+24*a*b^2*c^4*d^4+4*b^3*c^8-32*b^3*c^7*d+21...
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2144 vs. $2(603) = 1206$.

Time $= 0.69$ (sec) , antiderivative size $= 4346$, normalized size of antiderivative $= 6.99$

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
[1/48*(48*(a^3*c^14 - 5*a^3*c^12*d^2 + 10*a^3*c^10*d^4 - 10*a^3*c^8*d^6 +
5*a^3*c^6*d^8 - a^3*c^4*d^10)*f*x*cos(f*x + e)^4 + 192*(a^3*c^13*d - 5*a^3
*c^11*d^3 + 10*a^3*c^9*d^5 - 10*a^3*c^7*d^7 + 5*a^3*c^5*d^9 - a^3*c^3*d^11
)*f*x*cos(f*x + e)^3 + 288*(a^3*c^12*d^2 - 5*a^3*c^10*d^4 + 10*a^3*c^8*d^6
 - 10*a^3*c^6*d^8 + 5*a^3*c^4*d^10 - a^3*c^2*d^12)*f*x*cos(f*x + e)^2 + 19
2*(a^3*c^11*d^3 - 5*a^3*c^9*d^5 + 10*a^3*c^7*d^7 - 10*a^3*c^5*d^9 + 5*a^3*
c^3*d^11 - a^3*c*d^13)*f*x*cos(f*x + e) + 48*(a^3*c^10*d^4 - 5*a^3*c^8*d^6
 + 10*a^3*c^6*d^8 - 10*a^3*c^4*d^10 + 5*a^3*c^2*d^12 - a^3*d^14)*f*x + 3*(
63*a^3*c^4*d^9 - 36*a^3*c^2*d^11 + 8*a^3*d^13 - 4*(6*a^2*b + b^3)*c^9*d^4
+ 20*(2*a^3 + 3*a*b^2)*c^8*d^5 - 9*(8*a^2*b + 3*b^3)*c^7*d^6 - 5*(8*a^3 -
9*a*b^2)*c^6*d^7 - (9*a^2*b + 4*b^3)*c^5*d^8 + (63*a^3*c^8*d^5 - 36*a^3*c^
6*d^7 + 8*a^3*c^4*d^9 - 4*(6*a^2*b + b^3)*c^13 + 20*(2*a^3 + 3*a*b^2)*c^12
*d - 9*(8*a^2*b + 3*b^3)*c^11*d^2 - 5*(8*a^3 - 9*a*b^2)*c^10*d^3 - (9*a^2*
b + 4*b^3)*c^9*d^4)*cos(f*x + e)^4 + 4*(63*a^3*c^7*d^6 - 36*a^3*c^5*d^8 +
8*a^3*c^3*d^10 - 4*(6*a^2*b + b^3)*c^12*d + 20*(2*a^3 + 3*a*b^2)*c^11*d^2
- 9*(8*a^2*b + 3*b^3)*c^10*d^3 - 5*(8*a^3 - 9*a*b^2)*c^9*d^4 - (9*a^2*b +
4*b^3)*c^8*d^5)*cos(f*x + e)^3 + 6*(63*a^3*c^6*d^7 - 36*a^3*c^4*d^9 + 8*a^
3*c^2*d^11 - 4*(6*a^2*b + b^3)*c^11*d^2 + 20*(2*a^3 + 3*a*b^2)*c^10*d^3 -
9*(8*a^2*b + 3*b^3)*c^9*d^4 - 5*(8*a^3 - 9*a*b^2)*c^8*d^5 - (9*a^2*b + 4*b
^3)*c^7*d^6)*cos(f*x + e)^2 + 4*(63*a^3*c^5*d^8 - 36*a^3*c^3*d^10 + 8*a...
```

3.197.6 Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx=\int \frac{(a+b\sec{(e+fx)})^3}{(c+d\sec{(e+fx)})^5}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**5,x)
```

output
```
Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**5, x)
```

3.197.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

3.197.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3173 vs. 2(603) = 1206.

Time = 0.55 (sec) , antiderivative size = 3173, normalized size of antiderivative = 5.10

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="giac")
```

output

```
1/12*(3*(24*a^2*b*c^9 + 4*b^3*c^9 - 40*a^3*c^8*d - 60*a*b^2*c^8*d + 72*a^2
*b*c^7*d^2 + 27*b^3*c^7*d^2 + 40*a^3*c^6*d^3 - 45*a*b^2*c^6*d^3 + 9*a^2*b*
c^5*d^4 + 4*b^3*c^5*d^4 - 63*a^3*c^4*d^5 + 36*a^3*c^2*d^7 - 8*a^3*d^9)*(pi
*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x +
1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^13 - 4*c^11*d^2 +
6*c^9*d^4 - 4*c^7*d^6 + c^5*d^8)*sqrt(-c^2 + d^2)) + 12*(f*x + e)*a^3/c^5
- (72*a*b^2*c^11*tan(1/2*f*x + 1/2*e)^7 - 12*b^3*c^11*tan(1/2*f*x + 1/2*e)
^7 - 288*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e)^7 - 108*a*b^2*c^10*d*tan(1/2*f*
x + 1/2*e)^7 - 60*b^3*c^10*d*tan(1/2*f*x + 1/2*e)^7 + 240*a^3*c^9*d^2*tan(
1/2*f*x + 1/2*e)^7 + 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*e)^7 + 324*a*b^2*
c^9*d^2*tan(1/2*f*x + 1/2*e)^7 + 189*b^3*c^9*d^2*tan(1/2*f*x + 1/2*e)^7 -
600*a^3*c^8*d^3*tan(1/2*f*x + 1/2*e)^7 - 504*a^2*b*c^8*d^3*tan(1/2*f*x + 1
/2*e)^7 - 891*a*b^2*c^8*d^3*tan(1/2*f*x + 1/2*e)^7 - 183*b^3*c^8*d^3*tan(1
/2*f*x + 1/2*e)^7 + 240*a^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^7 + 459*a^2*b*c^7
*d^4*tan(1/2*f*x + 1/2*e)^7 + 801*a*b^2*c^7*d^4*tan(1/2*f*x + 1/2*e)^7 + 1
83*b^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^7 + 435*a^3*c^6*d^5*tan(1/2*f*x + 1/2*
e)^7 - 513*a^2*b*c^6*d^5*tan(1/2*f*x + 1/2*e)^7 - 189*a*b^2*c^6*d^5*tan(1/
2*f*x + 1/2*e)^7 - 189*b^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^7 - 249*a^3*c^5*d^
6*tan(1/2*f*x + 1/2*e)^7 + 153*a^2*b*c^5*d^6*tan(1/2*f*x + 1/2*e)^7 + 63*a
*b^2*c^5*d^6*tan(1/2*f*x + 1/2*e)^7 + 60*b^3*c^5*d^6*tan(1/2*f*x + 1/2*...
```

3.197.9 Mupad [B] (verification not implemented)

Time = 28.96 (sec) , antiderivative size = 21021, normalized size of antiderivative = 33.80

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

input

```
int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^5,x)
```

output
```
(atan(((((c + d)^9*(c - d)^9)^(1/2)*((tan(e/2 + (f*x)/2)*(64*a^6*c^18 + 12
8*a^6*d^18 + 16*b^6*c^18 - 128*a^6*c*d^17 - 128*a^6*c^17*d + 192*a^2*b^4*c
^18 + 576*a^4*b^2*c^18 - 1024*a^6*c^2*d^16 + 1024*a^6*c^3*d^15 + 3584*a^6*
c^4*d^14 - 3584*a^6*c^5*d^13 - 6968*a^6*c^6*d^12 + 7168*a^6*c^7*d^11 + 838
5*a^6*c^8*d^10 - 8960*a^6*c^9*d^9 - 7024*a^6*c^10*d^8 + 7168*a^6*c^11*d^7
+ 4848*a^6*c^12*d^6 - 3584*a^6*c^13*d^5 - 1920*a^6*c^14*d^4 + 1024*a^6*c^1
5*d^3 + 1152*a^6*c^16*d^2 + 16*b^6*c^10*d^8 + 216*b^6*c^12*d^6 + 761*b^6*c
^14*d^4 + 216*b^6*c^16*d^2 - 360*a*b^5*c^11*d^7 - 2910*a*b^5*c^13*d^5 - 36
00*a*b^5*c^15*d^3 - 3200*a^3*b^3*c^17*d - 144*a^5*b*c^5*d^13 - 504*a^5*b*c
^7*d^11 + 3666*a^5*b*c^9*d^9 - 6624*a^5*b*c^11*d^7 + 2016*a^5*b*c^13*d^5 -
 3840*a^5*b*c^15*d^3 + 72*a^2*b^4*c^10*d^8 + 3087*a^2*b^4*c^12*d^6 + 9552*
a^2*b^4*c^14*d^4 + 5472*a^2*b^4*c^16*d^2 - 64*a^3*b^3*c^5*d^13 - 144*a^3*b
^3*c^7*d^11 + 1376*a^3*b^3*c^9*d^9 - 3604*a^3*b^3*c^11*d^7 - 6224*a^3*b^3*
c^13*d^5 - 12640*a^3*b^3*c^15*d^3 + 720*a^4*b^2*c^6*d^12 - 2280*a^4*b^2*c^
8*d^10 + 1431*a^4*b^2*c^10*d^8 + 5256*a^4*b^2*c^12*d^6 + 4416*a^4*b^2*c^14
*d^4 + 8256*a^4*b^2*c^16*d^2 - 480*a*b^5*c^17*d - 1920*a^5*b*c^17*d))/(2*(
c^22*d + c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*
d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^
6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)) + (((32*a^3*c^27
 + 16*b^3*c^27 + 96*a^2*b*c^27 - 160*a^3*c^26*d - 16*b^3*c^26*d - 32*a^...
```

3.197. $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

3.198 $\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx$

3.198.1 Optimal result

Integrand size = 25, antiderivative size = 320

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = -\frac{2(a-b)\sqrt{a+b}d\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bf} + \frac{2\sqrt{a+b}(b(c-d)+ad)\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bf} - \frac{2\sqrt{a+b}c\cot(e+fx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

output
```
-2*(a-b)*d*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e)
)/(a-b))^(1/2)/b/f+2*(b*(c-d)+a*d)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(
1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))
^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/f-2*c*cot(f*x+e)*EllipticPi((a+b*
sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*
(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f
```

3.198.2 Mathematica [A] (verified)

Time = 10.20 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.18

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx$$
$$= \frac{\cos(e+fx)\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\left(2d\sin(e+fx)-\frac{\cos^2\left(\frac{1}{2}(e+fx)\right)\left(4(a+b)d\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+\cdots}{(a+b)\cdots}}\cdots\right)}{\cdots}\right)}{\cdots}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])*(2*d*Sin[e + f
*x] - (Cos[(e + f*x)/2]^2*(4*(a + b)*d*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x]
)]*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*EllipticE[ArcSi
n[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 4*(a*(c - d) - b*(c + d))*Sqrt[Cos
[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[
e + f*x]))]*EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 8*a*c*S
qrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1
 + Cos[e + f*x]))]*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b
)] + d*(b + a*Cos[e + f*x])*Sec[(e + f*x)/2]^3*(-Sin[(e + f*x)/2] + Sin[(3
*(e + f*x))/2])))/(b + a*Cos[e + f*x])))/(f*(d + c*Cos[e + f*x]))
```

3.198.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4404, 3042, 4271, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 4404}$$

$$\int \frac{\sec(e+fx)(bc+ad+bd\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}dx+ac\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx$$

$\big\downarrow$ 3042

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(bc+ad+bd\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+ac\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$\big\downarrow$ 4271

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(bc+ad+bd\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

$\big\downarrow$ 4493

$$(ad+b(c-d))\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx+bd\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}dx-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

$\big\downarrow$ 3042

$$(ad+b(c-d))\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+bd\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

$\big\downarrow$ 4319

$$bd\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(ad+b(c-d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

$\big\downarrow$ 4492

3.198. $\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx$

$$\frac{2\sqrt{a+b}(ad+b(c-d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}-\frac{2d(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bf}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

output
```
(-2*(a - b)*Sqrt[a + b]*d*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e +
 f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) + (2*Sqrt[a + b]*(b*(c - d
) + a*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b
]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[e + f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[
(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b)
)])/f
```

3.198.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4404
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[a*c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x]
+ Int[Csc[e + f*x]*((b*c + a*d + b*d*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]
), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.198.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(293) = 586$.

Time = 20.48 (sec) , antiderivative size = 1249, normalized size of antiderivative = 3.90

method	result	size
parts	Expression too large to display	1249
default	Expression too large to display	1840

input
```
int((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*c/f*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))
*a-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi(c
ot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+a*cos(f*x+e))/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/
2)/(b+a*cos(f*x+e))-2*d/f*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(
1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*a*cos(f*x+e)^2+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))
^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*b*cos(f*x+e)^2-(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a
-b)/(a+b))^(1/2))*a*cos(f*x+e)^2-(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),(
(a-b)/(a+b))^(1/2))*b*cos(f*x+e)^2+2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b
)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*a*cos(f*x+e)+2*EllipticF(cot(f*x+e)-csc(f*x+e),((a
-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+
e))/(cos(f*x+e)+1))^(1/2)*b*cos(f*x+e)-2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(
f*x+e),((a-b)/(a+b))^(1/2))*a*cos(f*x+e)-2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)...
```

3.198.5 Fricas [F]

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx=\int \sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)\,dx$$

input
```
integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

3.198.6 Sympy [F]

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx=\int \sqrt{a+b\sec{(e+fx)}}(c+d\sec{(e+fx)})\;dx$$

input
```
integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x)), x)
```

3.198.7 Maxima [F]

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx=\int \sqrt{b\sec{(fx+e)}+a}(d\sec{(fx+e)}+c)\,dx$$

input
```
integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

3.198.8 Giac [F]

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx=\int \sqrt{b\sec{(fx+e)}+a}(d\sec{(fx+e)}+c)\,dx$$

input
```
integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx=\int \sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)), x)
```

3.199 $\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

3.199.1 Optimal result

Integrand size = 27, antiderivative size = 220

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{cf} + \frac{2(bc-ad)\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{c(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

output
```
-2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e
))/(a-b))^(1/2)/c/f+2*(-a*d+b*c)*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/
2),2*d/(c+d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f
*x+e)/c/(c+d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

3.199.2 Mathematica [A] (verified)

Time = 8.25 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$
$$= \frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left(-((a-b)c(c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

output
```
(4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(-((a - b)*c*(c + d)*EllipticF[A
rcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]) + 2*a*(c^2 - d^2)*EllipticPi[-1
, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*d*(-(b*c) + a*d)*Elliptic
Pi[(c - d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b
*Sec[e + f*x]])/(c*(c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

3.199.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4414, 3042, 4271, 4461}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{ 4414}$$
$$\frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx}{c}+\frac{a\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx}{c}$$
$$\Big\downarrow \text{ 3042}$$

$$\frac{(bc-ad)\int \frac{\csc(e+fx+\frac{\pi}{2})}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}(c+d\csc(e+fx+\frac{\pi}{2}))}dx}{c}+\frac{a\int \frac{1}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx}{c}$$

$\big\downarrow$ 4271

$$\frac{(bc-ad)\int \frac{\csc(e+fx+\frac{\pi}{2})}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}(c+d\csc(e+fx+\frac{\pi}{2}))}dx}{c}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{cf}$$

$\big\downarrow$ 4461

$$\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\operatorname{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{cf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{cf}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

output
```
(-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
 + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(c*f) + (2*(b*c - a*d)*Ellipti
cPi[(2*d)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*
Sqrt[(a + b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + b*S
ec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])
```

3.199.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

3.199. $\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

rule 4414
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[a/c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x]
+ Simp[(b*c - a*d)/c   Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Cs
c[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4461
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.199.4 Maple [A] (verified)

Time = 6.16 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.82

method	result
default	$\frac{2(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)a\,c^2+\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)acd-\text{EllipticF}\left(\cot(fx+\right.\right.}{}$

input
```
int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/f/c/(c-d)/(c+d)*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(
a+b))^(1/2))*a*c^2+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*
c*d-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b*c^2-EllipticF(c
ot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b*c*d-2*EllipticPi(cot(f*x+e)-cs
c(f*x+e),-1,((a-b)/(a+b))^(1/2))*a*c^2+2*EllipticPi(cot(f*x+e)-csc(f*x+e),
-1,((a-b)/(a+b))^(1/2))*a*d^2-2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+
d),((a-b)/(a+b))^(1/2))*a*d^2+2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+
d),((a-b)/(a+b))^(1/2))*b*c*d)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x
+e))
```

3.199.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.199.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a+b\sec{(e+fx)}}}{c+d\sec{(e+fx)}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x)), x)
```

3.199.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec{(fx+e)}+a}}{d\sec{(fx+e)}+c}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

3.199.8 Giac [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{b\sec(fx+e)+a}}{d\sec(fx+e)+c}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)), x)
```

3.200 $\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$

3.200.1 Optimal result

Integrand size = 25, antiderivative size = 380

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}(3bc+4ad)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{3bf}$$

$$+\frac{2\sqrt{a+b}(ab(6c-4d)-b^2(3c-d)+3a^2d)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+}{a+b}}}{3bf}$$

$$-\frac{2a\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

$$+\frac{2bd\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{3f}$$

output
```
-2/3*(a-b)*(4*a*d+3*b*c)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b
*(1+sec(f*x+e))/(a-b))^(1/2)/b/f+2/3*(a*b*(6*c-4*d)-b^2*(3*c-d)+3*a^2*d)*c
ot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/
2)/b/f-2*a*c*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b
)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1
+sec(f*x+e))/(a-b))^(1/2)/f+2/3*b*d*(a+b*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

3.200.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 6063 vs. $2(380) = 760$.

Time = 23.57 (sec) , antiderivative size = 6063, normalized size of antiderivative = 15.96

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

output
```
Result too large to show
```

3.200.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4406, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4406}$$

$$\frac{2}{3}\int \frac{3ca^2+b(3bc+4ad)\sec^2(e+fx)+\left(3da^2+6bca+b^2d\right)\sec(e+fx)}{2\sqrt{a+b\sec(e+fx)}}dx+\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3ca^2+b(3bc+4ad)\sec^2(e+fx)+\left(3da^2+6bca+b^2d\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx+\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3ca^2+b(3bc+4ad)\csc\left(e+fx+\frac{\pi}{2}\right)^2+\left(3da^2+6bca+b^2d\right)\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+$$
$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$\downarrow$ 4546

$$\frac{1}{3}\left(\int \frac{3ca^2+\left(3da^2+6bca+b^2d-b(3bc+4ad)\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx+b(4ad+3bc)\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}d\right.$$
$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\int \frac{3ca^2+\left(3da^2+6bca+b^2d-b(3bc+4ad)\right)\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+b(4ad+3bc)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e\right.\right.}{\sqrt{a+b\csc\left(e+f\right.}}\right.$$
$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$\downarrow$ 4409

$$\frac{1}{3}\left(\left(3a^2d+ab(6c-4d)-b^2(3c-d)\right)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx+3a^2c\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx+b(4ad+3bc)\int\right.$$
$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\left(3a^2d+ab(6c-4d)-b^2(3c-d)\right)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+3a^2c\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+b(4a\right.$$
$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$\downarrow$ 4271

$$\frac{1}{3}\left(\left(3a^2d+ab(6c-4d)-b^2(3c-d)\right)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+b(4ad+3bc)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+\right.\right.}{\sqrt{a+b\csc\left(e+f\right.}}\right.$$
$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

3.200. $\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$

$$\Big\downarrow \text{4319}$$

$$\frac{1}{3}\left(b(4ad+3bc)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\left(3a^2d+ab(6c-4d)-b^2(3c-d)\right)\cot(e\right.$$

$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \text{4492}$$

$$\frac{1}{3}\left(\frac{2\sqrt{a+b}\left(3a^2d+ab(6c-4d)-b^2(3c-d)\right)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a}}{}\right.\right.}{bf}\right.$$

$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

input
```
Int[(a + b*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

output
```
((-2*(a - b)*Sqrt[a + b]*(3*b*c + 4*a*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqr
t[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e +
f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) + (2*Sqrt[a
 + b]*(a*b*(6*c - 4*d) - b^2*(3*c - d) + 3*a^2*d)*Cot[e + f*x]*EllipticF[A
rcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) -
(6*a*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Se
c[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f)/3 + (2*b*d*Sqrt[a + b*Se
c[e + f*x]]*Tan[e + f*x])/(3*f)
```

3.200.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4406
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Simp[1/m   Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
 + (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
 - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*
c - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.200. $\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$

3.200.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2983 vs. $2(345) = 690$.

Time = 24.79 (sec) , antiderivative size = 2984, normalized size of antiderivative = 7.85

method	result	size
parts	Expression too large to display	2984
default	Expression too large to display	2988

input
```
int((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2*c/f*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*
cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2*co
s(f*x+e)^2-2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)
*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
a*b*cos(f*x+e)^2-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(
a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*b^2*cos(f*x+e)^2+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))
^(1/2))*a*b*cos(f*x+e)^2+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a
+b))^(1/2))*b^2*cos(f*x+e)^2-2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x+e),-1
,((a-b)/(a+b))^(1/2))*a^2*cos(f*x+e)^2+2*EllipticF(cot(f*x+e)-csc(f*x+e),(
(a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*a^2*cos(f*x+e)-4*EllipticF(cot(f*x+e)-csc(f*x+
e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b*cos(f*x+e)-2*EllipticF(cot(f*x+e)-csc(
f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2*cos(f*x+e)+2*(1/(a+b)*(b+a*cos(f*x
+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(...
```

3.200.5 Fricas [F]

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*d*sec(f*x + e)^2 + a*c + (b*c + a*d)*sec(f*x + e))*sqrt(b*sec(
f*x + e) + a), x)
```

3.200.6 Sympy [F]

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int (a+b\sec(e+fx))^{\frac{3}{2}}\,(c+d\sec(e+fx))\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e)),x)
```

output
```
Integral((a + b*sec(e + f*x))**(3/2)*(c + d*sec(e + f*x)), x)
```

3.200.7 Maxima [F]

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c), x)
```

3.200.8 Giac [F]

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int \left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)dx$$

input
```
int((a + b/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)),x)
```

output
```
int((a + b/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)), x)
```

3.201 $\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$

3.201.1 Optimal result

Integrand size = 27, antiderivative size = 326

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \frac{2b\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{df}$$
$$-\frac{2a\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{cf}$$
$$-\frac{2(bc-ad)^2\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{cd(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

output
```
2*b*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^
(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b)
)^(1/2)/d/f-2*a*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(
a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b
*(1+sec(f*x+e))/(a-b))^(1/2)/c/f-2*(-a*d+b*c)^2*EllipticPi(1/2*(1-sec(f*x+
e))^(1/2)*2^(1/2),2*d/(c+d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+
b))^(1/2)*tan(f*x+e)/c/d/(c+d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1
/2)
```

3.201.2 Mathematica [A] (verified)

Time = 9.68 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx =$$
$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((a-b)^2c(c+d)\operatorname{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)\ldots\right.}{\ldots}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

output
```
(-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*C
os[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)^2*c*(c + d)*EllipticF[
ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*(a^2*(c^2 - d^2)*EllipticPi
[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + (b*c - a*d)^2*EllipticPi
[(c - d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))*Sqrt[a + b*
Sec[e + f*x]])/(c*(c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

3.201.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3042, 4416, 3042, 4409, 3042, 4271, 4319, 4461}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{4416}$$
$$\frac{\int \frac{da^2+b^2c\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{cd}-\frac{(bc-ad)^2\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx}{cd}$$
$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{da^2+b^2c\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{cd}-\frac{(bc-ad)^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{cd}$$

$$\Big\downarrow \text{4409}$$

$$\frac{a^2d\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx+b^2c\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{cd}-\frac{(bc-ad)^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{cd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2d\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+b^2c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{cd}-\frac{(bc-ad)^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{cd}$$

$$\Big\downarrow \text{4271}$$

$$\frac{b^2c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2ad\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}}{cd}-\frac{(bc-ad)^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{cd}$$

$$\Big\downarrow \text{4319}$$

$$\frac{\frac{2bc\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}-\frac{2ad\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-}}{}}{cd}-\frac{(bc-ad)^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{cd}$$

$$\Big\downarrow \text{4461}$$

$$\frac{\frac{2bc\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}-\frac{2ad\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-}}{}}{cd}$$
$$-\frac{2(bc-ad)^2\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{cdf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$

input
```
Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

3.201. $\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$

output
```
((2*b*Sqrt[a + b]*c*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[e + f*x]))/(a - b))])/f - (2*a*Sqrt[a + b]*d*Cot[e + f*x]*El
lipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x])
)/(a - b))])/f)/(c*d) - (2*(b*c - a*d)^2*EllipticPi[(2*d)/(c + d), ArcSin[
Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])/
(a + b)]*Tan[e + f*x])/(c*d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[-Tan[e
 + f*x]^2])
```

3.201.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.201. $\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$

rule 4416
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Simp[1/(c*d)   Int[(a^2*d + b^2*c*Csc[e + f*x])/
Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[(b*c - a*d)^2/(c*d)   Int[Csc[e + f
*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0]
```

rule 4461
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.201.4 Maple [A] (verified)

Time = 8.07 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.63

method	result
default	$\frac{2(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)a^2c^2+\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)a^2cd-2\,\text{EllipticF}\left(\cot(f\right.\right.}{}$

input
```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/f/c/(c+d)/(c-d)*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(
a+b))^(1/2))*a^2*c^2+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*
a^2*c*d-2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b*c^2-2*E
llipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b*c*d+EllipticF(cot(
f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b^2*c^2+EllipticF(cot(f*x+e)-csc(f*
x+e),((a-b)/(a+b))^(1/2))*b^2*c*d-2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((
a-b)/(a+b))^(1/2))*a^2*c^2+2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a
+b))^(1/2))*a^2*d^2-2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/
(a+b))^(1/2))*a^2*d^2+4*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b
)/(a+b))^(1/2))*a*b*c*d-2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a
-b)/(a+b))^(1/2))*b^2*c^2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))
```

3.201.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.201.6 Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(a+b\sec{(e+fx)})^{\frac{3}{2}}}{c+d\sec{(e+fx)}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x)), x)
```

3.201.7 Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{3}{2}}}{d\sec{(fx+e)}+c}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

3.201.8 Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{d\sec(fx+e)+c}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)),x)
```

output
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)), x)
```

3.202 $\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

3.202.1 Optimal result

Integrand size = 25, antiderivative size = 442

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}(35abc+23a^2d+9b^2d)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{15bf}$$

$$+\frac{2\sqrt{a+b}(a^2b(45c-23d)-ab^2(35c-17d)+b^3(5c-9d)+15a^3d)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e}}{\sqrt{a+b}}\right.\right.}{15bf}$$

$$-\frac{2a^2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

$$+\frac{2b(5bc+8ad)\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{15f}+\frac{2bd(a+b\sec(e+fx))^{3/2}\tan(e+fx)}{5f}$$

output
```
-2/15*(a-b)*(23*a^2*d+35*a*b*c+9*b^2*d)*cot(f*x+e)*EllipticE((a+b*sec(f*x+
e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(
a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/f+2/15*(a^2*b*(45*c-23*d)-a*
b^2*(35*c-17*d)+b^3*(5*c-9*d)+15*a^3*d)*cot(f*x+e)*EllipticF((a+b*sec(f*x+
e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(
a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/f-2*a^2*c*cot(f*x+e)*Ellipti
cPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^
(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f+2/5
*b*d*(a+b*sec(f*x+e))^(3/2)*tan(f*x+e)/f+2/15*b*(8*a*d+5*b*c)*(a+b*sec(f*x
+e))^(1/2)*tan(f*x+e)/f
```

3.202.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7138 vs. $2(442) = 884$.

Time = 26.04 (sec) , antiderivative size = 7138, normalized size of antiderivative = 16.15

$$\int (a + b\sec(e + fx))^{5/2}(c + d\sec(e + fx))\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

output
```
Result too large to show
```

3.202.3 Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4406, 27, 3042, 4544, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sec(e + fx))^{5/2}(c + d\sec(e + fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a + b\csc\left(e + fx + \frac{\pi}{2}\right)\right)^{5/2}\left(c + d\csc\left(e + fx + \frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4406}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a + b\sec(e + fx)}\left(5ca^2 + b(5bc + 8ad)\sec^2(e + fx) + \left(5da^2 + 10bca + 3b^2d\right)\sec(e + fx)\right)dx + \frac{2bd\tan(e + fx)(a + b\sec(e + fx))^{3/2}}{5f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{a + b\sec(e + fx)}\left(5ca^2 + b(5bc + 8ad)\sec^2(e + fx) + \left(5da^2 + 10bca + 3b^2d\right)\sec(e + fx)\right)dx + \frac{2bd\tan(e + fx)(a + b\sec(e + fx))^{3/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(5ca^2+b(5bc+8ad)\csc\left(e+fx+\frac{\pi}{2}\right)^2+\left(5da^2+10bca+3b^2d\right)\csc\left(e+fx+\frac{\pi}{2}\right.\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$\downarrow$ 4544

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{15ca^3+b\left(23da^2+35bca+9b^2d\right)\sec^2(e+fx)+\left(15da^3+45bca^2+17b^2da+5b^3c\right)\sec(e+fx)}{2\sqrt{a+b\sec(e+fx)}}dx+\frac{2b(8\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15ca^3+b\left(23da^2+35bca+9b^2d\right)\sec^2(e+fx)+\left(15da^3+45bca^2+17b^2da+5b^3c\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx+\frac{2b(8\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15ca^3+b\left(23da^2+35bca+9b^2d\right)\csc\left(e+fx+\frac{\pi}{2}\right)^2+\left(15da^3+45bca^2+17b^2da+5b^3c\right)\csc\left(e+fx+\frac{\pi}{2}\right.}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$\downarrow$ 4546

$$\frac{1}{5}\left(\frac{1}{3}\left(b\left(23a^2d+35abc+9b^2d\right)\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}dx+\int \frac{15ca^3+\left(15da^3+45bca^2+17b^2da+5b^3\right.}{\sqrt{a+b\,\mathrm{se}}}\right.\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(b\left(23a^2d+35abc+9b^2d\right)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\int \frac{15ca^3+\left(15da^3+45bca^2+1\right.}{}\right.\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$\downarrow$ 4409

3.202. $\int(a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^3c\int\frac{1}{\sqrt{a+b\sec(e+fx)}}dx+b\left(23a^2d+35abc+9b^2d\right)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx\right.\right.$$

$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^3c\int\frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+b\left(23a^2d+35abc+9b^2d\right)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}\right.\right.$$

$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$$\Big\downarrow \text{4271}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(b\left(23a^2d+35abc+9b^2d\right)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\left(15a^3d+a^2b(45c-23d)-ab^2(\right.\right.\right.$$

$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$$\Big\downarrow \text{4319}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(b\left(23a^2d+35abc+9b^2d\right)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{30a^2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-s}{}}}{}\right.\right.$$

$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$$\Big\downarrow \text{4492}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(-\frac{2(a-b)\sqrt{a+b}\left(23a^2d+35abc+9b^2d\right)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e}}{\sqrt{a+b}}\right.\right.}{bf}\right.\right.$$

$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

input
```
Int[(a + b*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

3.202. $\int(a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

output
```
(2*b*d*(a + b*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f) + (((-2*(a - b)*Sqrt
[a + b]*(35*a*b*c + 23*a^2*d + 9*b^2*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt
[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) + (2*Sqrt[a
+ b]*(a^2*b*(45*c - 23*d) - a*b^2*(35*c - 17*d) + b^3*(5*c - 9*d) + 15*a^3
*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e
+ f*x]))/(a - b))])/(b*f) - (30*a^2*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[
(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b)
)])/f)/3 + (2*b*(5*b*c + 8*a*d)*Sqrt[a + b*Sec[e + f*x]]*Tan[e + f*x])/(3*
f))/5
```

3.202.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4406
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Simp[1/m   Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
 + (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
 - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*
c - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4544
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[(
a + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m
)*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.202. $\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

3.202.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4160 vs. $2(403) = 806$.

Time = 32.02 (sec) , antiderivative size = 4161, normalized size of antiderivative = 9.41

method	result	size
parts	Expression too large to display	4161
default	Expression too large to display	4180

input
```
int((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/3*c/f*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))/(cos(f*x+e)+1)*(-14*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^
(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2*cos(f*x+e)+14
*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b*cos(f*
x+e)+14*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2
*cos(f*x+e)-18*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(
f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2
)*a^2*b*cos(f*x+e)-2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e
)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*b^3*cos(f*x+e)+7*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f
*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e
)+1))^(1/2)*a^2*b+7*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)
-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))
^(1/2)*a*b^2-9*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(
f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2
)*a^2*b-7*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e
),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b
^2-(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((...
```

3.202.5 Fricas [F]

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int (b\sec{(fx+e)}+a)^{\frac{5}{2}}(d\sec{(fx+e)}+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b^2*d*sec(f*x + e)^3 + a^2*c + (b^2*c + 2*a*b*d)*sec(f*x + e)^2
+ (2*a*b*c + a^2*d)*sec(f*x + e))*sqrt(b*sec(f*x + e) + a), x)
```

3.202.6 Sympy [F]

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int (a+b\sec{(e+fx)})^{\frac{5}{2}}\,(c+d\sec{(e+fx)})\,\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e)),x)
```

output
```
Integral((a + b*sec(e + f*x))**(5/2)*(c + d*sec(e + f*x)), x)
```

3.202.7 Maxima [F]

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int (b\sec{(fx+e)}+a)^{\frac{5}{2}}(d\sec{(fx+e)}+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c), x)
```

3.202.8 Giac [F]

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c), x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int \left(a+\frac{b}{\cos(e+f\,x)}\right)^{5/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((a + b/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)),x)
```

output
```
int((a + b/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)), x)
```

3.203 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$

3.203.1 Optimal result

Integrand size = 25, antiderivative size = 208

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \frac{2\sqrt{a+b}d\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bf} - \frac{2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{af}$$

output
```
2*d*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^
(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b)
)^(1/2)/b/f-2*c*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(
a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b
*(1+sec(f*x+e))/(a-b))^(1/2)/a/f
```

3.203.2 Mathematica [A] (verified)

Time = 3.78 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.70

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$
$$= \frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((-c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)+2\right.}{f\sqrt{a+b\sec(e+fx)}}$$

input
```
Integrate[(c + d*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((-c + d)*EllipticF[ArcSin[Tan[(
e + f*x)/2]], (a - b)/(a + b)] + 2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2
]], (a - b)/(a + b)])*Sec[e + f*x])/(f*Sqrt[a + b*Sec[e + f*x]])
```

3.203.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4409, 3042, 4271, 4319}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4409}$$
$$c\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx+d\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$c\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4271}$$

$$d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af}$$

$$\Big\downarrow \text{4319}$$

$$\frac{2d\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af}$$

input
```
Int[(c + d*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[a + b]*d*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*Ell
ipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))
/(a - b))])/(a*f)
```

3.203.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

3.203. $\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.203.4 Maple [A] (verified)

Time = 17.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.88

method	result
default	$\frac{2(\cos(fx+e)+1)\,(\text{EllipticF}(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}})c-\text{EllipticF}(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}})d-2\,\text{EllipticPi}(\cot(fx+e)-}{f(b+a\cos(fx+e))}$
parts	$\frac{2c(\cos(fx+e)+1)\,(\text{EllipticF}(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}})-2\,\text{EllipticPi}(\cot(fx+e)-\csc(fx+e),-1,\sqrt{\frac{a-b}{a+b}}))\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+}}}{f(b+a\cos(fx+e))}$

input
```
int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*c
-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*d-2*EllipticPi(cot(f
*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*c)*(1/(a+b)*(b+a*cos(f*x+e))/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)
/(b+a*cos(f*x+e))
```

3.203.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.203.6 Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{c+d\sec{(e+fx)}}{\sqrt{a+b\sec{(e+fx)}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*sec(e + f*x))/sqrt(a + b*sec(e + f*x)), x)
```

3.203.7 Maxima [F]

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{d\sec{(fx+e)}+c}{\sqrt{b\sec{(fx+e)}+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

3.203.8 Giac [F]

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{d\sec{(fx+e)}+c}{\sqrt{b\sec{(fx+e)}+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{c+\frac{d}{\cos(e+f\,x)}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(1/2), x)
```

3.204 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

3.204.1 Optimal result

Integrand size = 27, antiderivative size = 216

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{acf} - \frac{2d\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{c(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

output
```
-2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e
))/(a-b))^(1/2)/a/c/f-2*d*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/
(c+d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/c
/(c+d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 21.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx =$$

$$-\frac{2\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}(d+c\cos(e+fx))\left(c(c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)-2\left((c^2-d^2\cdots\right.\right.}{c(c-\cdots}$$

input
```
Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output
```
(-2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(d + c*Cos[e +
 f*x])*(c*(c + d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2
*((c^2 - d^2)*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] +
d^2*EllipticPi[(c - d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]
))*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sec[e + f*x]^(3/2)*Sqrt[1 + Sec[e
 + f*x]])/(c*(c - d)*(c + d)*f*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[a + b*Sec[e +
 f*x]]*(c + d*Sec[e + f*x]))
```

3.204.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4418, 3042, 4271, 4461}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4418}$$

$$\frac{\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx}{c}-\frac{d\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{c}$$

$$\Big\downarrow \text{4271}$$

$$-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{c}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{acf}$$

$$\Big\downarrow \text{4461}$$

$$-\frac{2d\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{cf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{acf}$$

input
```
Int[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output
```
(-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
 + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*c*f) - (2*d*EllipticPi[(2*d
)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a
+ b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + b*Sec[e + f
*x]]*Sqrt[-Tan[e + f*x]^2])
```

3.204.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4418
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Simp[1/c   Int[1/Sqrt[a + b*Csc[e + f*x]], x],
 x] - Simp[d/c   Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0]
```

rule 4461
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.204.4 Maple [A] (verified)

Time = 7.08 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.30

method	result
default	$\frac{2(\cos(fx+e)+1)\left(c^2 \operatorname{EllipticF}\left(\cot(fx+e)-\csc(fx+e), \sqrt{\frac{a-b}{a+b}}\right)+d \operatorname{EllipticF}\left(\cot(fx+e)-\csc(fx+e), \sqrt{\frac{a-b}{a+b}}\right)c-2c^2 \operatorname{EllipticPi}\left(\cot(f\right.\right.}{}$

input
```
int(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/c/(c+d)/(c-d)*(cos(f*x+e)+1)*(c^2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-
b)/(a+b))^(1/2))+d*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*c-
2*c^2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))+2*EllipticP
i(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*d^2-2*EllipticPi(cot(f*x+e
)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2))*d^2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e)
)^(1/2)/(b+a*cos(f*x+e))
```

3.204.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\text{Timed out}$$

input
```
integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.204.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{1}{\sqrt{a+b\sec\left(e+fx\right)}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

3.204.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{1}{\sqrt{b\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

3.204.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

output
```
int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.205 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$

3.205.1 Optimal result

Integrand size = 25, antiderivative size = 376

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1-\cdots}{\cdots}}}{ab\sqrt{a+b}f}$$
$$-\frac{2(bc-ad)\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{ab\sqrt{a+b}f}$$
$$-\frac{2\sqrt{a+b}c\cot(e+fx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a^2f}$$
$$+\frac{2b(bc-ad)\tan(e+fx)}{a(a^2-b^2)f\sqrt{a+b\sec(e+fx)}}$$

output
```
2*(-a*d+b*c)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(
1/2)/a/b/f/(a+b)^(1/2)-2*(-a*d+b*c)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^
(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*
(1+sec(f*x+e))/(a-b))^(1/2)/a/b/f/(a+b)^(1/2)-2*c*cot(f*x+e)*EllipticPi((a
+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*
(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a^2/f+2*b*(
-a*d+b*c)*tan(f*x+e)/a/(a^2-b^2)/f/(a+b*sec(f*x+e))^(1/2)
```

3.205.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1138 vs. $2(376) = 752$.

Time = 13.36 (sec) , antiderivative size = 1138, normalized size of antiderivative = 3.03

$$\int \frac{c + d\sec(e + fx)}{(a + b\sec(e + fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
((b + a*Cos[e + f*x])^2*Sec[e + f*x]*(c + d*Sec[e + f*x])*((2*(-(b*c) + a*
d)*Sin[e + f*x])/(a*(a^2 - b^2)) - (2*(-(b^2*c*Sin[e + f*x]) + a*b*d*Sin[e
 + f*x]))/(a*(a^2 - b^2)*(b + a*Cos[e + f*x]))))/(f*(d + c*Cos[e + f*x])*(
a + b*Sec[e + f*x])^(3/2)) - (2*(b + a*Cos[e + f*x])^(3/2)*Sqrt[Sec[e + f*
x]]*(c + d*Sec[e + f*x])*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f
*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)]*(-(a*b*c*Tan[(e + f*x)/2]) - b^2*c*Tan
[(e + f*x)/2] + a^2*d*Tan[(e + f*x)/2] + a*b*d*Tan[(e + f*x)/2] + 2*a*b*c*
Tan[(e + f*x)/2]^3 - 2*a^2*d*Tan[(e + f*x)/2]^3 - a*b*c*Tan[(e + f*x)/2]^5
 + b^2*c*Tan[(e + f*x)/2]^5 + a^2*d*Tan[(e + f*x)/2]^5 - a*b*d*Tan[(e + f*
x)/2]^5 - 2*a^2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)
]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[
(e + f*x)/2]^2)/(a + b)] + 2*b^2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]]
, (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f
*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - 2*a^2*c*EllipticPi[-1, ArcSin[
Tan[(e + f*x)/2]], (a - b)/(a + b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f
*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a +
b)] + 2*b^2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Ta
n[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x
)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (a + b)*(-(b*c) + a*d)*EllipticE
[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2...
```

3.205.3 Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4411, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4411}$$

$$\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}-\frac{2\int -\frac{-b(bc-ad)\sec^2(e+fx)-a(bc-ad)\sec(e+fx)+\left(a^2-b^2\right)c}{2\sqrt{a+b\sec(e+fx)}}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-b(bc-ad)\sec^2(e+fx)-a(bc-ad)\sec(e+fx)+\left(a^2-b^2\right)c}{\sqrt{a+b\sec(e+fx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{-b(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)^2-a(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)+\left(a^2-b^2\right)c}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 4546}$$

$$\frac{\int \frac{\left(a^2-b^2\right)c+(b(bc-ad)-a(bc-ad))\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx-b(bc-ad)\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\left(a^2-b^2\right)c+(b(bc-ad)-a(bc-ad))\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-b(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 4409}$$

$$\frac{c\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx-b(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-(a-b)(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

3.205. $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}dx$

$$\frac{c\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-(a-b)(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-b(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+\right.}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}}{a\left(a^2-b^2\right)}$$

$$\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$\downarrow$ 4271

$$\frac{-(a-b)(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-b(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2c\sqrt{a+b}\left(a^2-b^2\right)\cot(e+fx)\sqrt{\frac{b(}{}}}{}}{a\left(a^2-b^2\right)}$$

$$\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$\downarrow$ 4319

$$\frac{-b(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2c\sqrt{a+b}\left(a^2-b^2\right)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\text{arcsi}\right.}{af}}{a\left(a^2-\right.}$$

$$\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$\downarrow$ 4492

$$\frac{-\frac{2c\sqrt{a+b}\left(a^2-b^2\right)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af}-\frac{2(a-b)\sqrt{a+b}(bc-ad)\cot(e}{}}{}$$

$$\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

input
```
Int[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(3/2),x]
```

3.205. $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$

output
```
((2*(a - b)*Sqrt[a + b]*(b*c - a*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a +
 b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) - (2*(a - b)*Sqr
t[a + b]*(b*c - a*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*(a^2 - b^2)*c*
Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[e + f*x]))/(a - b))])/(a*f))/(a*(a^2 - b^2)) + (2*b*(b*c - a*d)*Tan[
e + f*x])/(a*(a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x]])
```

3.205.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4411
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2)
)   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
 a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && N
eQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.205.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2882 vs. $2(347) = 694$.

Time = 14.50 (sec) , antiderivative size = 2883, normalized size of antiderivative = 7.67

method	result	size
parts	Expression too large to display	2883
default	Expression too large to display	2987

input
```
int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*c/f/a/(a+b)/(a-b)*(-((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2
*csc(f*x+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e
))^2*csc(f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x
+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-
b)/(a+b))^(1/2))*a^2-((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*
csc(f*x+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e)
)^2*csc(f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+
e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b
)/(a+b))^(1/2))*a*b+((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))
^2*csc(f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e
))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)
/(a+b))^(1/2))*a*b+((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^
2*csc(f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e)
)^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/
(a+b))^(1/2))*b^2+2*((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))
^2*csc(f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e
))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x+e),-1...
```

3.205.5 Fricas [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)/(b^2*sec(f*x + e)^2
 + 2*a*b*sec(f*x + e) + a^2), x)
```

3.205.6 Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{c+d\sec\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*sec(e + f*x))/(a + b*sec(e + f*x))**(3/2), x)
```

3.205.7 Maxima [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(3/2), x)
```

3.205.8 Giac [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(3/2), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c + d\sec(e + fx)}{(a + b\sec(e + fx))^{3/2}}\,dx = \int \frac{c + \frac{d}{\cos(e+f\,x)}}{\left(a + \frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(3/2),x)
```

output
```
int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(3/2), x)
```

3.206 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx$

3.206.1 Optimal result

Integrand size = 25, antiderivative size = 495

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx = \frac{2(7a^2bc-3b^3c-4a^3d)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\cdots}{3a^2(a-b)b(a+b)^{3/2}f}$$
$$-\frac{2(6a^2bc-ab^2c-3b^3c-3a^3d+a^2bd)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\cdots}{3a^2(a-b)b(a+b)^{3/2}f}$$
$$-\frac{2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a^3f}$$
$$+\frac{2b(bc-ad)\tan(e+fx)}{3a(a^2-b^2)f(a+b\sec(e+fx))^{3/2}}+\frac{2b(7a^2bc-3b^3c-4a^3d)\tan(e+fx)}{3a^2(a^2-b^2)^2f\sqrt{a+b\sec(e+fx)}}$$

output
```
2/3*(-4*a^3*d+7*a^2*b*c-3*b^3*c)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/
2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+
sec(f*x+e))/(a-b))^(1/2)/a^2/(a-b)/b/(a+b)^(3/2)/f-2/3*(-3*a^3*d+6*a^2*b*c
+a^2*b*d-a*b^2*c-3*b^3*c)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b
)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x
+e))/(a-b))^(1/2)/a^2/(a-b)/b/(a+b)^(3/2)/f-2*c*cot(f*x+e)*EllipticPi((a+b
*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b
*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a^3/f+2/3*b*(
-a*d+b*c)*tan(f*x+e)/a/(a^2-b^2)/f/(a+b*sec(f*x+e))^(3/2)+2/3*b*(-4*a^3*d+
7*a^2*b*c-3*b^3*c)*tan(f*x+e)/a^2/(a^2-b^2)^2/f/(a+b*sec(f*x+e))^(1/2)
```

3.206.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1589 vs. $2(495) = 990$.

Time = 14.93 (sec) , antiderivative size = 1589, normalized size of antiderivative = 3.21

$$\int \frac{c + d\sec(e + fx)}{(a + b\sec(e + fx))^{5/2}} \, dx = \text{Too large to display}$$

input
```
Integrate[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(5/2),x]
```

output
```
((b + a*Cos[e + f*x])^3*Sec[e + f*x]^2*(c + d*Sec[e + f*x])*((2*(-7*a^2*b*
c + 3*b^3*c + 4*a^3*d)*Sin[e + f*x])/(3*a^2*(a^2 - b^2)^2) - (2*(b^3*c*Sin
[e + f*x] - a*b^2*d*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)*(b + a*Cos[e + f*x])
^2) - (2*(-8*a^2*b^2*c*Sin[e + f*x] + 4*b^4*c*Sin[e + f*x] + 5*a^3*b*d*Sin
[e + f*x] - a*b^3*d*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)^2*(b + a*Cos[e + f*x
]))))/(f*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(5/2)) - (2*(b + a*Cos[
e + f*x])^(5/2)*Sec[e + f*x]^(3/2)*(c + d*Sec[e + f*x])*Sqrt[(a + b - a*Ta
n[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)]*(-7*a^3
*b*c*Tan[(e + f*x)/2] - 7*a^2*b^2*c*Tan[(e + f*x)/2] + 3*a*b^3*c*Tan[(e +
f*x)/2] + 3*b^4*c*Tan[(e + f*x)/2] + 4*a^4*d*Tan[(e + f*x)/2] + 4*a^3*b*d*
Tan[(e + f*x)/2] + 14*a^3*b*c*Tan[(e + f*x)/2]^3 - 6*a*b^3*c*Tan[(e + f*x)
/2]^3 - 8*a^4*d*Tan[(e + f*x)/2]^3 - 7*a^3*b*c*Tan[(e + f*x)/2]^5 + 7*a^2*
b^2*c*Tan[(e + f*x)/2]^5 + 3*a*b^3*c*Tan[(e + f*x)/2]^5 - 3*b^4*c*Tan[(e +
 f*x)/2]^5 + 4*a^4*d*Tan[(e + f*x)/2]^5 - 4*a^3*b*d*Tan[(e + f*x)/2]^5 - 6
*a^4*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2
]^2)/(a + b)] + 12*a^2*b^2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a -
 b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]
^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - 6*b^4*c*EllipticPi[-1, ArcSin[Tan[(e
 + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b ...
```

3.206.3 Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4411, 27, 3042, 4548, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4411}$$

$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}-\frac{2\int -\frac{b(bc-ad)\sec^2(e+fx)-3a(bc-ad)\sec(e+fx)+3\left(a^2-b^2\right)c}{2(a+b\sec(e+fx))^{3/2}}dx}{3a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b(bc-ad)\sec^2(e+fx)-3a(bc-ad)\sec(e+fx)+3\left(a^2-b^2\right)c}{(a+b\sec(e+fx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)^2-3a(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)+3\left(a^2-b^2\right)c}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4548}$$

$$\frac{\frac{2b\left(-4a^3d+7a^2bc-3b^3c\right)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}-\frac{2\int -\frac{3c\left(a^2-b^2\right)^2-b\left(-4da^3+7bca^2-3b^3c\right)\sec^2(e+fx)-a\left(-3da^3+6bca^2-b^2da-2b^3c\right)\sec(e+fx)}{2\sqrt{a+b\sec(e+fx)}}dx}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3c\left(a^2-b^2\right)^2-b\left(-4da^3+7bca^2-3b^3c\right)\sec^2(e+fx)-a\left(-3da^3+6bca^2-b^2da-2b^3c\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b\left(-4a^3d+7a^2bc-3b^3c\right)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}}{3a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.206. $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}dx$

$$\frac{\frac{\int \frac{3c\left(a^2-b^2\right)^2-b\left(-4da^3+7bca^2-3b^3c\right)\csc\left(e+fx+\frac{\pi}{2}\right)^2-a\left(-3da^3+6bca^2-b^2da-2b^3c\right)\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b(-4a^3d+7a^2bc-3b^3c)\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}}{3a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 4546

$$\frac{\frac{\int \frac{3c\left(a^2-b^2\right)^2+\left(b\left(-4da^3+7bca^2-3b^3c\right)-a\left(-3da^3+6bca^2-b^2da-2b^3c\right)\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx-b(-4a^3d+7a^2bc-3b^3c)\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}dx}{a(a^2-b^2)}+\frac{2b(-4a\cdots}{af(\cdots}}{3a\left(a^2-b^2\right)}$$
$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{3c\left(a^2-b^2\right)^2+\left(b\left(-4da^3+7bca^2-3b^3c\right)-a\left(-3da^3+6bca^2-b^2da-2b^3c\right)\right)\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-b(-4a^3d+7a^2bc-3b^3c)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}d\cdots}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$
$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 4409

$$\frac{\frac{3c(a^2-b^2)^2\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx-b(-4a^3d+7a^2bc-3b^3c)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+(a-b)\left(3a^3d-a^2b(6c+d)+ab^2c+3b^3c\right)\int \frac{\sec(e+\cdots}{\sqrt{a+b\sec(\cdots}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$
$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3c(a^2-b^2)^2\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-b(-4a^3d+7a^2bc-3b^3c)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+(a-b)\left(3a^3d-a^2b(6c+d)+ab^2c+3b^3c\right)\int \frac{\mathrm{cs}\cdots}{\sqrt{a+\cdots}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$
$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 4271

3.206. $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx$

$$\frac{-b(-4a^3d+7a^2bc-3b^3c)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+(a-b)(3a^3d-a^2b(6c+d)+ab^2c+3b^3c)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{6c\sqrt{a+b}\left(a^2-b^2\right)^2 \cdots}{\cdots}}{a(a^2-b^2)} \over 3a\left(a^2-b^2\right)$$

$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)\left(a+b\sec(e+fx)\right)^{3/2}}$$

$\downarrow$ 4319

$$\frac{-b(-4a^3d+7a^2bc-3b^3c)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{6c\sqrt{a+b}\left(a^2-b^2\right)^2\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\cdots\right.}{af}}{a\cdots}$$

$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)\left(a+b\sec(e+fx)\right)^{3/2}}$$

$\downarrow$ 4492

$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)\left(a+b\sec(e+fx)\right)^{3/2}}+$$

$$\frac{-\frac{6c\sqrt{a+b}\left(a^2-b^2\right)^2\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af}+\frac{2(a-b)\sqrt{a+b}\left(-4a^3d+7a^2bc-3b^3c\right)\cot\cdots}{\cdots}}{\cdots}$$

input

```
Int[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(5/2),x]
```

output

```
(2*b*(b*c - a*d)*Tan[e + f*x])/(3*a*(a^2 - b^2)*f*(a + b*Sec[e + f*x])^(3/
2)) + (((2*(a - b)*Sqrt[a + b]*(7*a^2*b*c - 3*b^3*c - 4*a^3*d)*Cot[e + f*x
]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b
))])/(b*f) + (2*(a - b)*Sqrt[a + b]*(a*b^2*c + 3*b^3*c + 3*a^3*d - a^2*b*(
6*c + d))*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[e + f*x]))/(a - b))])/(b*f) - (6*Sqrt[a + b]*(a^2 - b^2)^2*c*Cot[e + f
*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e +
 f*x]))/(a - b))])/(a*f))/(a*(a^2 - b^2)) + (2*b*(7*a^2*b*c - 3*b^3*c - 4*
a^3*d)*Tan[e + f*x])/(a*(a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x]]))/(3*a*(a^2
 - b^2))
```

3.206. $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx$

3.206.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4411
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2)
)   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
 a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && N
eQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

3.206. $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx$

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.206.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7172 vs. $2(456) = 912$.

Time = 17.34 (sec) , antiderivative size = 7173, normalized size of antiderivative = 14.49

method	result	size
parts	Expression too large to display	7173
default	Expression too large to display	7241

input
```
int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.206.5 Fricas [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)/(b^3*sec(f*x + e)^3
 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e) + a^3), x)
```

3.206.6 Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx=\int \frac{c+d\sec\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(5/2),x)
```

output
```
Integral((c + d*sec(e + f*x))/(a + b*sec(e + f*x))**(5/2), x)
```

3.206.7 Maxima [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(5/2), x)
```

3.206.8 Giac [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(5/2), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx=\int \frac{c+\frac{d}{\cos(e+f\,x)}}{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(5/2),x)
```

output
```
int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(5/2), x)
```

3.207 $\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$

3.207.1 Optimal result

Integrand size = 29, antiderivative size = 389

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx =$$

$$-\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\cdots}}{\sqrt{a+b}f}$$

$$+\frac{2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\cdots}{(c-d)(\cdots}}}{\sqrt{\frac{a+b}{c+d}}f}$$

output
```
2*cot(f*x+e)*EllipticPi(((a+b)/(c+d))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*se
c(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(
f*x+e))*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b
*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/f/((a+b)/(c+d))^(1/2)-2*c
ot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*s
ec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec
(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(
1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/f/(a+b)^(1/2
)
```

3.207.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 36.77 (sec) , antiderivative size = 40517, normalized size of antiderivative = 104.16

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
Result too large to show
```

3.207.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 4420, 3042, 4424, 4470}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4420$$

$$c\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}dx+d\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \; 3042$$

$$c\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4424$$

$$d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx - \frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a-}}{\sqrt{c+}}\right.\right.}{f\sqrt{a+b}}$$

$$\Big\downarrow \text{4470}$$

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+}}{\sqrt{a+b\,\text{se}}}\right.\right.}{f\sqrt{\frac{a+b}{c+d}}}$$
$$\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a-}}{\sqrt{c+}}\right.\right.}{f\sqrt{a+b}}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(S
qrt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]]
)], ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e +
f*x]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x
]))/((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*f)
+ (2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)
/(c + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c
 + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)
*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a
 + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[(a + b)/(c + d)]*f)
```

3.207.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4420
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c +
 d*Csc[e + f*x]], x], x] + Simp[d   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*
x]]/Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Ne
Q[b*c - a*d, 0]
```

rule 4424
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4470
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*((a + b*Csc[
e + f*x])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]))*Sqrt[(-(b*c - a*d))*((1
 - Csc[e + f*x])/((c + d)*(a + b*Csc[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + Cs
c[e + f*x])/((c - d)*(a + b*Csc[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a +
b))), ArcSin[Sqrt[(a + b)/(c + d)]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc
[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.207.4 Maple [A] (verified)

Time = 11.87 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.31

method	result
default	$-\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\,\sqrt{\frac{d+c\cos(fx+e)}{(c+d)(\cos(fx+e)+1)}}\left(2\,\mathrm{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(\cot(fx+e)-\csc(fx+e)),-\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)ac-\mathrm{Ell}}{}$

input
```
int((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(1/
(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2*EllipticPi(((a-b)/(a+b))^(
1/2)*(cot(f*x+e)-csc(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b)
)^(1/2))*a*c-EllipticF(((a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),((a+b)*
(c-d)/(a-b)/(c+d))^(1/2))*a*c+EllipticF(((a-b)/(a+b))^(1/2)*(cot(f*x+e)-cs
c(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*d+EllipticF(((a-b)/(a+b))^(1/
2)*(cot(f*x+e)-csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*c-EllipticF(
((a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2
))*b*d+2*EllipticPi(((a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),(a+b)/(a-b
),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b*d)*(1/(a+b)*(b+a*cos(f*x+e))/
(cos(f*x+e)+1))^(1/2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(
f*x+e))
```

3.207.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.207.6 Sympy [F]

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \int \sqrt{a+b\sec{(e+fx)}}\sqrt{c+d\sec{(e+fx)}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x)), x)
```

3.207.7 Maxima [F]

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

3.207.8 Giac [F]

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx=\int \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2), x)
```

3.208 $\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

3.208.1 Optimal result

Integrand size = 29, antiderivative size = 198

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{}}{\sqrt{a+b}cf}$$

output
```
-2*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)/(a
+b*sec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b
*sec(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)
))^(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/c/f/(a+b
)^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 5.31 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \frac{4\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}(e+fx)\right)}{c-d}}\sqrt{\frac{(a+b)(d+c\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{-bc+ad}}\csc(e+fx)\left((a+b)c\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a+b)(d+c\cos}{}}}{}\right.\right.\right.}{(a+b)}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(4*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((a + b)*(d + c*Cos[e +
 f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Csc[e + f*x]*((a + b)*c*Ellipti
cF[ArcSin[Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) +
 a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))] - a*(c + d)*EllipticPi
[(b*c - a*d)/(a*c + b*c), ArcSin[Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e
 + f*x)/2]^2)/(-(b*c) + a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]
)*Sqrt[a + b*Sec[e + f*x]]*Sin[(e + f*x)/2]^2)/((a + b)*c*f*Sqrt[((c + d)*
(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[c + d*Sec[e + f
*x]])
```

3.208.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {3042, 4424}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 4424$$

$$-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{cf\sqrt{a+b}}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(S
qrt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]]
)], ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e +
f*x]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x
]))/((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*c*f
)
```

3.208.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4424
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.208.4 Maple [A] (verified)

Time = 10.63 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.63

method	result
default	$-\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\,\sqrt{\frac{d+c\cos(fx+e)}{(c+d)(\cos(fx+e)+1)}}\,\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\left(\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+e)+\csc(fx+e)),\sqrt{\right.\right.}{}$

input
```
int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(1/
(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(co
s(f*x+e)+1))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e))
,((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a-EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*
x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b-2*EllipticPi(((a-b)/(a
+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b
)/(a+b))^(1/2))*a)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x
+e))
```

3.208.5 Fricas [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)
```

3.208.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{a+b\sec\left(e+fx\right)}}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))/sqrt(c + d*sec(e + f*x)), x)
```

3.208.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{b\sec(fx+e)+a}}{\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)
```

3.208.8 Giac [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{b\sec(fx+e)+a}}{\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2), x)
```

3.209 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$

3.209.1 Optimal result

Integrand size = 29, antiderivative size = 598

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-\cdots}{(c-\cdots}}}{\sqrt{a+b}c^2f}$$

$$-\frac{2\sqrt{a+b}d\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)(1+\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}}{c(c-d)\sqrt{c+d}f\sqrt{-\frac{(bc-ad)(1+\sec(e+fx))}{(a-b)(c+d\sec(e+fx))}}}$$

$$-\frac{2(a-b)\sqrt{a+b}d\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(b\cdots}{(a\cdots}}}{c(c-d)\sqrt{c+d}(bc-ad)f}$$

output
```
-2*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)/(a
+b*sec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b
*sec(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)
))^(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/c^2/f/(a
+b)^(1/2)-2*d*cot(f*x+e)*EllipticE((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b
)^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1+sec(f*x
+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)/
c/(c-d)/f/(c+d)^(1/2)/(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^
(1/2)-2*(a-b)*d*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a
+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec
(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1
/2)*(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/c/(c-d)/(-a*
d+b*c)/f/(c+d)^(1/2)
```

3.209.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1708 vs. $2(598) = 1196$.

Time = 14.55 (sec) , antiderivative size = 1708, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

output
```
((d + c*Cos[e + f*x])^(3/2)*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]]*((4*b*c*
(b*c - a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b +
a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos
[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[
Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[
2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c +
d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(a*c
 + b*d)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*
Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e
 + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sq
rt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]
], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)
*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot[(
e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2
]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/
(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[
((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]],
(2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b +
 a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*a*d*((Sqrt[(-a + b)/(a + b
)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(...
```

3.209.3 Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3042, 4427, 3042, 4424, 4474, 3042, 4472, 4482}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4427}$$

$$\frac{\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}dx}{c}-\frac{d\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}dx}{c}$$

$\downarrow$ 3042

$$\frac{\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}$$

$\downarrow$ 4424

$$-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a-}}{\sqrt{c+}}\right.\right.}{c^2f\sqrt{a+b}}$$

$\downarrow$ 4474

$$-\frac{d\left(\frac{(bc-ad)\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}dx}{c-d}+\frac{(a-b)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}dx}{c-d}\right)}{c}-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a-}}{\sqrt{c+}}\right.\right.}{c^2f\sqrt{a+b}}$$

$\downarrow$ 3042

$$-\frac{d\left(\frac{(a-b)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c-d}+\frac{(bc-ad)\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}dx}{c-d}\right)}{c}-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a-}}{\sqrt{c+}}\right.\right.}{c^2f\sqrt{a+b}}$$

$\downarrow$ 4472

$$-\frac{d\left(\frac{(bc-ad)\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}dx}{c-d}+\frac{2(a-b)\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1}{(a-b)(c+d\sec(e+fx)}}}{f(c-d)\sqrt{c+d}(bc-ad)}\right.}{c}-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a-}}{\sqrt{c+}}\right.\right.}{c^2f\sqrt{a+b}}$$

$\downarrow$ 4482

3.209. $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$

$$-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{c^2f\sqrt{a+b}}$$

$$\frac{d\left(\frac{2(a-b)\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right.}{f(c-d)\sqrt{c+d}(bc-ad)}\right.}{c}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(S
qrt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]]
)], ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e +
f*x]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x
]))/((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*c^2
*f) - (d*((2*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a
 + b*Sec[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c -
 d))/((a - b)*(c + d))]*(1 + Sec[e + f*x])*Sqrt[((b*c - a*d)*(1 - Sec[e +
f*x]))/((a + b)*(c + d*Sec[e + f*x]))])/((c - d)*Sqrt[c + d]*f*Sqrt[-(((b*
c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]) + (2*(a - b
)*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e
+ f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a -
b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f
*x])))]*(c + d*Sec[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f)))/c
```

3.209.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4424
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4427
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(3/2), x_Symbol] :> Simp[1/c   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt
[c + d*Csc[e + f*x]], x], x] - Simp[d/c   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[
e + f*x]]/(c + d*Csc[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[c^2 - d^2, 0]
```

rule 4472
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4474
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(3/2), x_Symbol] :> Simp[(a - b)/(c -
d)   Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]),
x], x] + Simp[(b*c - a*d)/(c - d)   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/(S
qrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(3/2))), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^
2, 0]
```

rule 4482
```
Int[(sec[(e_.) + (f_.)*(x_)]*((A_) + (B_.)*sec[(e_.) + (f_.)*(x_)]))/(Sqrt[
(a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sec[(e_.) + (f_.)*(x_)]
)^(3/2)), x_Symbol] :> Simp[2*A*(1 + Sec[e + f*x])*(Sqrt[(b*c - a*d)*((1 -
Sec[e + f*x])/((a + b)*(c + d*Sec[e + f*x])))]/(f*(b*c - a*d)*Rt[(c + d)/(a
 + b), 2]*Tan[e + f*x]*Sqrt[(-(b*c - a*d))*((1 + Sec[e + f*x])/((a - b)*(c
+ d*Sec[e + f*x])))]))*EllipticE[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*
Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]])], (a + b)*((c - d)/((a - b)*(c + d)
))], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

3.209. $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$

3.209.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2443 vs. 2(553) = 1106.

Time = 14.38 (sec) , antiderivative size = 2444, normalized size of antiderivative = 4.09

method	result	size
default	Expression too large to display	2444

input
```
int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/((a-b)/(a+b))^(1/2)/(c-d)/(c+d)/c*((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b
*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/
2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*((c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(
1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)
*(((a-b)/(a+b))^(1/2)*a*c*d*(1-cos(f*x+e))^3*csc(f*x+e)^3-((a-b)/(a+b))^(1
/2)*a*d^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-((a-b)/(a+b))^(1/2)*b*c*d*(1-cos(f
*x+e))^3*csc(f*x+e)^3+((a-b)/(a+b))^(1/2)*b*d^2*(1-cos(f*x+e))^3*csc(f*x+e
)^3+2*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a
-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc
(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+cs
c(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*c^2-2*(-
(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b
))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^
2-c-d)/(c+d))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)
),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*d^2-(-(a*(1-cos(
f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(
-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+
d))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c
-d)/(a-b)/(c+d))^(1/2))*a*c^2-(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(
f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+...
```

3.209.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.209.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a+b\sec{(e+fx)}}}{(c+d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x))**(3/2), x)
```

3.209.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\sec{(fx+e)}+a}}{(d\sec{(fx+e)}+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)
```

3.209.8 Giac [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\sec{(fx+e)}+a}}{(d\sec{(fx+e)}+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2), x)
```

3.210 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx$

3.210.1 Optimal result

Integrand size = 29, antiderivative size = 899

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx = \frac{2(a-b)\sqrt{a+b}d(6bc^3-7ac^2d-2bcd^2+3ad^3)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-}{(a-}}}{3c^2(c-d)^2(c+d)^{3}}$$

$$+\frac{2\sqrt{a+b}(bc^2(3c^2+3cd-2d^2)-ad(9c^3-2c^2d-6cd^2+3d^3))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}}{3c^3(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b-}}$$

$$-\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},a\right.}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx}}$$

$$+\frac{2d^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

output
```
2/3*d^2*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/(d+c*cos(f*x+e))/(
c+d*sec(f*x+e))^(1/2)+2/3*(a-b)*d*(-7*a*c^2*d+3*a*d^3+6*b*c^3-2*b*c*d^2)*(
d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))^(1
/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a
+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*
d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)
/c^2/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^2/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*
x+e))^(1/2)+2/3*(b*c^2*(3*c^2+3*c*d-2*d^2)-a*d*(9*c^3-2*c^2*d-6*c*d^2+3*d^
3))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e
))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2
))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(
-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^
(1/2)/c^3/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec
(f*x+e))^(1/2)-2*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*
(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),
((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(
a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*
x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^3/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/
2)/(c+d*sec(f*x+e))^(1/2)
```

3.210.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1990 vs. $2(899) = 1798$.

Time = 7.33 (sec) , antiderivative size = 1990, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(5/2),x]
```

output
```
((d + c*Cos[e + f*x])^3*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]]*((2*d^2*Si
n[e + f*x])/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*x])^2) - (2*(6*b*c^3*d*Sin[e
 + f*x] - 7*a*c^2*d^2*Sin[e + f*x] - 2*b*c*d^3*Sin[e + f*x] + 3*a*d^4*Sin[
e + f*x]))/(3*c*(b*c - a*d)*(c^2 - d^2)^2*(d + c*Cos[e + f*x]))))/(f*(c +
d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(5/2)*Sec[e + f*x]^2*Sqrt[a
 + b*Sec[e + f*x]]*((4*(b*c - a*d)*(3*b^2*c^4 - 3*a*b*c^3*d - a^2*c^2*d^2
+ b^2*c^2*d^2 - a*b*c*d^3 + a^2*d^4)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c
- d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*
Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e
 + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)
/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e +
f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*
x]]) + 4*(b*c - a*d)*(3*a*b*c^4 - 3*a^2*c^3*d + 6*b^2*c^3*d - 7*a*b*c^2*d^
2 - a^2*c*d^3 - 2*b^2*c*d^3 + 4*a*b*d^4)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^
2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c -
a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e
+ f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin
[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[
e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*...
```

3.210.3 Rubi [A] (verified)

Time = 3.03 (sec) , antiderivative size = 826, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {3042, 4430, 3042, 3527, 27, 3042, 3532, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{\cos^2(e+fx)\sqrt{b+a\cos(e+fx)}}{(d+c\cos(e+fx))^{5/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^2\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\downarrow 3527$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int-\frac{-3a\left(c^2-d^2\right)\cos^2(e+fx)-\left(3bc^2-3adc-2bd^2\right)\cos(e+fx)+d(3bc-ad)}{2\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{3c(c^2-d^2)}+\frac{2d^2\sin(e+fx)\sqrt{c}}{3cf(c^2-d^2)(c\cos}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\downarrow 27$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{-3a\left(c^2-d^2\right)\cos^2(e+fx)-\left(3bc^2-3adc-2bd^2\right)\cos(e+fx)+d}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{-3a\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(-3bc^2+3adc+2bd^2\right)\sin\left(e+\right.}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^3}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\downarrow 3532$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\frac{\int\frac{d\left(3bc^3+2adc^2-3ad^3\right)-c(bc-3ad)\left(3c^2-2d^2\right)\cos(e+fx)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c^2}-}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\frac{\int\frac{d\left(3bc^3+2adc^2-3ad^3\right)-c(bc-3ad)\left(3c^2-2d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}}{c^2}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

3.210. $\int\frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}dx$

$$\downarrow 3290$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\frac{\int\frac{d\left(3bc^3+2adc^2-3ad^3\right)-c(bc-3ad)\left(3c^2-2d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}}{c^2}}{\ldots}\right.}{\sqrt{a\cos(e+fx)\ldots}}$$

$$\downarrow 3477$$

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\frac{\frac{cd\left(-7ac^2d+3ad^3+6bc^3-2bcd^2\right)\int\frac{\cos(e+fx)+1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e\ldots}}{c-d}}{\ldots}}{\ldots}\right.$$

$$\downarrow 3042$$

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\frac{\frac{cd\left(-7ac^2d+3ad^3+6bc^3-2bcd^2\right)\int\frac{\sin\left(e+fx+\frac{\pi}{2}\ldots\right.}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\ldots\right.}}{c-d}}{\ldots}}{\ldots}\right.$$

$$\downarrow 3297$$

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\frac{\frac{cd\left(-7ac^2d+3ad^3+6bc^3-2bcd^2\right)\int\frac{\sin\left(e+fx+\frac{\pi}{2}\ldots\right.}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\ldots\right.}}{c-d}}{\ldots}}{\ldots}\right.$$

$$\downarrow 3475$$

3.210. $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx$

$$\frac{\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2\sqrt{b+a\cos(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))^{3/2}}-\frac{-\frac{2(a-b)\sqrt{a+b}cd\left(6bc^3-7adc^2-2bd^2c+3ad^3\right)\sqrt{-\frac{(bc-ad)(1-c}{(a+b)(d+c\,c}}}{}}{}\right.}{}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(5/2),x]
```

output
```
(Sqrt[d + c*Cos[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*(-1/3*(((-2*(a - b)*Sqr
t[a + b]*c*d*(6*b*c^3 - 7*a*c^2*d - 2*b*c*d^2 + 3*a*d^3)*Sqrt[-(((b*c - a*
d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d
)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x]
)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sq
rt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
)/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f) - (2*Sqrt[a + b]*(c*(b*c - 3*a*d)*
(3*c^2 - 2*d^2) + d*(3*b*c^3 + 2*a*c^2*d - 3*a*d^3))*Sqrt[-(((b*c - a*d)*(
1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1
 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])*Cs
c[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a
 + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/((
c - d)*Sqrt[c + d]*(b*c - a*d)*f))/c^2 + (6*Sqrt[a + b]*(c^2 - d^2)*Sqrt[-
(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(
((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*C
os[e + f*x])*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt
[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])],
 ((a + b)*(c - d))/((a - b)*(c + d))])/(c^2*Sqrt[c + d]*f))/(c*(c^2 - d^2)
) + (2*d^2*Sqrt[b + a*Cos[e + f*x]]*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d +
c*Cos[e + f*x])^(3/2))))/(Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f...
```

3.210.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.210. $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx$

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.210. $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx$

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

3.210.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 18679 vs. $2(820) = 1640$.

Time = 17.71 (sec) , antiderivative size = 18680, normalized size of antiderivative = 20.78

method	result	size
default	Expression too large to display	18680

input
```
int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.210.5 Fricas [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\sec(fx+e)+a}}{(d\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)/(d^3*sec(f*x +
e)^3 + 3*c*d^2*sec(f*x + e)^2 + 3*c^2*d*sec(f*x + e) + c^3), x)
```

3.210.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x))**(5/2), x)
```

3.210.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\sec(fx+e)+a}}{(d\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(5/2), x)
```

3.210.8 Giac [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(5/2), x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(5/2),x)
```

output
```
\text{Hanged}
```

3.211 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx$

3.211.1 Optimal result

Integrand size = 29, antiderivative size = 744

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)E\Big(\arcsin\Big(\cdots}{c(c-d)\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}\cdots}$$

$$-\frac{2\sqrt{a+b}(bc-a(2c-d))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{Ell}\cdots}{c^2(c-d)\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\,\text{sec}}\cdots}$$

$$-\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)}\cdots\right.}{c^2\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+f}\cdots}$$

output
```
-2*(a-b)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(
f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))
^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1
/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x
+e))^(1/2)/c/(c-d)/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(
1/2)-2*(b*c-a*(2*c-d))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(
1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d
)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*c
os(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2
)*(a+b*sec(f*x+e))^(1/2)/c^2/(c-d)/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c
+d*sec(f*x+e))^(1/2)-2*a*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d
)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/
a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f
*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+
c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^2/f/(c+d)^(1/2)/(b+a*cos(f*x
+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)
```

3.211.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1750 vs. $2(744) = 1488$.

Time = 19.22 (sec) , antiderivative size = 1750, normalized size of antiderivative = 2.35

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(3/2)*(-(b*c*Sin[e + f*x]) +
a*d*Sin[e + f*x]))/((-c^2 + d^2)*f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x
])^(3/2)) + ((d + c*Cos[e + f*x])^(3/2)*(a + b*Sec[e + f*x])^(3/2)*((4*(b*
c - a*d)*(a*b*c - b^2*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[(
(c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a -
b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*Elli
pticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
- a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/(
(a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(a^
2*c - b^2*c)*(b*c - a*d)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt
[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a
- b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*El
lipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)
/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sq
rt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x]
)*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc
[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)
*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c -
 a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)...
```

3.211.3 Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 691, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 4430, 3042, 3277, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{3/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3277

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{a^2\int\frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}dx}{c^2}+\frac{(bc-ad)\int\frac{bc+2a\cos(e+fx)c+ad}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c^2}\right)}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{a^2\int\frac{\sqrt{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{c^2}+\frac{(bc-ad)\int\frac{bc+2a\sin\left(e+fx+\frac{\pi}{2}\right)c+ad}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}\right)}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3290

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{(bc-ad)\int\frac{bc+2a\sin\left(e+fx+\frac{\pi}{2}\right)c+ad}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}-\frac{2a\sqrt{a+b}\csc(e+fx)(c\cos(e+fx)+\cdots}{\cdots}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\,\mathrm{se}\cdots}}$$

$\Big\downarrow$ 3477

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{(bc-ad)\left(\frac{c(bc-ad)\int\frac{\cos(e+fx)+1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c-d}-\frac{(bc-a(2c-d))\int\frac{1}{\sqrt{b+a\cos(e+fx)}\sqrt{d+\cdots}}}{c-d}\cdots\right.}{c^2}\right.}{\sqrt{a\cos(e\cdots}}$$

$\Big\downarrow$ 3042

3.211. $\int\frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{(bc-ad)\left(\frac{c(bc-ad)\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)+1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c-d}-\frac{(bc-a(2c-d))\int\frac{}{\sqrt{b+a\sin\left(e\right.}}}{}\right.}{c^2}\right.}{\sqrt{}}$$

$\downarrow$ 3297

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{(bc-ad)\left(\frac{c(bc-ad)\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)+1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c-d}-\frac{2\sqrt{a+b}(bc-a(2c-d))\csc(e+}{}\right.}{}\right.}{}$$

$\downarrow$ 3475

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{(bc-ad)\left(-\frac{2\sqrt{a+b}(bc-a(2c-d))\csc(e+fx)(c\cos(e+fx)+d)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(c}{(a-b)(c\cos}}}{f(c-d)\sqrt{c+d}(bc-ad}\right.}{}\right.}{}$$

input
```
Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(3/2),x]
```

3.211. $\int\frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx$

output
```
(Sqrt[d + c*Cos[e + f*x]]*(((b*c - a*d)*((-2*(a - b)*Sqrt[a + b]*c*Sqrt[-(
((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-((
(b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Co
s[e + f*x])*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e +
f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)
*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f) - (2*Sqrt[a + b]*(b*c - a*
(2*c - d))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e +
 f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e +
f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*S
qrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)
*(c - d))/((a - b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f)))/c^2 -
(2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*C
os[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Co
s[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a
*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt
[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(c^2*Sqrt[c
+ d]*f))*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Se
c[e + f*x]])
```

3.211.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3277
```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Simp[d^2/b^2   Int[Sqrt[a + b*Sin[e + f*x
]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[(b*c - a*d)/b^2   Int[Simp[b*c +
 a*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.211. $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx$

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

3.211.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3568 vs. 2(675) = 1350.

Time = 13.97 (sec) , antiderivative size = 3569, normalized size of antiderivative = 4.80

method	result	size
default	Expression too large to display	3569

input
```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/((a-b)/(a+b))^(1/2)/(c-d)/(c+d)/c*((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b
*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/
2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*((c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(
1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)
*(-2*((a-b)/(a+b))^(1/2)*a*b*c*d*(-cot(f*x+e)+csc(f*x+e))+(-(a*(1-cos(f*x+
e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*
(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^
(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/
(a-b)/(c+d))^(1/2))*a^2*c*d-(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*
x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-
d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticE(((a-b)/(a+b))^
(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*b*c^2-(-
(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b
))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^
2-c-d)/(c+d))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e))
,((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*b*d^2+(-(a*(1-cos(f*x+e))^2*csc(f*x+e)
^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2
*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticE(
((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/
2))*b^2*c*d-(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f...
```

3.211.5 Fricas [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
integral((b*sec(f*x + e) + a)^(3/2)*sqrt(d*sec(f*x + e) + c)/(d^2*sec(f*x
+ e)^2 + 2*c*d*sec(f*x + e) + c^2), x)
```

3.211.6 Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{(a+b\sec(e+fx))^{\frac{3}{2}}}{(c+d\sec(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x))**(3/2), x)
```

3.211.7 Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(3/2), x)
```

3.211.8 Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(3/2), x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(3/2), x)
```

3.212 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

3.212.1 Optimal result

Integrand size = 29, antiderivative size = 919

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}(3bc^3-7ac^2d+bcd^2+3ad^3)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+f}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b+a\cos(e+f}}$$

$$-\frac{2\sqrt{a+b}(b^2c^3(3c+d)-2abc^2(3c^2+2cd-d^2)+a^2d(9c^3-2c^2d-6cd^2+3d^3))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-}}{3c^3(c-d)^2(c+d)^{3/2}(bc-}$$

$$-\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{EllipticPi}\left(\frac{(a+b)c}{a(c+d)}\right.}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+f}}$$

$$-\frac{2d(bc-ad)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

output
```
-2/3*d*(-a*d+b*c)*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/(d+c*cos
(f*x+e))/(c+d*sec(f*x+e))^(1/2)-2/3*(a-b)*(-7*a*c^2*d+3*a*d^3+3*b*c^3+b*c*
d^2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+
e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/
2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*
(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))
^(1/2)/c^2/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*se
c(f*x+e))^(1/2)-2/3*(b^2*c^3*(3*c+d)-2*a*b*c^2*(3*c^2+2*c*d-d^2)+a^2*d*(9*
c^3-2*c^2*d-6*c*d^2+3*d^3))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c
+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)
*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(
d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))
^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^3/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f/(b+a*co
s(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-2*a*(d+c*cos(f*x+e))^(3/2)*csc(f*x+
e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+
e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(
-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos
(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^3/f/(c+d)^
(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)
```

3.212.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1960 vs. $2(919) = 1838$.

Time = 7.17 (sec) , antiderivative size = 1960, normalized size of antiderivative = 2.13

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

output
```
((d + c*Cos[e + f*x])^3*Sec[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((2*(-(b*c
*d*Sin[e + f*x]) + a*d^2*Sin[e + f*x]))/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*
x])^2) + (2*(3*b*c^3*Sin[e + f*x] - 7*a*c^2*d*Sin[e + f*x] + b*c*d^2*Sin[e
 + f*x] + 3*a*d^3*Sin[e + f*x]))/(3*c*(c^2 - d^2)^2*(d + c*Cos[e + f*x])))
)/(f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*
x])^(5/2)*Sec[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((4*(b*c - a*d)*(3*a*b*c
^3 + a^2*c^2*d - 4*b^2*c^2*d + a*b*c*d^2 - a^2*d^3)*Sqrt[((c + d)*Cot[(e +
 f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2
)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x]
)*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c -
 d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d
 + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(3*a^2*c^3 - 3*b^2*c^3 + 4*a*b*c^2*d +
 a^2*c*d^2 - b^2*c*d^2 - 4*a*b*d^3)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c
 - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[
e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x
)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e +
 f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f
*x]]) - (Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + ...
```

3.212.3 Rubi [A] (verified)

Time = 3.21 (sec) , antiderivative size = 850, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {3042, 4430, 3042, 3468, 27, 3042, 3532, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4430$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{\cos(e+fx)(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{5/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3468

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int\frac{3a^2(c^2-d^2)\cos^2(e+fx)-\left(3cda^2-2b(3c^2-d^2)a+b^2cd\right)\cos(e+fx)+(bc-ad)(3bc-ad)}{2\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{3c(c^2-d^2)}-\frac{2d(b\dots}{3\dots}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 27

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int\frac{3a^2(c^2-d^2)\cos^2(e+fx)-\left(3cda^2-2b(3c^2-d^2)a+b^2cd\right)\cos(e+fx)+(bc-ad)(3bc-ad)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{3c(c^2-d^2)}-\frac{2d(bc\dots}{3c\dots}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int\frac{3a^2(c^2-d^2)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(-3cda^2+2b(3c^2-d^2)a-b^2cd\right)\sin\left(e+fx+\frac{\pi}{2}\right)+(bc-ad)(3bc-ad)}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3532

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\int\frac{3b^2c^4-4abdc^3-\left((9c^2d-6d^3)a^2-b(6c^3-2cd^2)a+b^2c^2d\right)\cos(e+fx)c-a^2d^2(2c^2-3d^2)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c^2}+\frac{3a^2(c^2\dots}{\dots}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\int\frac{3b^2c^4-4abdc^3-\left((9c^2d-6d^3)a^2-b(6c^3-2cd^2)a+b^2c^2d\right)\sin\left(e+fx+\frac{\pi}{2}\right)c-a^2d^2(2c^2-3d^2)}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}+\frac{3a^2\dots}{\dots}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

3.212. $\displaystyle\int\frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

$\downarrow$ 3290

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\int \frac{3b^2c^4-4abdc^3-\left(\left(9c^2d-6d^3\right)a^2-b\left(6c^3-2cd^2\right)a+b^2c^2d\right)\sin\left(e+fx+\frac{\pi}{2}\right)c-a^2d^2\left(2c^2-3d^2\right)}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}-\frac{6a\ldots}{\ldots}}{\ldots}\right.$$

$\downarrow$ 3477

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\frac{c(bc-ad)\left(-7ac^2d+3ad^3+3bc^3+bcd^2\right)\int \frac{\cos(e+fx)+1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c-d}-\frac{\left(a^2d\left(9c^3-2c^2d-6\ldots\right.\right.}{\ldots}}{c^2}}{\ldots}\right.$$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\frac{c(bc-ad)\left(-7ac^2d+3ad^3+3bc^3+bcd^2\right)\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)+1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c-d}-\frac{\left(a^2d\left(9c^3\ldots\right.\right.}{\ldots}}{\ldots}}{\ldots}\right.$$

$\downarrow$ 3297

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\frac{c(bc-ad)\left(-7ac^2d+3ad^3+3bc^3+bcd^2\right)\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)+1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c-d}-\frac{2\sqrt{a+b}\left(a^2\ldots\right.}{\ldots}}{\ldots}}{\ldots}\right.$$

$\downarrow$ 3475

3.212. $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{\frac{-\frac{2(a-b)\sqrt{a+b}c\left(3bc^3-7adc^2+bd^2c+3ad^3\right)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c}{(c-d)\sqrt{c+d}(bc-ad)f}}{}\right.$$

input
```
Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

output
```
(Sqrt[d + c*Cos[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((((-2*(a - b)*Sqrt[a +
 b]*c*(3*b*c^3 - 7*a*c^2*d + b*c*d^2 + 3*a*d^3)*Sqrt[-(((b*c - a*d)*(1 - C
os[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Co
s[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e +
 f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]
*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/((c - d
)*Sqrt[c + d]*(b*c - a*d)*f) - (2*Sqrt[a + b]*(b^2*c^3*(3*c + d) - 2*a*b*c
^2*(3*c^2 + 2*c*d - d^2) + a^2*d*(9*c^3 - 2*c^2*d - 6*c*d^2 + 3*d^3))*Sqrt
[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[
-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c
*Cos[e + f*x])*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e
 + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a -
 b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f))/c^2 - (6*a*Sqrt[a + b]
*(c^2 - d^2)*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e
 + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e
+ f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c
+ d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d +
 c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(c^2*Sqrt[c + d]
*f))/(3*c*(c^2 - d^2)) - (2*d*(b*c - a*d)*Sqrt[b + a*Cos[e + f*x]]*Sin[e +
 f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^(3/2))))/(Sqrt[b + a*Cos...
```

3.212.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.212. $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

3.212. $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

3.212.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 15313 vs. $2(840) = 1680$.

Time = 16.55 (sec) , antiderivative size = 15314, normalized size of antiderivative = 16.66

method	result	size
default	Expression too large to display	15314

input
```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.212.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.212.6 Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\sec{(e+fx)})^{\frac{3}{2}}}{(c+d\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(5/2),x)
```

output
```
Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x))**(5/2), x)
```

3.212.7 Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{3}{2}}}{(d\sec{(fx+e)}+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

3.212.8 Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx=\text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(5/2),x)
```

output
```
\text{Hanged}
```

3.213 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

3.213.1 Optimal result

Integrand size = 29, antiderivative size = 1122

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \frac{2(a-b)\sqrt{a+b}(2abcd(35c^4-8c^2d^2+5d^4)-a^2d^2(58c^4-41c^2d^2+15d^4)-b}{}$$

$$-\frac{2\sqrt{a+b}(b^2c^3(15c^3+10c^2d+9cd^2-2d^3)-2abc^2(15c^4+20c^3d-4c^2d^2-4cd^3+5d^4)+a^2d(60c^5-2c^4d}{}$$

$$-\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)}\right.}{c^4\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+f}}$$

$$+\frac{2d^2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c(c^2-d^2)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$

$$-\frac{2d(10bc^3-13ac^2d-2bcd^2+5ad^3)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{15c^2(c^2-d^2)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

output
```
2/5*d^2*(b+a*cos(f*x+e))*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/(
d+c*cos(f*x+e))^2/(c+d*sec(f*x+e))^(1/2)-2/15*d*(-13*a*c^2*d+5*a*d^3+10*b*
c^3-2*b*c*d^2)*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c^2/(c^2-d^2)^2/f/(d+c*co
s(f*x+e))/(c+d*sec(f*x+e))^(1/2)+2/15*(a-b)*(2*a*b*c*d*(35*c^4-8*c^2*d^2+5
*d^4)-a^2*d^2*(58*c^4-41*c^2*d^2+15*d^4)-b^2*(15*c^6+19*c^4*d^2-2*c^2*d^4)
)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))
^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))
*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(
-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1
/2)/c^3/(c-d)^3/(c+d)^(5/2)/(-a*d+b*c)^2/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec
(f*x+e))^(1/2)-2/15*(b^2*c^3*(15*c^3+10*c^2*d+9*c*d^2-2*d^3)-2*a*b*c^2*(15
*c^4+20*c^3*d-4*c^2*d^2-4*c*d^3+5*d^4)+a^2*d*(60*c^5-2*c^4*d-66*c^3*d^2+25
*c^2*d^3+30*c*d^4-15*d^5))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+
d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*
(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d
+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^
(1/2)*(a+b*sec(f*x+e))^(1/2)/c^4/(c-d)^3/(c+d)^(5/2)/(-a*d+b*c)/f/(b+a*cos
(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-2*a*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e
)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e
))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(...
```

3.213.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2385 vs. $2(1122) = 2244$.

Time = 7.98 (sec) , antiderivative size = 2385, normalized size of antiderivative = 2.13

$$\int \frac{(a + b\sec(e + fx))^{3/2}}{(c + d\sec(e + fx))^{7/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

output
```
((d + c*Cos[e + f*x])^4*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2)*((-2*(-(
b*c*d^2*Sin[e + f*x]) + a*d^3*Sin[e + f*x]))/(5*c^2*(c^2 - d^2)*(d + c*Cos
[e + f*x])^3) - (4*(5*b*c^3*d*Sin[e + f*x] - 8*a*c^2*d^2*Sin[e + f*x] - b*
c*d^3*Sin[e + f*x] + 4*a*d^4*Sin[e + f*x]))/(15*c^2*(c^2 - d^2)^2*(d + c*C
os[e + f*x])^2) + (2*(15*b^2*c^6*Sin[e + f*x] - 70*a*b*c^5*d*Sin[e + f*x]
+ 58*a^2*c^4*d^2*Sin[e + f*x] + 19*b^2*c^4*d^2*Sin[e + f*x] + 16*a*b*c^3*d
^3*Sin[e + f*x] - 41*a^2*c^2*d^4*Sin[e + f*x] - 2*b^2*c^2*d^4*Sin[e + f*x]
 - 10*a*b*c*d^5*Sin[e + f*x] + 15*a^2*d^6*Sin[e + f*x]))/(15*c^2*(b*c - a*
d)*(c^2 - d^2)^3*(d + c*Cos[e + f*x]))))/(f*(b + a*Cos[e + f*x])*(c + d*Se
c[e + f*x])^(7/2)) + ((d + c*Cos[e + f*x])^(7/2)*Sec[e + f*x]^2*(a + b*Sec
[e + f*x])^(3/2)*((4*(b*c - a*d)*(-15*a*b^2*c^6 + 5*a^2*b*c^5*d + 25*b^3*c
^5*d + 13*a^3*c^4*d^2 - 38*a*b^2*c^4*d^2 + 25*a^2*b*c^3*d^3 + 7*b^3*c^3*d^
3 - 18*a^3*c^2*d^4 - 11*a*b^2*c^2*d^4 + 2*a^2*b*c*d^5 + 5*a^3*d^6)*Sqrt[((
c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc
[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e +
 f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d +
 c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d)
)/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e
 + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(-15*a^2*b*c^6 + 15*b^3
*c^6 + 15*a^3*c^5*d - 55*a*b^2*c^5*d + 33*a^2*b*c^4*d^2 + 19*b^3*c^4*d^...
```

3.213.3 Rubi [A] (verified)

Time = 4.51 (sec) , antiderivative size = 1053, normalized size of antiderivative = 0.94, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.586$, Rules used = {3042, 4430, 3042, 3527, 27, 3042, 3526, 27, 3042, 3532, 25, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\cos^2(e+fx)(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{7/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^2\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3527}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int-\frac{\sqrt{b+a\cos(e+fx)}\left(-5a\left(c^2-d^2\right)\cos^2(e+fx)-\left(5bc^2-5adc-2bd^2\right)\cos(e+fx)+d(5bc-3ad)\right)}{2(d+c\cos(e+fx))^{5/2}}dx}{5c(c^2-d^2)}+\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\int\frac{\sqrt{b+a\cos(e+fx)}\left(-5a\left(c^2-d^2\right)\cos^2(e+fx)-\left(5bc^2-5adc\right.\right.}{(d+c\cos(e+fx))^{5/2}}}{5c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\int\frac{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(-5a\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(-5\right.\right.}{\left(d+c\sin\left(e+fx+\right.\right.}}{5c(c^2-d^2}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3526}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{2\int-\frac{3\left(5c^4+3d^2c^2\right)b^2-8acd\left(5c^2-d^2\right)b+15a^2\left(c^2-d^2\right)^2\cos}{}}{}\right.}{\sqrt{a\cos(e+fx)}}$$

$$\Big\downarrow \text{27}$$

3.213. $\int\frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}dx$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)\cdots}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}\cdots}{\cdots}\cdots\right.}{\sqrt{a\cos(e+fx)\cdots}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)\cdots}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}\cdots}{\cdots}\cdots\right.}{\sqrt{a\cos(e+f\cdots}}$$

$\downarrow$ 3532

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)\cdots}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}\cdots}{\cdots}\cdots\right.}{\sqrt{a\cdots}}$$

$\downarrow$ 25

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)\cdots}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}\cdots}{\cdots}\cdots\right.}{\sqrt{a\,c\cdots}}$$

$\downarrow$ 3042

3.213. $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}}{}\right.$$

$\downarrow$ 3290

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}}{}\right.$$

$\downarrow$ 3477

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}}{}\right.$$

$\downarrow$ 3297

3.213. $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{3/2}\sin(e+fx)}{5c(c^2-d^2)f(d+c\cos(e+fx))^{5/2}}-\frac{\frac{2d\left(10bc^3-13adc^2-2bd^2c+5ad^3\right)\sqrt{b+a\cos(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{3/2}}}{}\right.$$

$$\downarrow \text{3475}$$

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{3/2}\sin(e+fx)}{5c(c^2-d^2)f(d+c\cos(e+fx))^{5/2}}-\frac{\frac{2d\left(10bc^3-13adc^2-2bd^2c+5ad^3\right)\sqrt{b+a\cos(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{3/2}}}{}\right.$$

input
```
Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

3.213. $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

output
```
(Sqrt[d + c*Cos[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((2*d^2*(b + a*Cos[e +
f*x])^(3/2)*Sin[e + f*x])/(5*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^(5/2)) -
 (-1/3*(-(((-2*(a - b)*Sqrt[a + b]*c*(2*a*b*c*d*(35*c^4 - 8*c^2*d^2 + 5*d^
4) - a^2*d^2*(58*c^4 - 41*c^2*d^2 + 15*d^4) - b^2*(15*c^6 + 19*c^4*d^2 - 2
*c^2*d^4))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e +
 f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e +
f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*S
qrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)
*(c - d))/((a - b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f) + (2*S
qrt[a + b]*(b^2*c^3*(15*c^3 + 10*c^2*d + 9*c*d^2 - 2*d^3) - 2*a*b*c^2*(15*
c^4 + 20*c^3*d - 4*c^2*d^2 - 4*c*d^3 + 5*d^4) + a^2*d*(60*c^5 - 2*c^4*d -
66*c^3*d^2 + 25*c^2*d^3 + 30*c*d^4 - 15*d^5))*Sqrt[-(((b*c - a*d)*(1 - Cos
[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[
e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f
*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*S
qrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/((c - d)*
Sqrt[c + d]*(b*c - a*d)*f))/c^2) - (30*a*Sqrt[a + b]*(c^2 - d^2)^2*Sqrt[-(
((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-((
(b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Co
s[e + f*x])*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sq...
```

3.213.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.213. $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.213. $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

3.213. $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

3.213.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 54550 vs. $2(1037) = 2074$.

Time = 21.53 (sec) , antiderivative size = 54551, normalized size of antiderivative = 48.62

method	result	size
default	Expression too large to display	54551

input
```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.213.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.213.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.213.7 Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

3.213.8 Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(7/2),x)
```

output
```
\text{Hanged}
```

3.214 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

3.214.1 Optimal result

Integrand size = 29, antiderivative size = 891

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}(7ac^2-4bcd-3ad^2)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\,\mathrm{csc}}{3c^2(c-d)^2(c+d)^{3/2}f\sqrt{b+a\cos(e+fx)}\sqrt{c+}}$$

$$+\frac{2\sqrt{a+b}(b^2c^2(c+3d)-abc(7c^2+4cd-3d^2)+a^2(9c^3-2c^2d-6cd^2+3d^3))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc}{(a}}}{3c^3(c-d)^2(c+d)^{3/2}f\sqrt{}}$$

$$-\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{EllipticPi}\left(\frac{(a+b)c}{a(c+d)}\right.}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+f}}$$

$$+\frac{2(bc-ad)^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

output
```
2/3*(-a*d+b*c)^2*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/(d+c*cos(
f*x+e))/(c+d*sec(f*x+e))^(1/2)-2/3*(a-b)*(7*a*c^2-3*a*d^2-4*b*c*d)*(d+c*co
s(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a
+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1
/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)
*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^2/(
c-d)^2/(c+d)^(3/2)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)+2/3*(b^
2*c^2*(c+3*d)-a*b*c*(7*c^2+4*c*d-3*d^2)+a^2*(9*c^3-2*c^2*d-6*c*d^2+3*d^3))
*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e))^
(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*
(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-
a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/
2)/c^3/(c-d)^2/(c+d)^(3/2)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)
-2*a^2*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f
*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-
d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*
cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/
2)*(a+b*sec(f*x+e))^(1/2)/c^3/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d*se
c(f*x+e))^(1/2)
```

3.214.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2026 vs. $2(891) = 1782$.

Time = 7.03 (sec) , antiderivative size = 2026, normalized size of antiderivative = 2.27

$$\int \frac{(a + b\sec(e + fx))^{5/2}}{(c + d\sec(e + fx))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

output
```
((d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x])^(5/2)*((2*(b^2*c^2*Sin[e + f*
x] - 2*a*b*c*d*Sin[e + f*x] + a^2*d^2*Sin[e + f*x]))/(3*c*(c^2 - d^2)*(d +
 c*Cos[e + f*x])^2) + (2*(7*a*b*c^3*Sin[e + f*x] - 7*a^2*c^2*d*Sin[e + f*x
] - 4*b^2*c^2*d*Sin[e + f*x] + a*b*c*d^2*Sin[e + f*x] + 3*a^2*d^3*Sin[e +
f*x]))/(3*c*(c^2 - d^2)^2*(d + c*Cos[e + f*x]))))/(f*(b + a*Cos[e + f*x])^
2*(c + d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(5/2)*(a + b*Sec[e +
 f*x])^(5/2)*((4*(b*c - a*d)*(2*a^2*b*c^3 + b^3*c^3 + a^3*c^2*d - 8*a*b^2*
c^2*d + 2*a^2*b*c*d^2 + 3*b^3*c*d^2 - a^3*d^3)*Sqrt[((c + d)*Cot[(e + f*x)
/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a
*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc
[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]
*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*
Cos[e + f*x]]) + 4*(b*c - a*d)*(3*a^3*c^3 - 7*a*b^2*c^3 + 4*b^3*c^2*d + a^
3*c*d^2 + 3*a*b^2*c*d^2 - 4*a^2*b*d^3)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)
/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*C
sc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e +
f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(
e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos...
```

3.214.3 Rubi [A] (verified)

Time = 3.44 (sec) , antiderivative size = 853, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {3042, 4430, 3042, 3271, 27, 3042, 3532, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \ 4430$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{5/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3271

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int\frac{d^2a^3+3\left(c^2-d^2\right)\cos^2(e+fx)a^3-5bcda^2+7b^2c^2a-3b^3cd+\left(-3cda^3+b\left(9c^2-2d^2\right)a^2-5b^2cda+b^3c^2\right)\cdots}{2\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}}{3c(c^2-d^2)}\cdots\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 27

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int\frac{d^2a^3+3\left(c^2-d^2\right)\cos^2(e+fx)a^3-5bcda^2+7b^2c^2a-3b^3cd+\left(-3cda^3+b\left(9c^2-2d^2\right)a^2-5b^2cda+b^3c^2\right)\cdots}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}}{3c(c^2-d^2)}\cdots\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int\frac{d^2a^3+3\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2a^3-5bcda^2+7b^2c^2a-3b^3cd+\left(-3cda^3+b\left(9c^2-2d^2\right)a^2-5b^2cda+b^3\cdots\right.}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}}{3c(c^2-d^2)}\cdots\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3532

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{3a^3\left(c^2-d^2\right)\int\frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}dx}{c^2}+\frac{\int\frac{7ab^2c^4-3b^3dc^3-5a^2bdc^3+(bc-ad)\left(9a^2c^2+b^2c^2-4abdc-6a^2d\cdots\right.}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3\cdots}}}{c^2}}{3c(c^2-d^2)}\cdots\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{3a^3\left(c^2-d^2\right)\int\frac{\sqrt{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{c^2}+\frac{\int\frac{7ab^2c^4-3b^3dc^3-5a^2bdc^3+(bc-ad)\left(9a^2c^2+b^2c^2-4abdc-\cdots\right.}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e\cdots\right.\right.}}{c^2}}{3c(c^2-d^2)}\cdots\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

3.214. $\int\frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

$\downarrow$ 3290

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\int \frac{7ab^2c^4-3b^3dc^3-5a^2bdc^3+(bc-ad)\left(9a^2c^2+b^2c^2-4abdc-6a^2d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)c-a^3\left(2c^2d^2-3d^4\right)\cdots}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}\cdots}{c^2}\cdots}{\cdots}\right.$$

$\downarrow$ 3477

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\frac{\left(a^3(-d)\left(9c^3-2c^2d-6cd^2+3d^3\right)+a^2bc^2\left(9c^2+5cd-2d^2\right)-ab^2c^3(7c+5d)+b^3c^3(c+3d)\right)\int \frac{\cdots}{\sqrt{b+a\cos(e+\cdots}}}{c-d}\cdots}{c^2}\cdots}{\cdots}\right.$$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\left(a^3(-d)\left(9c^3-2c^2d-6cd^2+3d^3\right)+a^2bc^2\left(9c^2+5cd-2d^2\right)-ab^2c^3(7c+5d)+b^3c^3(c+3d)\right)\int \frac{\cdots}{\sqrt{b+a\sin\left(e+\cdots\right.}}}{c-d}\cdots}{\cdots}\right.$$

$\downarrow$ 3297

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{c\left(7ac^2-3ad^2-4bcd\right)(bc-ad)^2\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)+1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c-d}+\frac{2\sqrt{a+b}\left(a^3(-d)\left(9\cdots\right.\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3475

3.214. $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2\sqrt{b+a\cos(e+fx)}\sin(e+fx)(bc-ad)^2}{3c(c^2-d^2)f(d+c\cos(e+fx))^{3/2}}+\frac{\frac{2\sqrt{a+b}\left(-d\left(9c^3-2dc^2-6d^2c+3d^3\right)a^3+bc^2\left(9c^2+5dc-\right.\right.}{}}{}\right.$$

input
```
Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

output
```
(Sqrt[d + c*Cos[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((((-2*(a - b)*Sqrt[a +
 b]*c*(7*a*c^2 - 4*b*c*d - 3*a*d^2)*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x])
)/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))
/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*Ellipt
icE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*
Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/((c - d)*Sqrt[c + d
]*f) + (2*Sqrt[a + b]*(b^3*c^3*(c + 3*d) - a*b^2*c^3*(7*c + 5*d) + a^2*b*c
^2*(9*c^2 + 5*c*d - 2*d^2) - a^3*d*(9*c^3 - 2*c^2*d - 6*c*d^2 + 3*d^3))*Sq
rt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqr
t[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d +
 c*Cos[e + f*x])*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos
[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a
 - b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f))/c^2 - (6*a^2*Sqrt[a
+ b]*(c^2 - d^2)*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*C
os[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Co
s[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a
*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt
[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(c^2*Sqrt[c
+ d]*f))/(3*c*(c^2 - d^2)) + (2*(b*c - a*d)^2*Sqrt[b + a*Cos[e + f*x]]*Sin
[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^(3/2))))/(Sqrt[b + a...
```

3.214.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

3.214. $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

3.214.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 18990 vs. $2(812) = 1624$.

Time = 16.32 (sec) , antiderivative size = 18991, normalized size of antiderivative = 21.31

method	result	size
default	Expression too large to display	18991

input
```
int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.214.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.214.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.214.7 Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

3.214.8 Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(5/2),x)
```

output
```
\text{Hanged}
```

3.215 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

3.215.1 Optimal result

Integrand size = 29, antiderivative size = 1150

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \frac{2(a-b)\sqrt{a+b}(b^2c^2d(29c^2+3d^2)-abc(35c^4+34c^2d^2-5d^4)+a^2(58c^4d-}{}$$

$$+\frac{2\sqrt{a+b}(b^3c^4(5c^2+24cd+3d^2)-ab^2c^3(35c^3+42c^2d+21cd^2-2d^3)+a^2bc^2(45c^4+48c^3d+c^2d^2-8cd^3}{}$$

$$-\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\operatorname{EllipticPi}\left(\frac{(a+b)c}{a(c+d)}\right.}{c^4\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+f}}$$

$$-\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c(c^2-d^2)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$

$$+\frac{2(bc-ad)(5bc^3-13ac^2d+3bcd^2+5ad^3)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{15c^2(c^2-d^2)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

output
```
-2/5*d*(-a*d+b*c)*(b+a*cos(f*x+e))*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^
2-d^2)/f/(d+c*cos(f*x+e))^2/(c+d*sec(f*x+e))^(1/2)+2/15*(-a*d+b*c)*(-13*a*
c^2*d+5*a*d^3+5*b*c^3+3*b*c*d^2)*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c^2/(c^
2-d^2)^2/f/(d+c*cos(f*x+e))/(c+d*sec(f*x+e))^(1/2)+2/15*(a-b)*(b^2*c^2*d*(
29*c^2+3*d^2)-a*b*c*(35*c^4+34*c^2*d^2-5*d^4)+a^2*(58*c^4*d-41*c^2*d^3+15*
d^5))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x
+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1
/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)
*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e)
)^(1/2)/c^3/(c-d)^3/(c+d)^(5/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*s
ec(f*x+e))^(1/2)+2/15*(b^3*c^4*(5*c^2+24*c*d+3*d^2)-a*b^2*c^3*(35*c^3+42*c
^2*d+21*c*d^2-2*d^3)+a^2*b*c^2*(45*c^4+48*c^3*d+c^2*d^2-8*c*d^3+10*d^4)-a^
3*d*(60*c^5-2*c^4*d-66*c^3*d^2+25*c^2*d^3+30*c*d^4-15*d^5))*(d+c*cos(f*x+e
))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/
2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(
-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos
(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^4/(c-d)^3/
(c+d)^(5/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-2*a
^2*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x+e
))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d...
```

3.215.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2344 vs. $2(1150) = 2300$.

Time = 8.15 (sec) , antiderivative size = 2344, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

output
```
((d + c*Cos[e + f*x])^4*Sec[e + f*x]*(a + b*Sec[e + f*x])^(5/2)*((-2*(b^2*
c^2*d*Sin[e + f*x] - 2*a*b*c*d^2*Sin[e + f*x] + a^2*d^3*Sin[e + f*x]))/(5*
c^2*(c^2 - d^2)*(d + c*Cos[e + f*x])^3) + (2*(5*b^2*c^4*Sin[e + f*x] - 21*
a*b*c^3*d*Sin[e + f*x] + 16*a^2*c^2*d^2*Sin[e + f*x] + 3*b^2*c^2*d^2*Sin[e
 + f*x] + 5*a*b*c*d^3*Sin[e + f*x] - 8*a^2*d^4*Sin[e + f*x]))/(15*c^2*(c^2
 - d^2)^2*(d + c*Cos[e + f*x])^2) + (2*(35*a*b*c^5*Sin[e + f*x] - 58*a^2*c
^4*d*Sin[e + f*x] - 29*b^2*c^4*d*Sin[e + f*x] + 34*a*b*c^3*d^2*Sin[e + f*x
] + 41*a^2*c^2*d^3*Sin[e + f*x] - 3*b^2*c^2*d^3*Sin[e + f*x] - 5*a*b*c*d^4
*Sin[e + f*x] - 15*a^2*d^5*Sin[e + f*x]))/(15*c^2*(c^2 - d^2)^3*(d + c*Cos
[e + f*x]))))/(f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^(7/2)) + ((d
+ c*Cos[e + f*x])^(7/2)*Sec[e + f*x]*(a + b*Sec[e + f*x])^(5/2)*((4*(b*c -
 a*d)*(10*a^2*b*c^5 + 5*b^3*c^5 + 13*a^3*c^4*d - 48*a*b^2*c^4*d + 15*a^2*b
*c^3*d^2 + 27*b^3*c^3*d^2 - 18*a^3*c^2*d^3 - 16*a*b^2*c^2*d^3 + 7*a^2*b*c*
d^4 + 5*a^3*d^5)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*
(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d +
c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[Ar
cSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/
Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*
(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)
*(15*a^3*c^5 - 35*a*b^2*c^5 + 23*a^2*b*c^4*d + 29*b^3*c^4*d + 13*a^3*c^...
```

3.215.3 Rubi [A] (verified)

Time = 4.33 (sec) , antiderivative size = 1086, normalized size of antiderivative = 0.94, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.552$, Rules used = {3042, 4430, 3042, 3468, 27, 3042, 3526, 27, 3042, 3532, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{\cos(e+fx)(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{7/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3468

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int \frac{\sqrt{b+a\cos(e+fx)}\left(5a^2\left(c^2-d^2\right)\cos^2(e+fx)-\left(5cda^2-2b\left(5c^2-d^2\right)a+3b^2cd\right)\cos(e+fx)+(5bc-3ad)(\cdots}{2(d+c\cos(e+fx))^{5/2}}}{5c(c^2-d^2)}\cdots \right.$$
$$\overline{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 27

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int \frac{\sqrt{b+a\cos(e+fx)}\left(5a^2\left(c^2-d^2\right)\cos^2(e+fx)-\left(5cda^2-2b\left(5c^2-d^2\right)a+3b^2cd\right)\cos(e+fx)+(5bc-3ad)(b\cdots}{(d+c\cos(e+fx))^{5/2}}}{5c(c^2-d^2)}\cdots \right.$$
$$\overline{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int \frac{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(5a^2\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(-5cda^2+2b\left(5c^2-d^2\right)a-3b^2cd\right)\sin\left(e+fx+\frac{\pi}{2}\right)\cdots}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}}{5c(c^2-d^2)}\cdots \right.$$
$$\overline{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3526

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int -\frac{-15\left(c^2-d^2\right)^2\cos^2(e+fx)a^3+(bc-ad)\left(\left(13c^2d-5d^3\right)a^2-b\left(35c^3-3cd^2\right)a+24b^2c^2d\right)+\left(6cd\left(5c^2-\cdots\right.\right.}{2\sqrt{b+a\cos(e+fx)}(d+c\cos(\cdots}}{3c\left(c^2-d^2\right)}\cdots \right.$$
$$\overline{\sqrt{a\cdots}}$$

$\downarrow$ 27

3.215. $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{2(bc-ad)\left(-13ac^2d+5ad^3+5bc^3+3bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{-15\left(c^2-d^2\right)^2\cos^2(e+fx)a^3+}{}}{}}{}\right.$$

$\sqrt{a}$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{2(bc-ad)\left(-13ac^2d+5ad^3+5bc^3+3bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{-15\left(c^2-d^2\right)^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}{}}{}}{}\right.$$

$\downarrow$ 3532

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{2(bc-ad)\left(-13ac^2d+5ad^3+5bc^3+3bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{15\left(c^2d-d^3\right)^2a^3+c^2(bc-ad)\left(\right.}{}}{}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{2(bc-ad)\left(-13ac^2d+5ad^3+5bc^3+3bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{15\left(c^2d-d^3\right)^2a^3+c^2(bc-ad)\left(\right.}{}}{}}{}\right.$$

$\downarrow$ 3290

3.215. $\int\frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{2(bc-ad)\left(-13ac^2d+5ad^3+5bc^3+3bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{15\left(c^2d-d^3\right)^2a^3+c^2(bc-ad)\left(\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3477

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{2(bc-ad)\left(-13ac^2d+5ad^3+5bc^3+3bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}-\frac{-\frac{c(bc-ad)\left(-58a^2c^4d+41a^2c^2\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3042

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{\frac{2(bc-ad)\left(5bc^3-13adc^2+3bd^2c+5ad^3\right)\sqrt{b+a\cos(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{3/2}}-\frac{30a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+\cdots}{(a+b)(d+c\cos(e+\cdots}}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3297

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{\frac{2(bc-ad)\left(5bc^3-13adc^2+3bd^2c+5ad^3\right)\sqrt{b+a\cos(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{3/2}}-\frac{30a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+\cdots}{(a+b)(d+c\cos(e+\cdots}}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3475

3.215. $\int\frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{\frac{2(bc-ad)\left(5bc^3-13adc^2+3bd^2c+5ad^3\right)\sqrt{b+a\cos(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{3/2}}-\frac{30a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+}{(a+b)(d+c\cos(e+}}}{}}{}\right.$$

input
```
Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

output
```
(Sqrt[d + c*Cos[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((-2*d*(b*c - a*d)*(b +
 a*Cos[e + f*x])^(3/2)*Sin[e + f*x])/(5*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x
])^(5/2)) + (-1/3*(((2*(a - b)*Sqrt[a + b]*c*(35*a*b*c^5 - 58*a^2*c^4*d -
29*b^2*c^4*d + 34*a*b*c^3*d^2 + 41*a^2*c^2*d^3 - 3*b^2*c^2*d^3 - 5*a*b*c*d
^4 - 15*a^2*d^5)*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*C
os[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Co
s[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c
+ d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((
a + b)*(c - d))/((a - b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f) -
(2*Sqrt[a + b]*(b^3*c^4*(5*c^2 + 24*c*d + 3*d^2) - a*b^2*c^3*(35*c^3 + 42*
c^2*d + 21*c*d^2 - 2*d^3) + a^2*b*c^2*(45*c^4 + 48*c^3*d + c^2*d^2 - 8*c*d
^3 + 10*d^4) - a^3*d*(60*c^5 - 2*c^4*d - 66*c^3*d^2 + 25*c^2*d^3 + 30*c*d^
4 - 15*d^5))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e
 + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e
+ f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]
*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a +
b)*(c - d))/((a - b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f))/c^2 +
 (30*a^2*Sqrt[a + b]*(c^2 - d^2)^2*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))
/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/
((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*Elli...
```

3.215.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.215. $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

3.215. $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

3.215. $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

3.215.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 39099 vs. 2(1065) = 2130.

Time = 20.62 (sec) , antiderivative size = 39100, normalized size of antiderivative = 34.00

method	result	size
default	Expression too large to display	39100

input
```
int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.215.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.215.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.215.7 Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

3.215.8 Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx=\text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(7/2),x)
```

output
```
\text{Hanged}
```

3.216 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx$

3.216.1 Optimal result

Integrand size = 29, antiderivative size = 1428

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \frac{2(a-b)\sqrt{a+b}(2b^3c^3d(133c^4+62c^2d^2-3d^4)+2a^2bcd(406c^6+73c^4d^2+13}{}$$

$$+\frac{2\sqrt{a+b}(b^3c^4(35c^4+231c^3d+67c^2d^2+57cd^3-6d^4)-ab^2c^3(245c^5+413c^4d+439c^3d^2+53c^2d^3-12cd^4}{}$$

$$-\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\operatorname{EllipticPi}\left(\frac{(a+b)c}{a(c+d)}\right.}{c^5\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+f}}$$

$$+\frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}}$$

$$-\frac{2d(14bc^3-19ac^2d-2bcd^2+7ad^3)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{35c^2(c^2-d^2)^2f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$

$$-\frac{2(2abcd(91c^4-2c^2d^2+7d^4)-a^2d^2(162c^4-101c^2d^2+35d^4)-b^2(35c^6+67c^4d^2-6c^2d^4))\sqrt{a+b\sec(e}}{105c^3(c^2-d^2)^3f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

output
```
2/7*d^2*(b+a*cos(f*x+e))^2*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f
/(d+c*cos(f*x+e))^3/(c+d*sec(f*x+e))^(1/2)-2/35*d*(-19*a*c^2*d+7*a*d^3+14*
b*c^3-2*b*c*d^2)*(b+a*cos(f*x+e))*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c^2/(c
^2-d^2)^2/f/(d+c*cos(f*x+e))^2/(c+d*sec(f*x+e))^(1/2)-2/105*(2*a*b*c*d*(91
*c^4-2*c^2*d^2+7*d^4)-a^2*d^2*(162*c^4-101*c^2*d^2+35*d^4)-b^2*(35*c^6+67*
c^4*d^2-6*c^2*d^4))*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c^3/(c^2-d^2)^3/f/(d
+c*cos(f*x+e))/(c+d*sec(f*x+e))^(1/2)+2/105*(a-b)*(2*b^3*c^3*d*(133*c^4+62
*c^2*d^2-3*d^4)+2*a^2*b*c*d*(406*c^6+73*c^4*d^2+132*c^2*d^4-35*d^6)-a*b^2*
c^2*(245*c^6+852*c^4*d^2+41*c^2*d^4+14*d^6)-a^3*(582*c^6*d^2-485*c^4*d^4+3
92*c^2*d^6-105*d^8))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/
2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/
(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos
(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*
(a+b*sec(f*x+e))^(1/2)/c^4/(c-d)^4/(c+d)^(7/2)/(-a*d+b*c)^2/f/(b+a*cos(f*x
+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)+2/105*(b^3*c^4*(35*c^4+231*c^3*d+67*c^2*
d^2+57*c*d^3-6*d^4)-a*b^2*c^3*(245*c^5+413*c^4*d+439*c^3*d^2+53*c^2*d^3-12
*c*d^4+14*d^5)+a^2*b*c^2*(315*c^6+497*c^5*d+219*c^4*d^2-73*c^3*d^3+208*c^2
*d^4+56*c*d^5-70*d^6)-a^3*d*(525*c^7+57*c^6*d-699*c^5*d^2+214*c^4*d^3+672*
c^3*d^4-280*c^2*d^5-210*c*d^6+105*d^7))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*
EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e...
```

3.216.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2979 vs. $2(1428) = 2856$.

Time = 9.06 (sec) , antiderivative size = 2979, normalized size of antiderivative = 2.09

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(9/2),x]
```

output
```
((d + c*Cos[e + f*x])^5*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(5/2)*((2*(b^2
*c^2*d^2*Sin[e + f*x] - 2*a*b*c*d^3*Sin[e + f*x] + a^2*d^4*Sin[e + f*x]))/
(7*c^3*(c^2 - d^2)*(d + c*Cos[e + f*x])^4) + (2*(-14*b^2*c^4*d*Sin[e + f*x
] + 43*a*b*c^3*d^2*Sin[e + f*x] - 29*a^2*c^2*d^3*Sin[e + f*x] + 2*b^2*c^2*
d^3*Sin[e + f*x] - 19*a*b*c*d^4*Sin[e + f*x] + 17*a^2*d^5*Sin[e + f*x]))/(
35*c^3*(c^2 - d^2)^2*(d + c*Cos[e + f*x])^3) + (2*(35*b^2*c^6*Sin[e + f*x]
 - 224*a*b*c^5*d*Sin[e + f*x] + 234*a^2*c^4*d^2*Sin[e + f*x] + 67*b^2*c^4*
d^2*Sin[e + f*x] + 52*a*b*c^3*d^3*Sin[e + f*x] - 209*a^2*c^2*d^4*Sin[e + f
*x] - 6*b^2*c^2*d^4*Sin[e + f*x] - 20*a*b*c*d^5*Sin[e + f*x] + 71*a^2*d^6*
Sin[e + f*x]))/(105*c^3*(c^2 - d^2)^3*(d + c*Cos[e + f*x])^2) + (2*(245*a*
b^2*c^8*Sin[e + f*x] - 812*a^2*b*c^7*d*Sin[e + f*x] - 266*b^3*c^7*d*Sin[e
+ f*x] + 582*a^3*c^6*d^2*Sin[e + f*x] + 852*a*b^2*c^6*d^2*Sin[e + f*x] - 1
46*a^2*b*c^5*d^3*Sin[e + f*x] - 124*b^3*c^5*d^3*Sin[e + f*x] - 485*a^3*c^4
*d^4*Sin[e + f*x] + 41*a*b^2*c^4*d^4*Sin[e + f*x] - 264*a^2*b*c^3*d^5*Sin[
e + f*x] + 6*b^3*c^3*d^5*Sin[e + f*x] + 392*a^3*c^2*d^6*Sin[e + f*x] + 14*
a*b^2*c^2*d^6*Sin[e + f*x] + 70*a^2*b*c*d^7*Sin[e + f*x] - 105*a^3*d^8*Sin
[e + f*x]))/(105*c^3*(b*c - a*d)*(c^2 - d^2)^4*(d + c*Cos[e + f*x]))))/(f*
(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^(9/2)) + ((d + c*Cos[e + f*x])
^(9/2)*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(5/2)*((4*(b*c - a*d)*(-70*a^2*
b^2*c^8 - 35*b^4*c^8 - 77*a^3*b*c^7*d + 427*a*b^3*c^7*d + 162*a^4*c^6*d...
```

3.216.3 Rubi [A] (verified)

Time = 6.43 (sec) , antiderivative size = 1353, normalized size of antiderivative = 0.95, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.655$, Rules used = {3042, 4430, 3042, 3527, 27, 3042, 3526, 27, 3042, 3526, 27, 3042, 3532, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{\cos^2(e+fx)(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{9/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^2\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{9/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3527

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int -\frac{(b+a\cos(e+fx))^{3/2}\left(-7a\left(c^2-d^2\right)\cos^2(e+fx)-\left(7bc^2-7adc-2bd^2\right)\cos(e+fx)+d(7bc-5ad)\right)}{2(d+c\cos(e+fx))^{7/2}}dx}{7c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\int \frac{(b+a\cos(e+fx))^{3/2}\left(-7a\left(c^2-d^2\right)\cos^2(e+fx)-\left(7bc^2-7a\right.\right.}{(d+c\cos(e+fx))^{7/2}}}{7c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\int \frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(-7a\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\right.}{\left(d+c\sin\left(e+fx\right.\right.}}{7c(c^2-\right.}}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3526

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{2\int -\frac{\sqrt{b+a\cos(e+fx)}\left(35b^2c^4-112abdc^3+57a^2d^2c^2+25b^2c\right.}{}}{}\right.}{\sqrt{a\,\mathrm{co}}}$$

$\Big\downarrow$ 27

3.216. $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)+}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.}{\sqrt{a\cos}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)+}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.}{\sqrt{}}$$

$\downarrow$ 3526

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)+}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.}{}$$

$\downarrow$ 27

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)+}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.}{}$$

$\downarrow$ 3042

3.216. $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx$

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)+}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.$$

$\downarrow$ 3532

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)+}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)+}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.$$

$\downarrow$ 3290

3.216. $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx$

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{5/2}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^{7/2}}-\frac{\frac{2d\left(14bc^3-19adc^2-2bd^2c+7ad^3\right)(b+a\cos(e+fx))^{3/2}\sin(e+}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{5/2}}}{}\right.$$

$\downarrow$ 3477

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{5/2}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^{7/2}}-\frac{\frac{2d\left(14bc^3-19adc^2-2bd^2c+7ad^3\right)(b+a\cos(e+fx))^{3/2}\sin(e+}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{5/2}}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{5/2}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^{7/2}}-\frac{\frac{2d\left(14bc^3-19adc^2-2bd^2c+7ad^3\right)(b+a\cos(e+fx))^{3/2}\sin(e+}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{5/2}}}{}\right.$$

$\downarrow$ 3297

3.216. $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx$

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{5/2}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^{7/2}}-\frac{\frac{2d\left(14bc^3-19adc^2-2bd^2c+7ad^3\right)(b+a\cos(e+fx))^{3/2}\sin(e+}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{5/2}}}{}\right.$$

$\downarrow$ 3475

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{5/2}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^{7/2}}-\frac{\frac{2d\left(14bc^3-19adc^2-2bd^2c+7ad^3\right)(b+a\cos(e+fx))^{3/2}\sin(e+}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{5/2}}}{}\right.$$

input
```
Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(9/2),x]
```

3.216. $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx$

output
```
(Sqrt[d + c*Cos[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((2*d^2*(b + a*Cos[e +
f*x])^(5/2)*Sin[e + f*x])/(7*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^(7/2)) -
 ((2*d*(14*b*c^3 - 19*a*c^2*d - 2*b*c*d^2 + 7*a*d^3)*(b + a*Cos[e + f*x])^
(3/2)*Sin[e + f*x])/(5*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^(5/2)) - (-1/3
*(((-2*(a - b)*Sqrt[a + b]*c*(2*b^3*c^3*d*(133*c^4 + 62*c^2*d^2 - 3*d^4) +
 2*a^2*b*c*d*(406*c^6 + 73*c^4*d^2 + 132*c^2*d^4 - 35*d^6) - a*b^2*c^2*(24
5*c^6 + 852*c^4*d^2 + 41*c^2*d^4 + 14*d^6) - a^3*(582*c^6*d^2 - 485*c^4*d^
4 + 392*c^2*d^6 - 105*d^8))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a +
b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b
)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticE[ArcS
in[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e +
f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c -
 a*d)^2*f) - (2*Sqrt[a + b]*(b^3*c^4*(35*c^4 + 231*c^3*d + 67*c^2*d^2 + 57
*c*d^3 - 6*d^4) - a*b^2*c^3*(245*c^5 + 413*c^4*d + 439*c^3*d^2 + 53*c^2*d^
3 - 12*c*d^4 + 14*d^5) + a^2*b*c^2*(315*c^6 + 497*c^5*d + 219*c^4*d^2 - 73
*c^3*d^3 + 208*c^2*d^4 + 56*c*d^5 - 70*d^6) - a^3*d*(525*c^7 + 57*c^6*d -
699*c^5*d^2 + 214*c^4*d^3 + 672*c^3*d^4 - 280*c^2*d^5 - 210*c*d^6 + 105*d^
7))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x]))
)]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))
]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[...
```

3.216.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.216. $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx$

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

3.216.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 88655 vs. 2(1337) = 2674.

Time = 23.70 (sec) , antiderivative size = 88656, normalized size of antiderivative = 62.08

method	result	size
default	Expression too large to display	88656

input
```
int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.216.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.216.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

3.216.7 Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(9/2), x)
```

3.216.8 Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(9/2), x)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(9/2),x)
```

output
```
\text{Hanged}
```

3.217 $\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$

3.217.1 Optimal result

Integrand size = 29, antiderivative size = 652

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx =$$

$$-\frac{2c(c+d)\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{\frac{(bc-ad)(1+\sec(e+fx))}{(c-d)(a+b\sec(e+fx))}}(a+}{a(a+b)f\sqrt{c+d\sec(e+fx)}}$$

$$+\frac{2d(c+d)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{\frac{(bc-ad)(1+\sec(e+fx))}{(c-d)(a+b\sec(e+fx))}}(a+}{b(a+b)f\sqrt{c+d\sec(e+fx)}}$$

$$+\frac{2(bc-ad)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{\frac{(bc-ad)(-1+\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(1}{(c-d)(a+}}}{abf\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}}$$

output
```
-2*c*(c+d)*cot(f*x+e)*EllipticPi(((a+b)*(c+d*sec(f*x+e))/(c+d)/(a+b*sec(f*
x+e)))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*x
+e))^(3/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)*((a+b)
*(-a*d+b*c)*(-1+sec(f*x+e))*(c+d*sec(f*x+e))/(c+d)^2/(a+b*sec(f*x+e))^2)^(
1/2)/a/(a+b)/f/(c+d*sec(f*x+e))^(1/2)+2*d*(c+d)*cot(f*x+e)*EllipticPi(((a+
b)*(c+d*sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(
c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*x+e))^(3/2)*((-a*d+b*c)*(1+sec(f*x+e))
/(c-d)/(a+b*sec(f*x+e)))^(1/2)*(-(a+b)*(a*d-b*c)*(-1+sec(f*x+e))*(c+d*sec(
f*x+e))/(c+d)^2/(a+b*sec(f*x+e))^2)^(1/2)/b/(a+b)/f/(c+d*sec(f*x+e))^(1/2)
+2*(-a*d+b*c)*cot(f*x+e)*EllipticF(((a+b)*(c+d*sec(f*x+e))/(c+d)/(a+b*sec(
f*x+e)))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*((-a*d+b*c)*(-1+sec(f*x+e)
)/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(
f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/a/b/f/((a+b)*
(c+d*sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)
```

3.217.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 37.55 (sec) , antiderivative size = 50041, normalized size of antiderivative = 76.75

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
Result too large to show
```

3.217.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4433}$$

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}dx$$

input
```
Int[(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
$Aborted
```

3.217.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.217.4 Maple [A] (verified)

Time = 17.07 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.70

method	result
default	$-\frac{2\sqrt{c+d\sec(fx+e)}\,\sqrt{a+b\sec(fx+e)}\,\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\,\sqrt{\frac{d+c\cos(fx+e)}{(c+d)(\cos(fx+e)+1)}}\,\left(\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+e)+\csc(fx+e)),\sqrt{\right.\right.}{}}$

input
```
int((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/((a-b)/(a+b))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(1/
(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(co
s(f*x+e)+1))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e))
,((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*c^2-2*EllipticF(((a-b)/(a+b))^(1/2)*(-co
t(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*c*d+EllipticF(((a-b)
/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*d^
2-2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),(a+b)/(a-b),((
c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*d^2-2*EllipticPi(((a-b)/(a+b))^(1/2
)*(-cot(f*x+e)+csc(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^
(1/2))*c^2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))
```

3.217.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.217.6 Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(c+d\sec{(e+fx)})^{\frac{3}{2}}}{\sqrt{a+b\sec{(e+fx)}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**(3/2)/sqrt(a + b*sec(e + f*x)), x)
```

3.217.7 Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{(d\sec(fx+e)+c)^{\frac{3}{2}}}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((d*sec(f*x + e) + c)^(3/2)/sqrt(b*sec(f*x + e) + a), x)
```

3.217.8 Giac [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{(d\sec(fx+e)+c)^{\frac{3}{2}}}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e) + c)^(3/2)/sqrt(b*sec(f*x + e) + a), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))^(3/2)/(a + b/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))^(3/2)/(a + b/cos(e + f*x))^(1/2), x)
```

3.218 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx$

3.218.1 Optimal result

Integrand size = 29, antiderivative size = 198

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-}}{a\sqrt{c+d}f}$$

output
```
-2*cot(f*x+e)*EllipticPi((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)/(c
+d*sec(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d
*sec(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e))
)^(1/2)*(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/a/f/(c+d
)^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx = \frac{4\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(e+fx)\right)}{a-b}}\sqrt{\frac{(c+d)(b+a\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{bc-ad}}\csc(e+fx)\left(a(c+d)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(c+d)(b+a\,\mathrm{co}}{}}\right.\right.\right.}{a(c+}$$

input
```
Integrate[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(4*Sqrt[((a + b)*Cot[(e + f*x)/2]^2)/(a - b)]*Sqrt[((c + d)*(b + a*Cos[e +
 f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*(a*(c + d)*EllipticF[
ArcSin[Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(2*b*c - 2*a
*d)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))] - (a + b)*c*EllipticPi[(-(b*c
) + a*d)/(a*(c + d)), ArcSin[Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f
*x)/2]^2)/(2*b*c - 2*a*d)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))])*Sqrt[c
 + d*Sec[e + f*x]]*Sin[(e + f*x)/2]^2)/(a*(c + d)*f*Sqrt[((a + b)*(d + c*C
os[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Sqrt[a + b*Sec[e + f*x]])
```

3.218.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {3042, 4424}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4424}$$

$$-\frac{2\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\arcsin\left(\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\right.\right.}{af\sqrt{c+d}}$$

input
```
Int[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(S
qrt[c + d]*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]]
)], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*
x]))/((a + b)*(c + d*Sec[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]
))/((a - b)*(c + d*Sec[e + f*x])))]*(c + d*Sec[e + f*x]))/(a*Sqrt[c + d]*f
)
```

3.218.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4424
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.218.4 Maple [A] (verified)

Time = 11.56 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.63

method	result
default	$-\frac{2\sqrt{c+d\sec(fx+e)}\,\sqrt{a+b\sec(fx+e)}\left(\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+e)+\csc(fx+e)),\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)c-\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx\right.\right.}{}$

input
```
int((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/((a-b)/(a+b))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(El
lipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c
+d))^(1/2))*c-EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b
)*(c-d)/(a-b)/(c+d))^(1/2))*d-2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e
)+csc(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*c)*(1/
(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(co
s(f*x+e)+1))^(1/2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x
+e))
```

3.218.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.218.6 Sympy [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{\sqrt{c+d\sec{(e+fx)}}}{\sqrt{a+b\sec{(e+fx)}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(c + d*sec(e + f*x))/sqrt(a + b*sec(e + f*x)), x)
```

3.218.7 Maxima [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

3.218.8 Giac [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))^(1/2)/(a + b/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))^(1/2)/(a + b/cos(e + f*x))^(1/2), x)
```

3.219 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

3.219.1 Optimal result

Integrand size = 29, antiderivative size = 398

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{}}{a\sqrt{a+b}cf} - \frac{2b\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(}{(a-b)(c-}}}{a\sqrt{c+d}(bc-ad)f}$$

output
```
-2*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)/(a
+b*sec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b
*sec(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)
))^(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/a/c/f/(a
+b)^(1/2)-2*b*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b
)^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f
*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2
)*(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/a/(-a*d+b*c)/f
/(c+d)^(1/2)
```

3.219.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$= \frac{4i\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\sqrt{\frac{d+c\cos(e+fx)}{(c+d)(1+\cos(e+fx))}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{-a+b}{a+b}}\tan\left(\frac{1}{2}(e+fx)\right)\right),\right.\right.}{\sqrt{\frac{-a+b}{a+b}}f\sqrt{a+b\sec(e+fx)}\sqrt{}}$$

input
```
Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
((4*I)*Cos[(e + f*x)/2]^2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e +
f*x]))]*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x]))]*(EllipticF
[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], ((a + b)*(c - d))/((a
 - b)*(c + d))] - 2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(e + f*x)/2]], ((a + b)*(c - d))/((a - b)*(c + d))])*Sec[e +
 f*x])/(Sqrt[(-a + b)/(a + b)]*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e
 + f*x]])
```

3.219.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 4426, 3042, 4424, 4472}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4426}$$

$$\frac{\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}dx}{a} - \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a} - \frac{b\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}$$

$\Big\downarrow$ 4424

$$-\frac{b\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a} - \frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a-}}{\sqrt{c+}}\right.\right.}{acf\sqrt{a+b}}$$

$\Big\downarrow$ 4472

$$-\frac{2b\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a-}}{\sqrt{a+b}\sqrt{c-}}\right.\right.}{af\sqrt{c+d}(bc-ad)}$$
$$\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a-}}{\sqrt{c+}}\right.\right.}{acf\sqrt{a+b}}$$

input
```
Int[1/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
(-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(S
qrt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]]
)], ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e +
f*x]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x
]))/((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(a*Sqrt[a + b]*c
*f) - (2*b*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a +
 b*Sec[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d
))/((a - b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c +
d*Sec[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*
Sec[e + f*x])))]*(c + d*Sec[e + f*x]))/(a*Sqrt[c + d]*(b*c - a*d)*f)
```

3.219. $\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

3.219.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4424
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4426
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x
_)]*(d_.) + (c_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sq
rt[c + d*Csc[e + f*x]], x], x] - Simp[b/a   Int[Csc[e + f*x]/(Sqrt[a + b*Cs
c[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0]
```

rule 4472
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.219.4 Maple [A] (verified)

Time = 13.40 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.66

method	result
default	$\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\,\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\,\sqrt{\frac{d+c\cos(fx+e)}{(c+d)(\cos(fx+e)+1)}}\left(\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\cot(fx+e)-\csc(fx+e)),\sqrt{\frac{(a+}{(a-}}\right.\right.}{f\sqrt{\frac{a-b}{a+b}}\,(d+c\cos(fx+e))(b+a\cos(f}$

input
```
int(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(1/(
a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos
(f*x+e)+1))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),(
(a+b)*(c-d)/(a-b)/(c+d))^(1/2))-2*EllipticPi(((a-b)/(a+b))^(1/2)*(cot(f*x+
e)-csc(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2)))/(d+c
*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))
```

3.219.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

3.219.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec{(e+fx)}}\sqrt{c+d\sec{(e+fx)}}}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))), x)
```

3.219.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

3.219.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)), x)
```

3.220 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx$

3.220.1 Optimal result

Integrand size = 29, antiderivative size = 622

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = -\frac{2(a-b)\sqrt{a+b}d^2\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(1+s}{(a-b)(c+ds}}}{c(c-d)\sqrt{c+d}(bc-ad)^2f} - \frac{2\sqrt{a+b}(2c-d)d\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-}}{c^2(c-d)\sqrt{c+d}(bc-ad)f} - \frac{2\sqrt{a+b}\cot(e+fx)\operatorname{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc}{(a-}}}{ac^2\sqrt{c+d}f}$$

output
```
-2*(a-b)*d^2*cot(f*x+e)*EllipticE((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)
^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*
x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)
*(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/c/(c-d)/(-a*d+b
*c)^2/f/(c+d)^(1/2)-2*(2*c-d)*d*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(
f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))
^(1/2))*(c+d*sec(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d
*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1
/2)/c^2/(c-d)/(-a*d+b*c)/f/(c+d)^(1/2)-2*cot(f*x+e)*EllipticPi((c+d)^(1/2)
*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),(a+b)*c/a/(c+d)
,((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)
*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sec(f*x+e))/
(a-b)/(c+d*sec(f*x+e)))^(1/2)/a/c^2/f/(c+d)^(1/2)
```

3.220.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1761 vs. $2(622) = 1244$.

Time = 19.32 (sec) , antiderivative size = 1761, normalized size of antiderivative = 2.83

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)),x]
```

output
```
(Sqrt[b + a*Cos[e + f*x]]*(d + c*Cos[e + f*x])^(3/2)*Sec[e + f*x]^2*((-4*b
*c*d*(b*c - a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*
(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d +
c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[Ar
cSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/
Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*
(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)
*(b*c^2 - a*c*d - 2*b*d^2)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sq
rt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-
a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*
EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(
b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^
4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (
Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*
x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*C
sc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a +
b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
 - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/
((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) - 2*a*d^2*(
(Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x...
```

3.220.3 Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {3042, 4430, 3042, 3533, 25, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{\cos^2(e+fx)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^2}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \; 3533$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int -\frac{d^2+2c\cos(e+fx)d}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c^2}+\frac{\int \frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}dx}{c^2}\right)}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int \frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}dx}{c^2}-\frac{\int \frac{d^2+2c\cos(e+fx)d}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c^2}\right)}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int \frac{\sqrt{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{c^2}-\frac{\int \frac{d^2+2c\sin\left(e+fx+\frac{\pi}{2}\right)d}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}\right)}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \; 3290$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(-\frac{\int \frac{d^2+2c\sin\left(e+fx+\frac{\pi}{2}\right)d}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}-\frac{2\sqrt{a+b}\csc(e+fx)(c\cos(e+fx)+d)\sqrt{-}}{}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e}}$$

$$\Big\downarrow \; 3477$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(-\frac{\frac{d(2c-d)\int \frac{1}{\sqrt{b+a\cos(e+fx)}\sqrt{d+c\cos(e+fx)}}dx}{c-d}-\frac{cd^2\int \frac{\cos(e+fx)+1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c-d}}{c^2}-\right.}{\sqrt{a\cos(e+fx)-}}$$

$$\Big\downarrow \; 3042$$

3.220. $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(-\frac{\frac{d(2c-d)\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\sqrt{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{c-d}-\frac{cd^2\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)+1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx \cdots\right.\right.}}{c-d}}{c^2}\cdots\right.}{\sqrt{a\cos(e+\cdots}}$$

$\downarrow$ 3297

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(-\frac{2d\sqrt{a+b}(2c-d)\csc(e+fx)(c\cos(e+fx)+d)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}\text{Ell}\cdots}{f(c-d)\sqrt{c+d}(bc-ad)}\cdots\right.}{\cdots}$$

$\downarrow$ 3475

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(-\frac{2cd^2(a-b)\sqrt{a+b}\csc(e+fx)(c\cos(e+fx)+d)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}E\cdots}{f(c-d)\sqrt{c+d}(bc-ad)^2}\cdots\right.}{\cdots}$$

input
```
Int[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)),x]
```

3.220. $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx$

output
```
(Sqrt[d + c*Cos[e + f*x]]*(-(((2*(a - b)*Sqrt[a + b]*c*d^2*Sqrt[-(((b*c -
a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a
*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*
x])*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(
Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d)
)])/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f) + (2*Sqrt[a + b]*(2*c - d)*d*Sqr
t[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt
[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d +
c*Cos[e + f*x])*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[
e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f))/c^2) - (2*Sqrt[a + b]
*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*
Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(
d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSi
n[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f
*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(a*c^2*Sqrt[c + d]*f))*Sqrt[
a + b*Sec[e + f*x]])/(Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])
```

3.220.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

3.220. $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx$

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3533
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[1/b^2   Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e +
 f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.220. $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx$

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

3.220.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2866 vs. $2(577) = 1154$.

Time = 17.18 (sec) , antiderivative size = 2867, normalized size of antiderivative = 4.61

method	result	size
default	Expression too large to display	2867

input
```
int(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/f/((a-b)/(a+b))^(1/2)/(a*d-b*c)/(c-d)/(c+d)/c*((a*(1-cos(f*x+e))^2*csc(
f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)
^2-1))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*((c*(1-cos(f*x+e))^2*csc(f*
x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/((1-cos(f*x+e))^2*csc(f*x+e)^2
-1))^(1/2)*(((a-b)/(a+b))^(1/2)*a*c*d^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-((a-
b)/(a+b))^(1/2)*a*d^3*(1-cos(f*x+e))^3*csc(f*x+e)^3-((a-b)/(a+b))^(1/2)*b*
c*d^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+((a-b)/(a+b))^(1/2)*b*d^3*(1-cos(f*x+e
))^3*csc(f*x+e)^3+2*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*
csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos
(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(
-cot(f*x+e)+csc(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/
2))*a*c^2*d-2*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*
x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e
))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f
*x+e)+csc(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*
d^3-2*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a
-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc
(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+cs
c(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b*c^3+2*(-
(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(...
```

3.220.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{b\sec{(fx+e)}+a}(d\sec{(fx+e)}+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)/(b*d^2*sec(f*x
+ e)^3 + a*c^2 + (2*b*c*d + a*d^2)*sec(f*x + e)^2 + (b*c^2 + 2*a*c*d)*sec(
f*x + e)), x)
```

3.220.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a+b\sec{(e+fx)}}\,(c+d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))**(3/2)), x)
```

3.220.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{b\sec{(fx+e)}+a}(d\sec{(fx+e)}+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^(3/2)), x)
```

3.220.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^(3/2)), x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(3/2)), x)
```

3.221 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$

3.221.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \frac{\sqrt[3]{d+c\cos(e+fx)}\sqrt[3]{a+b\sec(e+fx)}\,\text{Int}\left(\frac{\sqrt[3]{b+a\cos(e+fx)}}{\sqrt[3]{d+c\cos(e+fx)}},x\right)}{\sqrt[3]{b+a\cos(e+fx)}\sqrt[3]{c+d\sec(e+fx)}}$$

output
```
(d+c*cos(f*x+e))^(1/3)*(a+b*sec(f*x+e))^(1/3)*Unintegrable((b+a*cos(f*x+e)
)^(1/3)/(d+c*cos(f*x+e))^(1/3),x)/(b+a*cos(f*x+e))^(1/3)/(c+d*sec(f*x+e))^
(1/3)
```

3.221.2 Mathematica [N/A]

Not integrable

Time = 14.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3), x]
```

3.221.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4431, 3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt[3]{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4431}$$

$$\frac{\sqrt[3]{a+b\sec(e+fx)}\sqrt[3]{c\cos(e+fx)+d}\int \frac{\sqrt[3]{b+a\cos(e+fx)}}{\sqrt[3]{d+c\cos(e+fx)}}dx}{\sqrt[3]{a\cos(e+fx)+b}\sqrt[3]{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{a+b\sec(e+fx)}\sqrt[3]{c\cos(e+fx)+d}\int \frac{\sqrt[3]{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt[3]{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{\sqrt[3]{a\cos(e+fx)+b}\sqrt[3]{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3304}$$

$$\frac{\sqrt[3]{a+b\sec(e+fx)}\sqrt[3]{c\cos(e+fx)+d}\int \frac{\sqrt[3]{b+a\cos(e+fx)}}{\sqrt[3]{d+c\cos(e+fx)}}dx}{\sqrt[3]{a\cos(e+fx)+b}\sqrt[3]{c+d\sec(e+fx)}}$$

input
```
Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3),x]
```

---

output
```
$Aborted
```

3.221.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4431
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sin[e + f*x]^(m + n)*(a + b*Csc[e + f*x
])^m*((c + d*Csc[e + f*x])^n/((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n
))   Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^Simpli
fy[m + n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d,
0] && EqQ[m + n, 0] &&  !IntegerQ[2*m]
```

3.221.4 Maple [N/A] (verified)

Not integrable

Time = 0.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{1}{3}}}{(c+d\sec(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x)
```

output
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x)
```

3.221.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="fric
as")
```

output
```
Timed out
```

3.221.6 Sympy [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt[3]{a+b\sec{(e+fx)}}}{\sqrt[3]{c+d\sec{(e+fx)}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(1/3),x)
```

output
```
Integral((a + b*sec(e + f*x))**(1/3)/(c + d*sec(e + f*x))**(1/3), x)
```

3.221.7 Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{1}{3}}}{(d\sec{(fx+e)}+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(1/3), x)
```

3.221.8 Giac [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{1}{3}}}{(d\sec(fx+e)+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(1/3), x)
```

3.221.9 Mupad [N/A]

Not integrable

Time = 99.95 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{1/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(1/3),x)
```

output
```
int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(1/3), x)
```

3.222 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$

3.222.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \text{Int}\left(\frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}},x\right)$$

output
```
Unintegrable((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

3.222.2 Mathematica [N/A]

Not integrable

Time = 87.53 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

3.222.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4433}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{4433}$$

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}dx$$

input
```
Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

output
```
$Aborted
```

3.222.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.222.4 Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{1}{3}}}{(c+d\sec(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

output
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

3.222.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="fric
as")
```

output
```
Timed out
```

3.222.6 Sympy [N/A]

Not integrable

Time = 10.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(4/3),x)
```

output
```
Integral((a + b*sec(e + f*x))**(1/3)/(c + d*sec(e + f*x))**(4/3), x)
```

3.222.7 Maxima [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{1}{3}}}{(d\sec(fx+e)+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

3.222.8 Giac [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{1}{3}}}{(d\sec(fx+e)+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

3.222.9 Mupad [N/A]

Not integrable

Time = 105.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{1/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{4/3}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(4/3),x)
```

output
```
int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(4/3), x)
```

3.222. $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$

3.223 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$

3.223.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Int}\left(\frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}},x\right)$$

output
```
Unintegrable((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

3.223.2 Mathematica [N/A]

Not integrable

Time = 84.79 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

3.223.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4433}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/3}}dx$$

$$\Big\downarrow \text{4433}$$

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}dx$$

input
```
Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3),x]
```

output
```
$Aborted
```

3.223.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.223.4 Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{1}{3}}}{(c+d\sec(fx+e))^{\frac{7}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

output
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

3.223.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="fric
as")
```

output
```
Timed out
```

3.223.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(7/3),x)
```

output
```
Timed out
```

3.223.7 Maxima [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{1}{3}}}{(d\sec(fx+e)+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

3.223.8 Giac [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{1}{3}}}{(d\sec(fx+e)+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(7/3),x)
```

output
```
\text{Hanged}
```

3.224 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$

3.224.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx=\frac{(d+c\cos(e+fx))^{2/3}(a+b\sec(e+fx))^{2/3}\text{Int}\left(\frac{(b+a\cos(e+fx))^{2/3}}{(d+c\cos(e+fx))^{2/3}},x\right)}{(b+a\cos(e+fx))^{2/3}(c+d\sec(e+fx))^{2/3}}$$

output
```
(d+c*cos(f*x+e))^(2/3)*(a+b*sec(f*x+e))^(2/3)*Unintegrable((b+a*cos(f*x+e)
)^(2/3)/(d+c*cos(f*x+e))^(2/3),x)/(b+a*cos(f*x+e))^(2/3)/(c+d*sec(f*x+e))^
(2/3)
```

3.224.2 Mathematica [N/A]

Not integrable

Time = 14.91 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3), x]
```

3.224.3 Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4431, 3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{2/3}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{4431}$$

$$\frac{(a+b\sec(e+fx))^{2/3}(c\cos(e+fx)+d)^{2/3}\int \frac{(b+a\cos(e+fx))^{2/3}}{(d+c\cos(e+fx))^{2/3}}dx}{(a\cos(e+fx)+b)^{2/3}(c+d\sec(e+fx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a+b\sec(e+fx))^{2/3}(c\cos(e+fx)+d)^{2/3}\int \frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{2/3}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{(a\cos(e+fx)+b)^{2/3}(c+d\sec(e+fx))^{2/3}}$$

$$\Big\downarrow \text{3304}$$

$$\frac{(a+b\sec(e+fx))^{2/3}(c\cos(e+fx)+d)^{2/3}\int \frac{(b+a\cos(e+fx))^{2/3}}{(d+c\cos(e+fx))^{2/3}}dx}{(a\cos(e+fx)+b)^{2/3}(c+d\sec(e+fx))^{2/3}}$$

input
```
Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3),x]
```

output
```
$Aborted
```

3.224.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4431
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sin[e + f*x]^(m + n)*(a + b*Csc[e + f*x
])^m*((c + d*Csc[e + f*x])^n/((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n
))   Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^Simpli
fy[m + n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d,
0] && EqQ[m + n, 0] &&  !IntegerQ[2*m]
```

3.224.4 Maple [N/A] (verified)

Not integrable

Time = 0.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{2}{3}}}{(c+d\sec(fx+e))^{\frac{2}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x)
```

output
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x)
```

3.224.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="fric
as")
```

output
```
Timed out
```

3.224.6 Sympy [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx = \int \frac{(a+b\sec{(e+fx)})^{\frac{2}{3}}}{(c+d\sec{(e+fx)})^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(2/3),x)
```

output
```
Integral((a + b*sec(e + f*x))**(2/3)/(c + d*sec(e + f*x))**(2/3), x)
```

3.224.7 Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{2}{3}}}{(d\sec{(fx+e)}+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(2/3), x)
```

3.224.8 Giac [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx=\int \frac{(b\sec{(fx+e)}+a)^{\frac{2}{3}}}{(d\sec{(fx+e)}+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(2/3), x)
```

3.224.9 Mupad [N/A]

Not integrable

Time = 107.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{2/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(2/3),x)
```

output
```
int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(2/3), x)
```

3.225 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$

3.225.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx=\text{Int}\left(\frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}},x\right)$$

output
```
Unintegrable((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

3.225.2 Mathematica [N/A]

Not integrable

Time = 89.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3), x]
```

3.225.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4433}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{2/3}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/3}}dx$$

$$\Big\downarrow \text{4433}$$

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}dx$$

input
```
Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3),x]
```

output
```
$Aborted
```

3.225.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.225.4 Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{2}{3}}}{(c+d\sec(fx+e))^{\frac{5}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

output
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

3.225.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="fric
as")
```

output
```
Timed out
```

3.225.6 Sympy [N/A]

Not integrable

Time = 61.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{\frac{2}{3}}}{(c+d\sec(e+fx))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(5/3),x)
```

output
```
Integral((a + b*sec(e + f*x))**(2/3)/(c + d*sec(e + f*x))**(5/3), x)
```

3.225.7 Maxima [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{2}{3}}}{(d\sec(fx+e)+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(5/3), x)
```

3.225.8 Giac [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{2}{3}}}{(d\sec(fx+e)+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(5/3), x)
```

3.225.9 Mupad [N/A]

Not integrable

Time = 107.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{2/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(5/3),x)
```

output
```
int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(5/3), x)
```

3.225. $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$

3.226 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$

3.226.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \text{Int}\left(\frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}},x\right)$$

output
```
Unintegrable((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

3.226.2 Mathematica [N/A]

Not integrable

Time = 87.72 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3), x]
```

3.226.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4433}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{2/3}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{8/3}}dx$$

$$\Big\downarrow \text{4433}$$

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}dx$$

input
```
Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3),x]
```

output
```
$Aborted
```

3.226.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.226.4 Maple [N/A] (verified)

Not integrable

Time = 0.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{2}{3}}}{(c+d\sec(fx+e))^{\frac{8}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

output
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

3.226.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="fric
as")
```

output
```
Timed out
```

3.226.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(8/3),x)
```

output
```
Timed out
```

3.226.7 Maxima [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{2}{3}}}{(d\sec(fx+e)+c)^{\frac{8}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(8/3), x)
```

3.226.8 Giac [N/A]

Not integrable

Time = 5.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{2}{3}}}{(d\sec(fx+e)+c)^{\frac{8}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(8/3), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(8/3),x)
```

output
```
\text{Hanged}
```

3.227 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$

3.227.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\frac{(d+c\cos(e+fx))^{4/3}(a+b\sec(e+fx))^{4/3}\text{Int}\left(\frac{(b+a\cos(e+fx))^{4/3}}{(d+c\cos(e+fx))^{4/3}},x\right)}{(b+a\cos(e+fx))^{4/3}(c+d\sec(e+fx))^{4/3}}$$

output
```
(d+c*cos(f*x+e))^(4/3)*(a+b*sec(f*x+e))^(4/3)*Unintegrable((b+a*cos(f*x+e)
)^(4/3)/(d+c*cos(f*x+e))^(4/3),x)/(b+a*cos(f*x+e))^(4/3)/(c+d*sec(f*x+e))^
(4/3)
```

3.227.2 Mathematica [N/A]

Not integrable

Time = 75.84 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

3.227.3 Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4431, 3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{4431}$$

$$\frac{(a+b\sec(e+fx))^{4/3}(c\cos(e+fx)+d)^{4/3}\int \frac{(b+a\cos(e+fx))^{4/3}}{(d+c\cos(e+fx))^{4/3}}dx}{(a\cos(e+fx)+b)^{4/3}(c+d\sec(e+fx))^{4/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a+b\sec(e+fx))^{4/3}(c\cos(e+fx)+d)^{4/3}\int \frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}dx}{(a\cos(e+fx)+b)^{4/3}(c+d\sec(e+fx))^{4/3}}$$

$$\Big\downarrow \text{3304}$$

$$\frac{(a+b\sec(e+fx))^{4/3}(c\cos(e+fx)+d)^{4/3}\int \frac{(b+a\cos(e+fx))^{4/3}}{(d+c\cos(e+fx))^{4/3}}dx}{(a\cos(e+fx)+b)^{4/3}(c+d\sec(e+fx))^{4/3}}$$

input
```
Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

output
```
$Aborted
```

3.227.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4431
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sin[e + f*x]^(m + n)*(a + b*Csc[e + f*x
])^m*((c + d*Csc[e + f*x])^n/((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n
))   Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^Simpli
fy[m + n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d,
0] && EqQ[m + n, 0] &&  !IntegerQ[2*m]
```

3.227.4 Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{4}{3}}}{(c+d\sec(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x)
```

output
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x)
```

3.227.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="fric
as")
```

output
```
Timed out
```

3.227.6 Sympy [N/A]

Not integrable

Time = 147.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{(a+b\sec{(e+fx)})^{\frac{4}{3}}}{(c+d\sec{(e+fx)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(4/3),x)
```

output
```
Integral((a + b*sec(e + f*x))**(4/3)/(c + d*sec(e + f*x))**(4/3), x)
```

3.227.7 Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{4}{3}}}{(d\sec{(fx+e)}+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

3.227.8 Giac [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

3.227.9 Mupad [N/A]

Not integrable

Time = 110.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{4/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{4/3}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(4/3),x)
```

output
```
int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(4/3), x)
```

3.228 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$

3.228.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx=\text{Int}\left(\frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}},x\right)$$

output
```
Unintegrable((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

3.228.2 Mathematica [N/A]

Not integrable

Time = 98.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

3.228.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4433}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/3}}dx$$

$$\Big\downarrow \text{4433}$$

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}dx$$

input
```
Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3),x]
```

output
```
$Aborted
```

3.228.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.228.4 Maple [N/A] (verified)

Not integrable

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{4}{3}}}{(c+d\sec(fx+e))^{\frac{7}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

output
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

3.228.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="fric
as")
```

output
```
Timed out
```

3.228.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(7/3),x)
```

output
```
Timed out
```

3.228.7 Maxima [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

3.228.8 Giac [N/A]

Not integrable

Time = 3.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(7/3),x)
```

output
```
\text{Hanged}
```

3.229 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$

3.229.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \text{Int}\left(\frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}},x\right)$$

output
```
Unintegrable((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

3.229.2 Mathematica [N/A]

Not integrable

Time = 130.90 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3), x]
```

3.229.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4433}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{10/3}}dx$$

$$\Big\downarrow \text{4433}$$

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}dx$$

input
```
Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3),x]
```

output
```
$Aborted
```

3.229.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.229.4 Maple [N/A] (verified)

Not integrable

Time = 0.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{4}{3}}}{(c+d\sec(fx+e))^{\frac{10}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

output
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

3.229.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="fri
cas")
```

output
```
Timed out
```

3.229.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(10/3),x)
```

output
```
Timed out
```

3.229.7 Maxima [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{10}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="max
ima")
```

output
```
integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(10/3), x)
```

3.229.8 Giac [N/A]

Not integrable

Time = 4.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{10}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="gia
c")
```

output
```
integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(10/3), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(10/3),x)
```

output
```
\text{Hanged}
```

3.230 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m dx$

3.230.1 Optimal result

Integrand size = 27, antiderivative size = 106

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m dx = -\frac{\text{AppellF1}\left(np,\frac{1}{2},\frac{1}{2}-m,1+np,\sec(e+fx),-\sec(e+fx)\right)(c(d\sec(e+fx))^p)^n(1+\sec(e+fx))^{-\frac{1}{2}-m}\cdots}{fnp\sqrt{1-\sec(e+fx)}}$$

output
```
-AppellF1(n*p,1/2-m,1/2,n*p+1,-sec(f*x+e),sec(f*x+e))*(c*(d*sec(f*x+e))^p)
^n*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/n/p/(1-sec(f*x+
e))^(1/2)
```

3.230.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2425 vs. 2(106) = 212.

Time = 13.88 (sec) , antiderivative size = 2425, normalized size of antiderivative = 22.88

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m dx = \text{Result too large to show}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
(3*2^(1 + m)*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n*p)*(Cos[(e + f*x)/2]^2*Sec[e
 + f*x])^(m + n*p)*(c*(d*Sec[e + f*x])^p)^n*(a*(1 + Sec[e + f*x]))^m*Tan[(
e + f*x)/2])/(f*(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1 + m
 + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(n*p)*(Cos[(e + f*x)/2]^2*Sec[e
 + f*x])^(m + n*p))/(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1
 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
 + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n*p)*AppellF1[1/2, m + n*p, 1 - n*p, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n*p
)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n*p)*Tan[(e + f*x)/2]^2)/(3*Appel
lF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 +...
```

3.230.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4436, 3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^m \left(c(d\sec(e+fx))^p\right)^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a\sec(e+fx)+a)^m \left(c(d\sec(e+fx))^p\right)^n dx$$

$$\Big\downarrow \ 4436$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int (d\sec(e+fx))^{np}(\sec(e+fx)a+a)^m dx$$

$$\Big\downarrow \ 3042$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m dx$$

$\Big\downarrow$ 4315

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int(d\sec(e+fx))^{np}(\sec(e+fx)+1)^m dx$$

$\Big\downarrow$ 3042

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$\Big\downarrow$ 4314

$$-\frac{d\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int\frac{(d\sec(e+fx))^{n}}{\sqrt{1}}}{f\sqrt{1-\sec(e+fx)}}$$

$\Big\downarrow$ 150

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(np,\frac{1}{2},\frac{1}{2}-m,np+1,\sec(e+fx),-\sec(e+\right.}{fnp\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
-((AppellF1[n*p, 1/2, 1/2 - m, 1 + n*p, Sec[e + f*x], -Sec[e + f*x]]*(c*(d
*Sec[e + f*x])^p)^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*T
an[e + f*x])/(f*n*p*Sqrt[1 - Sec[e + f*x]]))
```

3.230.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

3.230.4 Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+a\sec(fx+e))^m \,dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)
```

3.230.5 Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m \,dx = \int ((d\sec(fx+e))^p c)^n (a\sec(fx+e)+a)^m \,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

3.230.6 Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m \, dx$$
$$= \int (a(\sec(e+fx)+1))^m (c(d\sec(e+fx))^p)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*(c*(d*sec(e + f*x))**p)**n, x)
```

3.230.7 Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m \, dx$$
$$= \int ((d\sec(fx+e))^p c)^n (a\sec(fx+e)+a)^m \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

3.230.8 Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m \, dx$$
$$= \int ((d\sec(fx+e))^p c)^n (a\sec(fx+e)+a)^m \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m\,dx$$
$$=\int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^m,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^m, x)
```

3.231 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx$

3.231.1 Optimal result

Integrand size = 27, antiderivative size = 275

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx$$
$$= \frac{a^3(7+4np)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp(2+np)\sqrt{\sin^2(e+fx)}}$$
$$- \frac{a^3(1+4np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}}$$
$$+ \frac{a^3(5+2np)(c(d\sec(e+fx))^p)^n\tan(e+fx)}{f(1+np)(2+np)}$$
$$+ \frac{(c(d\sec(e+fx))^p)^n(a^3+a^3\sec(e+fx))\tan(e+fx)}{f(2+np)}$$

output
```
a^3*(4*n*p+7)*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*s
ec(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(n*p+2)/(sin(f*x+e)^2)^(1/2)-a^3*(4*n*p+1
)*cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c
*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+a^3*(2
*n*p+5)*(c*(d*sec(f*x+e))^p)^n*tan(f*x+e)/f/(n*p+1)/(n*p+2)+(c*(d*sec(f*x+
e))^p)^n*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f/(n*p+2)
```

3.231.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.69

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3\,dx = \frac{a^3\cot(e+fx)(c(d\sec(e+fx))^p)^n\left((2+9np+4n^2p^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)\right.}{}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^3,x]
```

output
```
(a^3*Cot[e + f*x]*(c*(d*Sec[e + f*x])^p)^n*((2 + 9*n*p + 4*n^2*p^2)*Hyperg
eometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^
2] + n*p*((6 + 3*n*p + (1 + n*p)*Sec[e + f*x])*Tan[e + f*x]^2 + (7 + 4*n*p
)*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2]*Sec[e +
 f*x]*Sqrt[-Tan[e + f*x]^2])))/(f*n*p*(1 + n*p)*(2 + n*p))
```

3.231.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4436, 3042, 4301, 3042, 4485, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3 (c(d\sec(e+fx))^p)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sec(e+fx)+a)^3 (c(d\sec(e+fx))^p)^n dx$$

$$\Big\downarrow \text{4436}$$

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \int (d\sec(e+fx))^{np}(\sec(e+fx)a+a)^3dx$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np} \left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3 dx$$

$$\Big\downarrow \text{4301}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\int(d\sec(e+fx))^{np}(\sec(e+fx)a+a)(2a(np+1)+a(2np+5)\sec(e+fx))dx}{np+2}+\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\left(2a(np+1)+a(2np+5)\csc\left(\right.\right.}{np+2}\right.$$

$\downarrow$ 4485

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\left(\frac{\int(d\sec(e+fx))^{np}\left((np+2)(4np+1)a^2+(np+1)(4np+7)\sec(e+fx)a^2\right)dx}{np+1}+\frac{a^2(2np+5)\tan(e+fx)(d\sec}{f(np+1)}\right.}{np+2}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left((np+2)(4np+1)a^2+(np+1)(4np+7)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{np+1}+\frac{a^2(2np+5)\tan(e}{f(}\right.}{np+2}\right.$$

$\downarrow$ 4274

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\left(\frac{a^2(np+2)(4np+1)\int(d\sec(e+fx))^{np}dx+\frac{a^2(np+1)(4np+7)\int(d\sec(e+fx))^{np+1}dx}{d}}{np+1}+\frac{a^2(2np+5)\tan(e+}{f(n}\right.}{np+2}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\left(\frac{a^2(np+2)(4np+1)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}dx+\frac{a^2(np+1)(4np+7)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np+1}dx}{d}}{np+1}+a^2(2np+\right.}{np+2}\right.$$

$\downarrow$ 4259

3.231. $\int(c(d\sec(e+fx))^p)^n(a+a\sec(e+fx))^3\,dx$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\left(\frac{a^2(np+2)(4np+1)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx+\frac{a^2(np+1)(4np+7)\left(\frac{\cos(e+fx)}{d}\right.}{}}{np+1}\right.}{np+2}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\left(\frac{a^2(np+2)(4np+1)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-np}dx+\frac{a^2(np+1)(4np+7)\left(\frac{\cos(e\right.}{d}}{}}{np+1}\right.}{np+2}\right.$$

$\downarrow$ 3122

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)(d\sec(e+fx))^{np}}{f(np+2)}+\frac{a\left(\frac{a^2(np+1)(4np+7)\sin(e+fx)(d\sec}{}\right.}{}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^3,x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*(((d*Sec[e + f*x])^(n*p)*(a^3 + a^3*Sec[e + f*x]
)*Tan[e + f*x])/(f*(2 + n*p)) + (a*(((a^2*(1 + n*p)*(7 + 4*n*p)*Hypergeome
tric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(n*
p)*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a^2*d*(2 + n*p)*(1 + 4*n*
p)*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(d*Sec
[e + f*x])^(-1 + n*p)*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2]))/(1
 + n*p) + (a^2*(5 + 2*n*p)*(d*Sec[e + f*x])^(n*p)*Tan[e + f*x])/(f*(1 + n*
p))))/(2 + n*p)))/(d*Sec[e + f*x])^(n*p)
```

3.231. $\int (c(d\sec(e+fx))^p)^n\,(a+a\sec(e+fx))^3\,dx$

3.231.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

3.231.4 Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+a\sec(fx+e))^3 \, dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x)
```

3.231.5 Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx = \int (a\sec(fx+e)+a)^3 ((d\sec(fx+e))^p c)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
 a^3)*((d*sec(f*x + e))^p*c)^n, x)
```

3.231.6 Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx = a^3 \left(\int (c(d\sec(e+fx))^p)^n \, dx + \int 3(c(d\sec(e+fx))^p)^n \sec(e+fx) \, dx + \int 3(c(d\sec(e+fx))^p)^n \sec^2(e+fx) \, dx + \int (c(d\sec(e+fx))^p)^n \sec^3(e+fx) \, dx \right)$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**3,x)
```

output
```
a**3*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral(3*(c*(d*sec(e + f
*x))**p)**n*sec(e + f*x), x) + Integral(3*(c*(d*sec(e + f*x))**p)**n*sec(e
 + f*x)**2, x) + Integral((c*(d*sec(e + f*x))**p)**n*sec(e + f*x)**3, x))
```

3.231.7 Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx = \int (a\sec(fx+e)+a)^3((d\sec(fx+e))^p c)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

3.231.8 Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx = \int (a\sec(fx+e)+a)^3((d\sec(fx+e))^p c)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{a}{\cos(e+f\,x)}\right)^3 dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^3,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^3, x)
```

3.232 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 dx$

3.232.1 Optimal result

Integrand size = 27, antiderivative size = 205

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 dx = \frac{2a^2 \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n \sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}} - \frac{a^2(1+2np)\cos(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}} + \frac{a^2(c(d\sec(e+fx))^p)^n \tan(e+fx)}{f(1+np)}$$

output
```
2*a^2*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e
))^p)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a^2*(2*n*p+1)*cos(f*x+e)*hyp
ergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^
p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+a^2*(c*(d*sec(f*x+e))^
p)^n*tan(f*x+e)/f/(n*p+1)
```

3.232.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.74

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2\,dx$$
$$= \frac{a^2\cot(e+fx)\,(c(d\sec(e+fx))^p)^n \left((1+2np)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)\sqrt{-\tan^2}\right.}{}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^2,x]
```

output
```
(a^2*Cot[e + f*x]*(c*(d*Sec[e + f*x])^p)^n*((1 + 2*n*p)*Hypergeometric2F1[
1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2] + n*p*(Ta
n[e + f*x]^2 + 2*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e +
f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2])))/(f*n*p*(1 + n*p))
```

3.232.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {3042, 4436, 3042, 4275, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2 (c(d\sec(e+fx))^p)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sec(e+fx)+a)^2 (c(d\sec(e+fx))^p)^n dx$$

$$\Big\downarrow \text{4436}$$

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \int (d\sec(e+fx))^{np}(\sec(e+fx)a+a)^2dx$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np} \left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2 dx$$

$$\Big\downarrow \text{4275}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int (d\sec(e+fx))^{np}\left(\sec^2(e+fx)a^2+a^2\right)dx+\frac{2a^2\int (d\sec(e+fx))^{np+1}dx}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{2a^2\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np+1}dx}{d}+\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right.\right.\right.$$

$$\Big\downarrow \text{4259}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\left(\frac{\cos(e+fx)}{d}\right)^{np}(\ldots}{\ldots}\right.$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\left(\frac{\cos(e+fx)}{d}\right)^{np}(\ldots}{\ldots}\right.$$

$$\Big\downarrow \text{3122}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\sin(e+fx)(d\,\mathrm{se}\ldots}{\ldots}\right.$$

$$\Big\downarrow \text{4534}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{a^2(2np+1)\int (d\sec(e+fx))^{np}dx}{np+1}+\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{np}\,\mathrm{Hypergeome}\ldots}{fnp\sqrt{\sin^2(\ldots}}\right.$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{a^2(2np+1)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}dx}{np+1}+\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{np}\,\mathrm{Hyper}\ldots}{fnp\sqrt{\ldots}}\right.$$

$$\Big\downarrow \text{4259}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{a^2(2np+1)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int \left(\frac{\cos(e+fx)}{d}\right)^{-np}dx}{np+1}+\frac{2a^2\sin(e+fx)\ldots}{\ldots}\right.$$

3.232. $\int \left(c(d\sec(e+fx))^p\right)^n(a+a\sec(e+fx))^2\,dx$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{a^2(2np+1)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-np}dx}{np+1}+\frac{2a^2\sin(e+\ldots}{}\right.$$

$$\Big\downarrow \text{3122}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{np}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+\ldots\right)}{fnp\sqrt{\sin^2(e+fx)}}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^2,x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*((2*a^2*Hypergeometric2F1[1/2, -1/2*(n*p), (2 -
n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(n*p)*Sin[e + f*x])/(f*n*p*Sqrt[S
in[e + f*x]^2]) - (a^2*d*(1 + 2*n*p)*Hypergeometric2F1[1/2, (1 - n*p)/2, (
3 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e + f*x])/(f*(
1 - n*p)*(1 + n*p)*Sqrt[Sin[e + f*x]^2]) + (a^2*(d*Sec[e + f*x])^(n*p)*Tan
[e + f*x])/(f*(1 + n*p))))/(d*Sec[e + f*x])^(n*p)
```

3.232.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.232.4 Maple [F]

$$\int (c(d\sec (fx+e))^{p})^{n} (a+a\sec (fx+e))^{2} dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x)
```

3.232.5 Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 dx = \int (a\sec (fx+e)+a)^2((d\sec (fx+e))^p c)^n dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*((d*sec(f*x + e))
^p*c)^n, x)
```

3.232.6 Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2\,dx = a^2\left(\int (c(d\sec(e+fx))^p)^n\,dx + \int 2(c(d\sec(e+fx))^p)^n\sec(e+fx)\,dx + \int (c(d\sec(e+fx))^p)^n\sec^2(e+fx)\,dx\right)$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**2,x)
```

output
```
a**2*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral(2*(c*(d*sec(e + f
*x))**p)**n*sec(e + f*x), x) + Integral((c*(d*sec(e + f*x))**p)**n*sec(e +
 f*x)**2, x))
```

3.232.7 Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2\,dx = \int (a\sec(fx+e)+a)^2((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

3.232.8 Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2\,dx = \int (a\sec(fx+e)+a)^2((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 \,dx$$
$$= \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{a}{\cos(e+f\,x)}\right)^2 \,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^2,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^2, x)
```

3.233 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx$

3.233.1 Optimal result

Integrand size = 25, antiderivative size = 156

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \frac{a\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}} - \frac{a\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+}{f(1-np)\sqrt{\sin^2(e+fx)}}$$

output
```
a*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p
)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a*cos(f*x+e)*hypergeom([1/2, -1/
2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/
f/(-n*p+1)/(sin(f*x+e)^2)^(1/2)
```

3.233.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \frac{a\csc(e+fx)\left((1+np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)+np\,\text{Hypergeometr}}{fnp(1+np)}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x]),x]
```

output
```
(a*Csc[e + f*x]*((1 + n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1
+ (n*p)/2, Sec[e + f*x]^2] + n*p*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 +
n*p)/2, Sec[e + f*x]^2])*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(
f*n*p*(1 + n*p))
```

3.233.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4436, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)\left(c(d\sec(e+fx))^p\right)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sec(e+fx)+a)\left(c(d\sec(e+fx))^p\right)^n dx$$

$$\Big\downarrow \text{4436}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int (d\sec(e+fx))^{np}(\sec(e+fx)a+a)dx$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)dx$$

$$\Big\downarrow \text{4274}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(a\int (d\sec(e+fx))^{np}dx+\frac{a\int (d\sec(e+fx))^{np+1}dx}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}dx+\frac{a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np+1}dx}{d}\right)$$

$$\Big\downarrow \text{4259}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx+\frac{a\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\right.$$

$\Big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-np}dx+\frac{a\left(\frac{\cos(e+fx)}{d}\right)\right.$$

$\Big\downarrow$ 3122

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\sin(e+fx)(d\sec(e+fx))^{np}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx\right.}{fnp\sqrt{\sin^2(e+fx)}}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x]),x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*((a*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)
/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(n*p)*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e
 + f*x]^2]) - (a*d*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e
+ f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[
e + f*x]^2])))/(d*Sec[e + f*x])^(n*p)
```

3.233.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

3.233.4 Maple [F]

$$\int \left(c(d\sec \left(fx+e\right))^{p}\right)^{n} \left(a+a\sec \left(fx+e\right)\right) dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x)
```

3.233.5 Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx)) \, dx = \int (a\sec \left(fx+e\right)+a)\left(\left(d\sec \left(fx+e\right)\right)^{p} c\right)^{n} \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

3.233.6 Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = a\left(\int (c(d\sec(e+fx))^p)^n\,dx + \int (c(d\sec(e+fx))^p)^n \sec(e+fx)\,dx\right)$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e)),x)
```

output
```
a*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral((c*(d*sec(e + f*x))*
*p)**n*sec(e + f*x), x))
```

3.233.7 Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

3.233.8 Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a + \frac{a}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x)),x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x)), x)
```

3.234 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx$

3.234.1 Optimal result

Integrand size = 27, antiderivative size = 208

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \frac{(c(d\sec(e+fx))^p)^n\sin(e+fx)}{f(a+a\sec(e+fx))}$$
$$-\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+f}{af\sqrt{\sin^2(e+fx)}}$$
$$+\frac{(1-np)\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2-np),\frac{1}{2}(4-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{af(2-np)\sqrt{\sin^2(e+fx)}}$$

output
```
(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(a+a*sec(f*x+e))-cos(f*x+e)*hypergeom(
[1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*si
n(f*x+e)/a/f/(sin(f*x+e)^2)^(1/2)+(-n*p+1)*cos(f*x+e)^2*hypergeom([1/2, -1
/2*n*p+1],[-1/2*n*p+2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/a/f
/(-n*p+2)/(sin(f*x+e)^2)^(1/2)
```

3.234.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.76

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx$$
$$= \frac{\cot\left(\frac{1}{2}(e+fx)\right)(c(d\sec(e+fx))^p)^n\left(-\left((-1+np)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)\sqrt{-}\right.\right.}{}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]),x]
```

output
```
(Cot[(e + f*x)/2]*(c*(d*Sec[e + f*x])^p)^n*(-((-1 + n*p)*Hypergeometric2F1
[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]) + n*p*(
1 - Cos[e + f*x] + Cos[e + f*x]*Hypergeometric2F1[1/2, (-1 + n*p)/2, (1 +
n*p)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2])))/(a*f*n*p*(1 + Sec[e + f*x
]))
```

3.234.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3042, 4436, 3042, 4307, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{4436}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\int \frac{(d\sec(e+fx))^{np}}{\sec(e+fx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4307}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\int(d\sec(e+fx))^{np-1}(a-a\sec(e+$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np-1}(a-a\csc$$

$\downarrow$ 4274

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\left(a\int(d\sec(e+fx))^{np-1}dx-\frac{a\int(d\sec}{}\right.}{a^2}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\left(a\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np-1}dx-\right.}{a^2}\right.$$

$\downarrow$ 4259

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\right.}{}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\right.}{}\right.$$

$\downarrow$ 3122

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\left(\frac{a\sin(e+fx)(d\sec(e+fx))^{np-1}\text{Hypergeome}}{f(1-np)\sqrt{}}\right.}{}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]),x]
```

3.234. $\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx$

output
```
((c*(d*Sec[e + f*x])^p)^n*(-((d*(1 - n*p)*(-((a*d*Hypergeometric2F1[1/2, (
2 - n*p)/2, (4 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-2 + n*p)*Sin[e
 + f*x])/(f*(2 - n*p)*Sqrt[Sin[e + f*x]^2])) + (a*Hypergeometric2F1[1/2, (
1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e
 + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2])))/a^2) + (d*(d*Sec[e + f*x])^(
-1 + n*p)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))))/(d*Sec[e + f*x])^(n*p)
```

3.234.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Simp[d*((n - 1)/(a*b))   Int[(d*Csc[e + f*x])^
(n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ
[a^2 - b^2, 0]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] && !IntegerQ[n]
```

3.234.4 Maple [F]

$$\int \frac{(c(d\sec (fx+e))^{p})^{n}}{a+a\sec (fx+e)}dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x)
```

3.234.5 Fricas [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \int \frac{((d\sec (fx+e))^{p} c)^{n}}{a\sec (fx+e)+a}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

3.234.6 Sympy [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{(c(d\sec (e+fx))^{p})^{n}}{\sec (e+fx)+1}\,dx}{a}$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n/(a+a*sec(f*x+e)),x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n/(sec(e + f*x) + 1), x)/a
```

3.234.7 Maxima [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx=\int \frac{((d\sec{(fx+e)})^p\,c)^n}{a\sec{(fx+e)}+a}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

3.234.8 Giac [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx=\int \frac{((d\sec{(fx+e)})^p\,c)^n}{a\sec{(fx+e)}+a}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx=\int \frac{\left(c\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^p\right)^n}{a+\frac{a}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x)),x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x)), x)
```

3.235 $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$

3.235.1 Optimal result

Integrand size = 27, antiderivative size = 248

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \frac{2(2-np)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}} - \frac{(3-2np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{3a^2f\sqrt{\sin^2(e+fx)}} - \frac{2(2-np)(c(d\sec(e+fx))^p)^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(c(d\sec(e+fx))^p)^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

output
```
2/3*(-n*p+2)*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*se
c(f*x+e))^p)^n*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)-1/3*(-2*n*p+3)*cos(f*
x+e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(
f*x+e))^p)^n*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)-2/3*(-n*p+2)*(c*(d*sec(
f*x+e))^p)^n*tan(f*x+e)/a^2/f/(1+sec(f*x+e))-1/3*(c*(d*sec(f*x+e))^p)^n*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^2
```

3.235.2 Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.87

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{(c(d\sec(e+fx))^p)^n\left(-np(1+np)\tan(e+fx)+(1+\sec(e+fx))\left(2np(-2+np)(1+np)\tan(e+fx)\right.\right.}{}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*(-(n*p*(1 + n*p)*Tan[e + f*x]) + (1 + Sec[e + f*
x])*(2*n*p*(-2 + n*p)*(1 + n*p)*Tan[e + f*x] + ((-1 + n*p)*(1 + n*p)*(-3 +
 2*n*p)*Cot[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e +
f*x]^2] - 2*n^2*p^2*(-2 + n*p)*Csc[e + f*x]*Hypergeometric2F1[1/2, (1 + n*
p)/2, (3 + n*p)/2, Sec[e + f*x]^2])*(1 + Sec[e + f*x])*Sqrt[-Tan[e + f*x]^
2])))/(3*a^2*f*n*p*(1 + n*p)*(1 + Sec[e + f*x])^2)
```

3.235.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {3042, 4436, 3042, 4304, 25, 3042, 4508, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a\sec(e+fx)+a)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a\sec(e+fx)+a)^2}dx$$
$$\Big\downarrow \text{4436}$$
$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\int \frac{(d\sec(e+fx))^{np}}{(\sec(e+fx)a+a)^2}dx$$
$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int\frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$\Big\downarrow$ 4304

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(-\frac{\int-\frac{(d\sec(e+fx))^{np}(a(3-np)-a(1-np)\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{\tan(e+fx)(d\sec(e+fx))^{np}}{3f(a\sec(e+fx)+a)^2}\right)$$

$\Big\downarrow$ 25

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\int\frac{(d\sec(e+fx))^{np}(a(3-np)-a(1-np)\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{\tan(e+fx)(d\sec(e+fx))^{np}}{3f(a\sec(e+fx)+a)^2}\right)$$

$\Big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\int\frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a(3-np)-a(1-np)\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\tan(e+fx)(d\sec(e+fx))^{np}}{3f(a\sec(e+fx)+a)^2}\ldots\right.$$

$\Big\downarrow$ 4508

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{\int(d\sec(e+fx))^{np}\left((3-2np)(1-np)a^2+2np(2-np)\sec(e+fx)a^2\right)dx}{a^2}-\frac{2(2-np)\tan(e+fx)(d\sec(e+fx))\ldots}{f(\sec(e+fx)+1)\ldots}}{3a^2\ldots}\right.$$

$\Big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left((3-2np)(1-np)a^2+2np(2-np)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{a^2}-\frac{2(2-np)\tan(e+fx)(d\sec\ldots}{f(\sec(e+fx)+1\ldots}}{3a^2\ldots}\right.$$

$\Big\downarrow$ 4274

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{a^2(3-2np)(1-np)\int(d\sec(e+fx))^{np}dx+\frac{2a^2np(2-np)\int(d\sec(e+fx))^{np+1}dx}{d}}{a^2}-\frac{2(2-np)\tan(e+fx)(d\sec(\ldots}{f(\sec(e+fx)+1)\ldots}}{3a^2\ldots}\right.$$

$\Big\downarrow$ 3042

3.235. $\int\frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}dx$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{a^2(3-2np)(1-np)\int\left(d\csc(e+fx+\frac{\pi}{2})\right)^{np}dx+\frac{2a^2np(2-np)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np+1}dx}{d}}{a^2}-\frac{2(2-np)\tan(e+}{f(\sec(e}}{3a^2}\right.$$

$\Big\downarrow$ 4259

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{a^2(3-2np)(1-np)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx+\frac{2a^2np(2-np)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec}{}}{a^2}}{3a^2}\right.$$

$\Big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{a^2(3-2np)(1-np)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-np}dx+\frac{2a^2np(2-np)\left(\frac{\cos(e+fx)}{d}\right)^{np}}{}}{a^2}}{3a^2}\right.$$

$\Big\downarrow$ 3122

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{\frac{2a^2(2-np)\sin(e+fx)(d\sec(e+fx))^{np}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}-\frac{a^2d(3-2np)\sin(e+fx)(d}{}}{a^2}}{3}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*(-1/3*((d*Sec[e + f*x])^(n*p)*Tan[e + f*x])/(f*(
a + a*Sec[e + f*x])^2) + (((2*a^2*(2 - n*p)*Hypergeometric2F1[1/2, -1/2*(n
*p), (2 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(n*p)*Sin[e + f*x])/(f*
Sqrt[Sin[e + f*x]^2]) - (a^2*d*(3 - 2*n*p)*Hypergeometric2F1[1/2, (1 - n*p
)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e + f*x]
)/(f*Sqrt[Sin[e + f*x]^2]))/a^2 - (2*(2 - n*p)*(d*Sec[e + f*x])^(n*p)*Tan[
e + f*x])/(f*(1 + Sec[e + f*x]))))/(3*a^2)))/(d*Sec[e + f*x])^(n*p)
```

3.235. $\int\frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$

3.235.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

3.235. $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.235.4 Maple [F]

$$\int \frac{\left(c\left(d\sec\left(fx+e\right)\right)^{p}\right)^{n}}{\left(a+a\sec\left(fx+e\right)\right)^{2}}dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x)
```

3.235.5 Fricas [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx=\int \frac{\left(\left(d\sec\left(fx+e\right)\right)^{p}c\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(((d*sec(f*x + e))^p*c)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e)
 + a^2), x)
```

3.235.6 Sympy [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx=\frac{\int \frac{\left(c\left(d\sec\left(e+fx\right)\right)^{p}\right)^{n}}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^{2}}$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n/(a+a*sec(f*x+e))**2,x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1)
, x)/a**2
```

3.235.7 Maxima [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{((d\sec(fx+e))^p c)^n}{(a\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a)^2, x)
```

3.235.8 Giac [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{((d\sec(fx+e))^p c)^n}{(a\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a)^2, x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x))^2,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x))^2, x)
```

3.236 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx$

3.236.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\begin{aligned}&\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx\\&=(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n \operatorname{Int}((d\sec(e+fx))^{np}(a+b\sec(e+fx))^m,x)\end{aligned}$$

output
```
(c*(d*sec(f*x+e))^p)^n*Unintegrable((d*sec(f*x+e))^(n*p)*(a+b*sec(f*x+e))^
m,x)/((d*sec(f*x+e))^(n*p))
```

3.236.2 Mathematica [N/A]

Not integrable

Time = 6.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx = \int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m,x]
```

output
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m, x]
```

3.236.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4436, 3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^m (c(d\sec(e+fx))^p)^n \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sec(e+fx))^m (c(d\sec(e+fx))^p)^n \, dx$$

$$\Big\downarrow \text{4436}$$

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \int (d\sec(e+fx))^{np}(a+b\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np} \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{4357}$$

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \int (d\sec(e+fx))^{np}(a+b\sec(e+fx))^m dx$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

3.236.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

3.236.4 Maple [N/A] (verified)

Not integrable

Time = 1.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (c(d\sec (fx+e))^{p})^{n} (a+b\sec (fx+e))^{m} \, dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x)
```

3.236.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx = \int ((d\sec (fx+e))^{p} c)^{n} (b\sec (fx+e)+a)^{m} \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

3.236.6 Sympy [N/A]

Not integrable

Time = 5.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m\,dx = \int (c(d\sec{(e+fx)})^p)^n (a+b\sec{(e+fx)})^m\,dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**m, x)
```

3.236.7 Maxima [N/A]

Not integrable

Time = 4.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m\,dx = \int ((d\sec{(fx+e)})^p c)^n (b\sec{(fx+e)}+a)^m\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

3.236.8 Giac [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m\,dx = \int ((d\sec{(fx+e)})^p c)^n (b\sec{(fx+e)}+a)^m\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

3.236.9 Mupad [N/A]

Not integrable

Time = 15.97 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m\,dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{b}{\cos(e+f\,x)}\right)^m dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^m,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^m, x)
```

3.237 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \, dx$

3.237.1 Optimal result

Integrand size = 27, antiderivative size = 296

$$\begin{aligned}
&\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \, dx \\
&= \frac{b(b^2(1+np)+3a^2(2+np)) \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n \sin(e+fx)}{fnp(2+np)\sqrt{\sin^2(e+fx)}} \\
&\quad - \frac{a(3b^2np+a^2(1+np))\cos(e+fx) \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n \sin(e+fx)}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}} \\
&\quad + \frac{ab^2(5+2np)(c(d\sec(e+fx))^p)^n \tan(e+fx)}{f(1+np)(2+np)} \\
&\quad + \frac{b^2(c(d\sec(e+fx))^p)^n (a+b\sec(e+fx)) \tan(e+fx)}{f(2+np)}
\end{aligned}$$

output
```
b*(b^2*(n*p+1)+3*a^2*(n*p+2))*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f
*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(n*p+2)/(sin(f*x+e)^2)^(1
/2)-a*(3*b^2*n*p+a^2*(n*p+1))*cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2],[-1
/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)
/(sin(f*x+e)^2)^(1/2)+a*b^2*(2*n*p+5)*(c*(d*sec(f*x+e))^p)^n*tan(f*x+e)/f/
(n*p+1)/(n*p+2)+b^2*(c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))*tan(f*x+e)/f/(
n*p+2)
```

3.237.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.94

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3\,dx =$$
$$-\frac{\csc^3(e+fx)\left(a^3(6+11np+6n^2p^2+n^3p^3)\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+f\right.\right.}{}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^3,x]
```

output
```
-((Csc[e + f*x]^3*(a^3*(6 + 11*n*p + 6*n^2*p^2 + n^3*p^3)*Cos[e + f*x]^3*H
ypergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + b*n*p*(3*a*b
*(3 + 4*n*p + n^2*p^2)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + (n*p)/2, 2
+ (n*p)/2, Sec[e + f*x]^2] + (2 + n*p)*(3*a^2*(3 + n*p)*Cos[e + f*x]^2*Hyp
ergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2] + b^2*(1 + n
*p)*Hypergeometric2F1[1/2, (3 + n*p)/2, (5 + n*p)/2, Sec[e + f*x]^2]))*(c
*(d*Sec[e + f*x])^p)^n*(-Tan[e + f*x]^2)^(3/2))/(f*n*p*(1 + n*p)*(2 + n*p)
*(3 + n*p)))
```

3.237.3 Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4436, 3042, 4329, 3042, 4535, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^3 (c(d\sec(e+fx))^p)^n\;dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\sec(e+fx))^3 (c(d\sec(e+fx))^p)^n dx$$

$$\Big\downarrow \; 4436$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\int (d\sec(e+fx))^{np}(a+b\sec(e+fx))^3dx$$

$$\Big\downarrow \; 3042$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$\downarrow$ 4329

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{\int(d\sec(e+fx))^{np}\left(ab^2d(2np+5)\sec^2(e+fx)+bd\left(3(np+2)a^2+b^2(np+1)\right)\text{se}}{d(np+2)}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(ab^2d(2np+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+bd(3(np+2)a^2+b^2(}{d(np+2)}\right.$$

$\downarrow$ 4535

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{\int(d\sec(e+fx))^{np}\left(ab^2d(2np+5)\sec^2(e+fx)+ad\left((np+2)a^2+b^2np\right)\right)dx+b(3}{d(np+2)}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{b\left(3a^2(np+2)+b^2(np+1)\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np+1}dx+\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right.\right.}{d(np+2)}\right.$$

$\downarrow$ 4259

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(ab^2d(2np+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad((np+2)a^2+b^2n}{}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(ab^2d(2np+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad((np+2)a^2+b^2n}{}\right.$$

$\downarrow$ 3122

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(ab^2d(2np+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad((np+2)a^2+b^2n}{}\right.$$

3.237. $\int(c(d\sec(e+fx))^p)^n(a+b\sec(e+fx))^3\,dx$

$\downarrow$ 4534

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{ad(np+2)\left(a^2(np+1)+3b^2np\right)\int(d\sec(e+fx))^{np}dx}{np+1}+\frac{bd\left(3a^2(np+2)+b^2(np+1)\right)\sin(e+fx)(d\sec(e+fx)}{fnp\sqrt{s}}}{d(np+}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{ad(np+2)\left(a^2(np+1)+3b^2np\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}dx}{np+1}+\frac{bd\left(3a^2(np+2)+b^2(np+1)\right)\sin(e+fx)(d\sec(e+}{f}}{d(np}\right.$$

$\downarrow$ 4259

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{ad(np+2)\left(a^2(np+1)+3b^2np\right)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx}{np+1}+\frac{bd\left(3a^2(np+2)+b^2(\right.}{}}{}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{ad(np+2)\left(a^2(np+1)+3b^2np\right)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-np}dx}{np+1}+\frac{bd\left(3a^2(np+2)\right.}{}}{}\right.$$

$\downarrow$ 3122

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{-\frac{ad^2(np+2)\left(a^2(np+1)+3b^2np\right)\sin(e+fx)(d\sec(e+fx))^{np-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np)\right)}{f(1-np)(np+1)\sqrt{\sin^2(e+fx)}}}{}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^3,x]
```

3.237. $\int\left(c(d\sec(e+fx))^p\right)^n(a+b\sec(e+fx))^3\,dx$

output
```
((c*(d*Sec[e + f*x])^p)^n*((b^2*(d*Sec[e + f*x])^(n*p)*(a + b*Sec[e + f*x]
)*Tan[e + f*x])/(f*(2 + n*p)) + ((b*d*(b^2*(1 + n*p) + 3*a^2*(2 + n*p))*Hy
pergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f
*x])^(n*p)*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a*d^2*(2 + n*p)*(
3*b^2*n*p + a^2*(1 + n*p))*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2
, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e + f*x])/(f*(1 - n*p)*(
1 + n*p)*Sqrt[Sin[e + f*x]^2]) + (a*b^2*d*(5 + 2*n*p)*(d*Sec[e + f*x])^(n*
p)*Tan[e + f*x])/(f*(1 + n*p)))/(d*(2 + n*p))))/(d*Sec[e + f*x])^(n*p)
```

3.237.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 && !(IGtQ[n, 2] && !IntegerQ[m])
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] && !IntegerQ[n]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.237.4 Maple [F]

$$\int (c(d\sec (fx+e))^p)^n (a+b\sec (fx+e))^3 \, dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x)
```

3.237.5 Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \, dx = \int (b\sec (fx+e)+a)^3 ((d\sec (fx+e))^p c)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x +
e) + a^3)*((d*sec(f*x + e))^p*c)^n, x)
```

3.237.6 Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \,dx = \int (c(d\sec{(e+fx)})^p)^n (a+b\sec{(e+fx)})^3 \,dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**3,x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**3, x)
```

3.237.7 Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \,dx = \int (b\sec{(fx+e)}+a)^3((d\sec{(fx+e)})^p c)^n \,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

3.237.8 Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \,dx = \int (b\sec{(fx+e)}+a)^3((d\sec{(fx+e)})^p c)^n \,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \, dx = \int \left(c\left(\frac{d}{\cos(e+fx)}\right)^p\right)^n \left(a+\frac{b}{\cos(e+fx)}\right)^3 \, dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^3,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^3, x)
```

3.238 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx$

3.238.1 Optimal result

Integrand size = 27, antiderivative size = 211

$$\begin{aligned}
&\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx \\
&= \frac{2ab \,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n \sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}} \\
&\quad - \frac{(b^2np+a^2(1+np))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}} \\
&\quad + \frac{b^2(c(d\sec(e+fx))^p)^n\tan(e+fx)}{f(1+np)}
\end{aligned}$$

output
```
2*a*b*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e
))^p)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-(b^2*n*p+a^2*(n*p+1))*cos(f*
x+e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(
f*x+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+b^2*(c*(d*sec(
f*x+e))^p)^n*tan(f*x+e)/f/(n*p+1)
```

3.238.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx$$
$$= \frac{\csc(e+fx)\left(a^2(2+3np+n^2p^2)\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)+bnp(b(1\right.}{}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^2,x]
```

output
```
(Csc[e + f*x]*(a^2*(2 + 3*n*p + n^2*p^2)*Cos[e + f*x]^2*Hypergeometric2F1[
1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + b*n*p*(b*(1 + n*p)*Hypergeome
tric2F1[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sec[e + f*x]^2] + 2*a*(2 + n*p)*Cos
[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2]
))*Sec[e + f*x]*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(f*n*p*(1
+ n*p)*(2 + n*p))
```

3.238.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {3042, 4436, 3042, 4275, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^2 (c(d\sec(e+fx))^p)^n \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sec(e+fx))^2 (c(d\sec(e+fx))^p)^n dx$$

$$\Big\downarrow \text{4436}$$

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \int (d\sec(e+fx))^{np}(a+b\sec(e+fx))^2 dx$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np} \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2 dx$$

$\downarrow$ 4275

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int(d\sec(e+fx))^{np}\left(a^2+b^2\sec^2(e+fx)\right)dx+\frac{2ab\int(d\sec(e+fx))^{np+1}dx}{d}\right)$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\int\left(d\csc\left(e+fx\right.\right.}{d}\right.$$

$\downarrow$ 4259

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\left(\frac{\cos(e+fx)}{d}\right)^{np}(c}{}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\left(\frac{\cos(e+fx)}{d}\right)^{np}(c}{}\right.$$

$\downarrow$ 3122

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\sin(e+fx)(d\,\mathrm{se}}{}\right.$$

$\downarrow$ 4534

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\left(a^2+\frac{b^2np}{np+1}\right)\int(d\sec(e+fx))^{np}dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^{np}\,\mathrm{Hyperge}}{fnp\sqrt{\mathrm{s}}}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\left(a^2+\frac{b^2np}{np+1}\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^{np}\,\mathrm{F}}{}\right.$$

$\downarrow$ 4259

3.238. $\int\left(c(d\sec(e+fx))^p\right)^n(a+b\sec(e+fx))^2\,dx$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\left(a^2+\frac{b^2np}{np+1}\right)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx+\frac{2ab}{\ldots}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\left(a^2+\frac{b^2np}{np+1}\right)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-np}dx\ldots\right.$$

$\downarrow$ 3122

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(-\frac{d\left(a^2+\frac{b^2np}{np+1}\right)\sin(e+fx)(d\sec(e+fx))^{np-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\ldots\right.}{f(1-np)\sqrt{\sin^2(e+fx)}}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^2,x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*((2*a*b*Hypergeometric2F1[1/2, -1/2*(n*p), (2 -
n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(n*p)*Sin[e + f*x])/(f*n*p*Sqrt[S
in[e + f*x]^2]) - (d*(a^2 + (b^2*n*p)/(1 + n*p))*Hypergeometric2F1[1/2, (1
 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e
+ f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2]) + (b^2*(d*Sec[e + f*x])^(n*p)*T
an[e + f*x])/(f*(1 + n*p))))/(d*Sec[e + f*x])^(n*p)
```

3.238.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.238.4 Maple [F]

$$\int \left(c(d\sec \left(fx+e\right))^{p}\right)^{n} \left(a+b\sec \left(fx+e\right)\right)^{2} dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x)
```

3.238.5 Fricas [F]

$$\int \left(c(d\sec(e+fx))^p\right)^n (a+b\sec(e+fx))^2 \, dx = \int \left(b\sec \left(fx+e\right)+a\right)^{2} \left(\left(d\sec \left(fx+e\right)\right)^{p} c\right)^{n} \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*((d*sec(f*x + e))
^p*c)^n, x)
```

3.238.6 Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx = \int (c(d\sec{(e+fx)})^p)^n (a+b\sec{(e+fx)})^2 \, dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**2, x)
```

3.238.7 Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx = \int (b\sec{(fx+e)}+a)^2((d\sec{(fx+e)})^p c)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

3.238.8 Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx = \int (b\sec{(fx+e)}+a)^2((d\sec{(fx+e)})^p c)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2\,dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{b}{\cos(e+f\,x)}\right)^2\,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^2,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^2, x)
```

3.239 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx$

3.239.1 Optimal result

Integrand size = 25, antiderivative size = 156

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx$$
$$= \frac{b\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}$$
$$- \frac{a\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+}{f(1-np)\sqrt{\sin^2(e+fx)}}$$

output
```
b*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p
)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a*cos(f*x+e)*hypergeom([1/2, -1/
2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/
f/(-n*p+1)/(sin(f*x+e)^2)^(1/2)
```

3.239.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx$$
$$= \frac{\csc(e+fx)\left(a(1+np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)+bnp\,\text{Hypergeomet}\right.}{fnp(1+np)}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x]),x]
```

output
```
(Csc[e + f*x]*(a*(1 + n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1
+ (n*p)/2, Sec[e + f*x]^2] + b*n*p*Hypergeometric2F1[1/2, (1 + n*p)/2, (3
+ n*p)/2, Sec[e + f*x]^2])*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])
/(f*n*p*(1 + n*p))
```

3.239.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4436, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))\left(c(d\sec(e+fx))^p\right)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sec(e+fx))\left(c(d\sec(e+fx))^p\right)^n\,dx$$

$$\Big\downarrow \text{4436}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int (d\sec(e+fx))^{np}(a+b\sec(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4274}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(a\int (d\sec(e+fx))^{np}dx+\frac{b\int (d\sec(e+fx))^{np+1}dx}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}dx+\frac{b\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np+1}dx}{d}\right)$$

$$\Big\downarrow \text{4259}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx+\frac{b\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\right.$$

$\big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-np}dx+\frac{b\left(\frac{\cos(e+fx)}{d}\right)}{}\right.$$

$\big\downarrow$ 3122

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{b\sin(e+fx)(d\sec(e+fx))^{np}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx\right.}{fnp\sqrt{\sin^2(e+fx)}}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x]),x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*((b*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)
/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(n*p)*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e
 + f*x]^2]) - (a*d*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e
+ f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[
e + f*x]^2])))/(d*Sec[e + f*x])^(n*p)
```

3.239.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

3.239.4 Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+b\sec(fx+e))\,dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x)
```

3.239.5 Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

3.239.6 Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e)),x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x)), x)
```

3.239.7 Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

3.239.8 Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a + \frac{b}{\cos(e+f\,x)}\right) dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x)),x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x)), x)
```

3.240 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx$

3.240.1 Optimal result

Integrand size = 27, antiderivative size = 206

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = -\frac{b\,\text{AppellF1}\left(\frac{1}{2},\frac{np}{2},1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}\,(c(d\sec(e+fx))^p)^n\sin(e+fx)}{(a^2-b^2)\,f} + \frac{a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1+np),1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+np)}\,(c(d\sec(e}{(a^2-b^2)\,f}$$

output
```
-b*AppellF1(1/2,1/2*n*p,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*(co
s(f*x+e)^2)^(1/2*n*p)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b^2)/f+a*Appe
llF1(1/2,1/2*n*p-1/2,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*cos(f*
x+e)*(cos(f*x+e)^2)^(1/2*n*p-1/2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b
^2)/f
```

3.240.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5411 vs. $2(206) = 412$.

Time = 32.62 (sec) , antiderivative size = 5411, normalized size of antiderivative = 26.27

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x]),x]
```

output
```
Result too large to show
```

3.240.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3042, 4436, 3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}dx$$

$$\Big\downarrow \text{4436}$$

$$(d\sec(e+fx))^{-np}\,(c(d\sec(e+fx))^p)^n\int \frac{(d\sec(e+fx))^{np}}{a+b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}\,(c(d\sec(e+fx))^p)^n\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4356}$$

$$\cos^{np}(e+fx)\,(c(d\sec(e+fx))^p)^n\int \frac{\cos^{1-np}(e+fx)}{b+a\cos(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{1-np}}{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3302}$$

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(b\int\frac{\cos^{1-np}(e+fx)}{b^2-a^2\cos^2(e+fx)}dx-a\int\frac{\cos^{2-np}(e+fx)}{b^2-a^2\cos^2(e+fx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(b\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{1-np}}{b^2-a^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx-a\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{2-np}}{b^2-a^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx\right)$$

$$\Big\downarrow \text{3668}$$

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(\frac{b\cos^{-np}(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\int-\frac{\left(1-\sin^2(e+fx)\right)^{-\frac{np}{2}}}{-\sin^2(e+fx)a^2+a^2-b^2}d\sin(e+fx)}{f}-\frac{a\cos^{1-np}(e+\right.$$

$$\Big\downarrow \text{25}$$

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(\frac{a\cos^{1-np}(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np-1)}\int\frac{\left(1-\sin^2(e+fx)\right)^{\frac{1}{2}(1-np)}}{-\sin^2(e+fx)a^2+a^2-b^2}d\sin(e+fx)}{f}-\frac{b\cos^{-np}(e\right.$$

$$\Big\downarrow \text{333}$$

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(\frac{a\sin(e+fx)\cos^{1-np}(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np-1)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(np-1),1,\frac{3}{2},\sin^2(e\right.}{f\left(a^2-b^2\right)}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x]),x]
```

output
```
Cos[e + f*x]^(n*p)*(c*(d*Sec[e + f*x])^p)^n*(-((b*AppellF1[1/2, (n*p)/2, 1
, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(Cos[e + f*x]^2)^
((n*p)/2)*Sin[e + f*x])/((a^2 - b^2)*f*Cos[e + f*x]^(n*p))) + (a*AppellF1[
1/2, (-1 + n*p)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2
)]*Cos[e + f*x]^(1 - n*p)*(Cos[e + f*x]^2)^((-1 + n*p)/2)*Sin[e + f*x])/((
a^2 - b^2)*f))
```

3.240. $\int\frac{\left(c(d\sec(e+fx))^p\right)^n}{a+b\sec(e+fx)}\,dx$

3.240.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

3.240. $\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx$

3.240.4 Maple [F]

$$\int \frac{(c(d\sec(fx+e))^p)^n}{a+b\sec(fx+e)}dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x)
```

3.240.5 Fricas [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{((d\sec(fx+e))^p\,c)^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

3.240.6 Sympy [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n/(a+b*sec(f*x+e)),x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n/(a + b*sec(e + f*x)), x)
```

3.240.7 Maxima [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{\left((d\sec\left(fx+e\right))^p c\right)^n}{b\sec\left(fx+e\right)+a}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

3.240.8 Giac [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{\left((d\sec\left(fx+e\right))^p c\right)^n}{b\sec\left(fx+e\right)+a}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{\left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n}{a+\frac{b}{\cos(e+f\,x)}}\,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x)),x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x)), x)
```

3.241 $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx$

3.241.1 Optimal result

Integrand size = 27, antiderivative size = 322

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = -\frac{2ab\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-2+np),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}(c(d\sec(e+fx))^p)^n\sin(e}{(a^2-b^2)^2 f} + \frac{a^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-3+np),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+np)}(c(d\sec(e}{(a^2-b^2)^2 f} + \frac{b^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1+np),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+np)}(c(d\sec(e}{(a^2-b^2)^2 f}$$

output
```
-2*a*b*AppellF1(1/2,1/2*n*p-1,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2
))*(cos(f*x+e)^2)^(1/2*n*p)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b^2)^2/
f+a^2*AppellF1(1/2,1/2*n*p-3/2,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^
2))*cos(f*x+e)*(cos(f*x+e)^2)^(1/2*n*p-1/2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x
+e)/(a^2-b^2)^2/f+b^2*AppellF1(1/2,1/2*n*p-1/2,2,3/2,sin(f*x+e)^2,a^2*sin(
f*x+e)^2/(a^2-b^2))*cos(f*x+e)*(cos(f*x+e)^2)^(1/2*n*p-1/2)*(c*(d*sec(f*x+
e))^p)^n*sin(f*x+e)/(a^2-b^2)^2/f
```

3.241.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 14108 vs. $2(322) = 644$.

Time = 46.81 (sec) , antiderivative size = 14108, normalized size of antiderivative = 43.81

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x])^2,x]
```

output
```
Result too large to show
```

3.241.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4436, 3042, 4356, 3042, 3303, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{4436}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\int \frac{(d\sec(e+fx))^{np}}{(a+b\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4356}$$

$$\cos^{np}(e+fx)\,(c(d\sec(e+fx))^p)^n\int \frac{\cos^{2-np}(e+fx)}{(b+a\cos(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{2-np}}{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3303}$$

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\int\left(\frac{b^2\cos^{2-np}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2}-\frac{2ab\cos^{3-np}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2}+\frac{a^2\cos^{4-np}(e+fx)}{\left(a^2\cos^2(e+fx)-b^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(-\frac{2ab\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\cos^{-np}(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(np-2),2,\frac{3}{2},\sin^2(e+\right.}{f\left(a^2-b^2\right)^2}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x])^2,x]
```

output
```
Cos[e + f*x]^(n*p)*(c*(d*Sec[e + f*x])^p)^n*((-2*a*b*AppellF1[1/2, (-2 + n
*p)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(Cos[e +
f*x]^2)^((n*p)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f*Cos[e + f*x]^(n*p)) + (a^
2*AppellF1[1/2, (-3 + n*p)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)
/(a^2 - b^2)]*Cos[e + f*x]^(1 - n*p)*(Cos[e + f*x]^2)^((-1 + n*p)/2)*Sin[e
 + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1[1/2, (-1 + n*p)/2, 2, 3/2, Sin[
e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]^(1 - n*p)*(Cos[
e + f*x]^2)^((-1 + n*p)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f))
```

3.241.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3303
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

3.241. $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx$

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

3.241.4 Maple [F]

$$\int \frac{(c(d\sec(fx+e))^p)^n}{(a+b\sec(fx+e))^2}dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x)
```

3.241.5 Fricas [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{((d\sec(fx+e))^p\,c)^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(((d*sec(f*x + e))^p*c)^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e)
 + a^2), x)
```

3.241.6 Sympy [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n/(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n/(a + b*sec(e + f*x))**2, x)
```

3.241.7 Maxima [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{((d\sec(fx+e))^p\,c)^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a)^2, x)
```

3.241.8 Giac [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{((d\sec(fx+e))^p\,c)^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a)^2, x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{\left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n}{\left(a+\frac{b}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x))^2,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x))^2, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```